

# From Exhaustive Robustness to Local Sensitivity: An Extension of the EORS Framework

Bartosz Paradowski<sup>1,2</sup>[0000–0002–9434–8109]

<sup>1</sup> National Institute of Telecommunications, ul. Szachowa 1, 04-894 Warsaw, Poland

<sup>2</sup> West Pomeranian University of Technology in Szczecin, ul. Żołnierska 49, 71-210  
Szczecin, Poland

{B.Paradowski}@il-pib.pl

**Abstract.** The Exhaustive Objective Ranking Solution (EORS) supports objective multi-criteria decision making by exhaustively exploring the admissible weight space and aggregating the resulting preference values into representative outcomes, complemented by stability indicators. While this global perspective reveals distributional robustness, it may obscure locally unstable behavior around decision-relevant weight configurations. This paper extends the KDE-based variant of EORS with a local sensitivity analysis that quantifies neighborhood-level responsiveness of (i) modal preference values and (ii) degrees of confidence to marginal perturbations of criterion weights, using simplex-projected finite differences. To improve interpretability, we introduce a criterion sensitivity index that integrates preference and confidence sensitivities and a fragility index that summarizes alternative-level stability. A theoretical study case illustrates how the proposed diagnostics identify criteria driving local instability and alternatives whose outcomes are most sensitive to reweighting, thereby complementing the global EORS ranking with actionable stability insights.

**Keywords:** MCDM · sensitivity analysis · weighting · local sensitivity.

## 1 Introduction

Multi-criteria decision-making (MCDM) problems often require selecting or ranking alternatives under competing criteria, where the final outcome depends critically on the assumed importance (weights) of the criteria [10]. In practice, weights are rarely known with certainty [11] and may vary across decision-makers or scenarios depending on the decision context [13]. Consequently, robust decision support requires not only producing a single ranking, but also characterizing how stable that outcome is under admissible changes in weights.

Sensitivity analysis provides a principled way to evaluate such stability [9]. Classical approaches often focus on global robustness by examining broad regions of the weight space and reporting aggregate stability patterns [12], but they may overlook transitions that occur locally around decision-relevant weight configurations. For decision-makers, these local effects are important: small changes in

weights, consistent with minor preference shifts, can lead to meaningful changes in preference values, confidence measures, or even the induced ordering of the alternatives [4].

Sensitivity analysis has been extensively studied, and a wide range of methods has been proposed for assessing the robustness of MCDM outcomes [1, 14, 2]. Nevertheless, many commonly used approaches remain largely global in nature and may not capture fine-grained effects caused by marginal perturbations of weights at the level of individual alternatives [15]. This limitation has motivated interest in local sensitivity analysis, which focuses on neighborhood behavior in the weight space and can reveal decision-relevant instability that may be obscured by aggregate robustness summaries. For example, Kizielewicz et al. proposed a local perspective on weight perturbations that highlights local decision dynamics [5] in the Characteristic Objects METHod (COMET); however, it does not provide a unified, alternative-level index that summarizes overall fragility in a single interpretable measure.

The Exhaustive Objective Ranking Solution (EORS) offers an objective, distribution-based perspective by exhaustively exploring the admissible weight space and aggregating the resulting preference values into representative outcomes. In its KDE-based variant, kernel density estimation is used to obtain smooth preference distributions and to define modal preference values and associated confidence indicators. While this global exploration supports transparency and interpretability, it does not directly explain which criteria drive local instability around the most relevant regions of the weight simplex.

In this study, we therefore propose a local sensitivity analysis framework for KDE-based EORS [7] that quantifies neighborhood-level responsiveness to marginal weight perturbations, alongside complementary confidence-based diagnostics. The main contributions of this study are as follows:

- A local weight-space sensitivity framework based on simplex-projected finite differences is introduced, ensuring feasibility under normalization constraints.
- The approach evaluates both KDE-based preference modes and degrees of confidence (DoC), uncovering instabilities not visible in global robustness analysis.
- The proposed Criterion Sensitivity Index (CSI) and Fragility Index (FI) provide concise, criterion- and alternative-level measures of structural instability.
- By anchoring the analysis at barycentric weight configurations derived from global EORS exploration, the method connects distributional robustness with local structural diagnostics.

The remainder of the paper is organized as follows. Section 2 presents a detailed description of the KDE-based EORS method. Section 3 introduces the proposed local sensitivity analysis framework building on EORS. Section 4 reports the results and discussion, and provides guidance on how to interpret the proposed metrics. Finally, Section 5 concludes the study and outlines future re-

search directions to enhance the practical applicability and theoretical robustness of the EORS approach.

## 2 Exhaustive Objective Ranking Solution (EORS)

The Exhaustive Objective Ranking Solution (EORS) is a multi-criteria decision-making (MCDM) approach designed to produce robust and transparent rankings by systematically exploring the admissible weight space [8]. EORS evaluates the preference values of alternatives over all considered weight vectors and aggregates the results to produce a representative ranking, while also quantifying the structural stability of each alternative.

In this study, we employ the KDE-based variant of EORS [7]. Kernel density estimation (KDE) serves as a smoothing tool to obtain a continuous representation of the preference values over the weight space. Since preference values are deterministically computed for each weight vector, KDE does not estimate a stochastic probability distribution; instead, it smooths the discrete evaluations to facilitate analysis.

1. Admissible weight vectors are systematically generated using a specified increment  $step_w$  (here,  $step_w = 0.05$ ), excluding vectors with zero weight. For three criteria, examples of weight vectors include

$$\begin{aligned} & [0.9, 0.05, 0.05] \\ & \quad \dots \\ & [0.3, 0.3, 0.4] \\ & \quad \dots \\ & [0.05, 0.05, 0.9] \end{aligned} \tag{1}$$

Systematic generation ensures full reproducibility, although random sampling is also supported by the method.

2. The preference values of the alternatives are computed for each generated weight vector using the selected decision-making method. In this study, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [6] was employed for this purpose.
3. Let  $x_{i,j}$  denote the preference value of alternative  $i$  under the  $j$ -th weight vector. A smooth representation over the sampled weight space is obtained via kernel density estimation,

$$\hat{f}_i(x) = \frac{1}{N_i h} \sum_{j=1}^{N_i} K\left(\frac{x - x_{i,j}}{h}\right), \tag{2}$$

where  $N_i$  is the number of sampled weight vectors,  $K$  is a Gaussian kernel, and  $h$  is set using Scott's rule. Since preference values are deterministically generated,  $\hat{f}_i(x)$  is interpreted as an intensity function summarizing the distribution of preference values across the weight space, rather than as a probability density of a stochastic model.

4. The mode of the smoothed values defines the most representative preference value:

$$P_i = \arg \max_x \hat{f}_i(x) \quad (3)$$

5. Around  $P_i$ , a confidence range interval (CRI) reflects decision-maker confidence in the stability of the representative preference level:

$$\text{CRI} = \left[ P_i - \frac{ci}{2}, P_i + \frac{ci}{2} \right], \quad ci = 0.1. \quad (4)$$

This interval is interpreted as a decision-maker tolerance band rather than a statistical confidence interval.

6. Degrees of confidence (DoC) quantify how concentrated an alternative's preference values are around  $P_i$  over the sampled weight space:

$$\text{DoC}_S(i) = \frac{N_{i,\max}}{N_i}, \quad (5)$$

$$\text{DoC}_C(i) = \frac{N_{i,\max}}{\sum_{j=1}^m N_{j,\max}}, \quad (6)$$

$$\text{DoC}_{\text{agg}}(i) = \alpha \text{DoC}_S(i) + (1 - \alpha) \text{DoC}_C(i), \quad \alpha = 0.5, \quad (7)$$

where  $N_{i,\max}$  is the number of preference values of alternative  $i$  falling within  $\text{CRI}_i$ . DoC values reflect structural stability and decision-analytic confidence, rather than statistical confidence. The parameter  $\alpha \in [0, 1]$  controls the trade-off between the single and cross perspectives (here,  $\alpha = 0.5$ ).

7. Finally, alternatives are ranked according to the preference values produced by the chosen MCDM method, and the DoC measures provide complementary information about the robustness of each alternative's position.

### 3 Local weight-space sensitivity and structural stability analysis

This section develops a local sensitivity framework for KDE-based EORS that quantifies how preference estimates, degrees of confidence (DoC), and induced rankings change under small perturbations of criterion weights. In contrast to global robustness checks, which explore broad regions of the admissible weight simplex, the proposed analysis focuses on neighborhood behavior around representative weight vectors, thereby providing a more fine-grained account of stability and interpretability.

#### 3.1 Weight space and global preference structure

Let

$$\mathcal{W} = \left\{ \mathbf{w} \in \mathbb{R}_+^k : \sum_{j=1}^k w_j = 1 \right\} \quad (8)$$

denote the admissible weight simplex. The set  $\mathcal{W}$  is convex and compact, and each  $\mathbf{w} \in \mathcal{W}$  represents a feasible trade-off among the  $k$  criteria. In this study, EORS was applied globally over the entire weight space using a step size of 0.05.

Let  $\{\mathbf{w}^{(r)}\}_{r=1}^N \subset \mathcal{W}$  be a set of sampled weight vectors. For each alternative  $i \in \{1, \dots, m\}$  we denote by

$$P_i^{(r)} = P_i(\mathbf{w}^{(r)}) \quad (9)$$

the preference value obtained under  $\mathbf{w}^{(r)}$ .

Given  $\{P_i^{(r)}\}_{r=1}^N$ , we estimate the preference density using kernel density estimation (KDE), denoted by  $\hat{f}_i(x)$ . The representative (modal) performance level is then defined as

$$P_i^* = \arg \max_x \hat{f}_i(x) \quad (10)$$

### 3.2 Reference weight vector

To identify a decision-relevant anchor for local analysis, we first isolate the subset of weights under which alternative  $i$  attains a preference value close to its modal level. Let  $P_i^*$  denote the modal preference (as obtained from KDE). We define

$$\mathcal{R}_i = \left\{ r : P_i^* - \frac{CRI}{2} \leq P_i^{(r)} \leq P_i^* + \frac{CRI}{2} \right\}, \quad (11)$$

where  $CRI > 0$  is the confidence-range parameter used in EORS (Eq. 4).

**Barycentric anchor** We then define the reference weight vector (anchor) for alternative  $i$  as the barycenter of this region,

$$\mathbf{w}_0^{(i)} = \frac{1}{|\mathcal{R}_i|} \sum_{r \in \mathcal{R}_i} \mathbf{w}^{(r)} \quad (12)$$

Since  $\mathcal{W}$  is convex,  $\mathbf{w}_0^{(i)} \in \mathcal{W}$ .

The vector  $\mathbf{w}_0^{(i)}$  represents a typical trade-off configuration under which alternative  $i$  exhibits near-modal behavior. Local sensitivity evaluated around  $\mathbf{w}_0^{(i)}$  is therefore aligned with the weight that support the observed global preference structure.

### 3.3 Feasible perturbations and local neighborhood

To preserve the admissibility of weight vectors after perturbation, we employ the Euclidean projection onto the simplex [3]

$$H_{\mathcal{W}}(\mathbf{z}) = \arg \min_{\mathbf{w} \in \mathcal{W}} \|\mathbf{w} - \mathbf{z}\|_2, \quad (13)$$

which maps any  $\mathbf{z} \in \mathbb{R}^k$  onto the closest feasible weight vector in  $\mathcal{W}$ .

To formalize locality, we define

$$\mathcal{N}_\epsilon(\mathbf{w}_0^{(i)}) = \left\{ \mathbf{w} \in \mathcal{W} : \|\mathbf{w} - \mathbf{w}_0^{(i)}\|_2 \leq \epsilon \right\}, \quad (14)$$

where  $\epsilon > 0$  represents admissible weight uncertainty.

### 3.4 Local KDE mode sensitivity

We quantify the responsiveness of the most likely preference outcome to marginal changes in single weights using simplex-projected finite differences.

Because the simplex constraint  $\sum_j w_j = 1$  must be satisfied, increasing one weight necessarily induces a redistribution across other coordinates. Therefore, the resulting sensitivity measure does not correspond to a classical partial derivative, but rather to a finite difference computed after nonlinear projection onto the admissible simplex.

For each criterion  $j$ , we define the simplex-projected perturbation

$$\mathbf{w}_\delta^{(i,j)} = \Pi_{\mathcal{W}} \left( \mathbf{w}_0^{(i)} + \delta \mathbf{e}_j \right), \quad (15)$$

where  $\delta > 0$  is a small step size and  $\mathbf{e}_j$  denotes the  $j$ -th canonical basis vector.

The local preference sensitivity is then defined as the simplex-projected finite difference quotient

$$S_{i,j}^{(P)} = \frac{P_i(\mathbf{w}_\delta^{(i,j)}) - P_i(\mathbf{w}_0^{(i)})}{\delta}. \quad (16)$$

For finite  $\delta$ , this quantity measures the rate of change in preference induced by an admissible marginal increase of weight  $j$ , accounting for the normalization constraint through projection.

Collecting the sensitivity measures over all alternatives and criteria yields the mode-sensitivity matrix

$$S^{(P)} \in \mathbb{R}^{m \times k}. \quad (17)$$

Large  $|S_{i,j}^{(P)}|$  indicates that the representative preference value of alternative  $i$  is locally unstable with respect to criterion  $j$ , while values close to zero indicate local robustness; the sign reflects the direction of influence.

### 3.5 Local degree-of-confidence sensitivity

Preference-mode stability does not necessarily imply decision reliability. Therefore, we additionally evaluate how DoC measures vary under the same simplex-projected perturbations. Let  $\text{DoC}_i(\mathbf{w})$  denote the chosen DoC indicator for alternative  $i$ .

Using identical perturbations  $\mathbf{w}_\delta^{(i,j)}$ , we compute

$$S_{i,j}^{(DoC)} = \frac{\text{DoC}_i(\mathbf{w}_\delta^{(i,j)}) - \text{DoC}_i(\mathbf{w}_0^{(i)})}{\delta}. \quad (18)$$

Stacking these quantities yields the DoC-sensitivity matrix

$$S^{(DoC)} \in \mathbb{R}^{m \times k}. \quad (19)$$

High magnitudes indicate fragile confidence with respect to admissible marginal weight changes. Discrepancies between  $S^{(P)}$  and  $S^{(DoC)}$  suggest that changes in distribution concentration or ranking overlap may occur even when modal preferences remain comparatively stable.

### 3.6 Normalization

To ensure comparability across criteria and alternatives, and to enable subsequent aggregation, we apply max-absolute normalization. Since the fragility analysis focuses on the magnitude of sensitivity rather than direction, signs are removed prior to scaling.

$$\hat{S}_{i,j}^{(P)} = \frac{|S_{i,j}^{(P)}|}{\max_{i,j} |S_{i,j}^{(P)}|}, \quad \hat{S}_{i,j}^{(DoC)} = \frac{|S_{i,j}^{(DoC)}|}{\max_{i,j} |S_{i,j}^{(DoC)}|}. \quad (20)$$

Then,

$$\hat{S}_{i,j}^{(\cdot)} \in [0, 1]. \quad (21)$$

### 3.7 Criterion sensitivity index

To summarize criterion-level influence in a single interpretable quantity, we aggregate the normalized sensitivities of performance and confidence. The resulting index highlights which criteria drive local changes in outcomes around the reference configuration.

The criterion sensitivity index (CSI) is defined as

$$CSI_{i,j} = \alpha \hat{S}_{i,j}^{(P)} + (1 - \alpha) \hat{S}_{i,j}^{(DoC)}, \quad (22)$$

where  $\alpha \in [0, 1]$  controls the relative emphasis on performance versus robustness.

By construction,

$$CSI \in [-1, 1]^{m \times k}. \quad (23)$$

### 3.8 Fragility index

Finally, we provide a single stability score per alternative by aggregating criterion effects into a norm.

$$FI_i = \sqrt{\sum_{j=1}^k CSI_{i,j}^2} \quad (24)$$

This corresponds to the Euclidean norm of the local sensitivity vector in the CSI space.

A low  $FI_i$  indicates a robust alternative whose local outcome is relatively insensitive to weight perturbations, whereas a high  $FI_i$  indicates fragility, i.e., a strong dependence on the exact choice of weights.

### 3.9 Geometric interpretation

We define the blended functional

$$F_i(\mathbf{w}) = \alpha P_i(\mathbf{w}) + (1 - \alpha) \text{DoC}_i(\mathbf{w}). \quad (25)$$

The proposed sensitivity analysis evaluates the response of  $F_i$  to admissible perturbations of the form

$$\mathbf{w}_\delta^{(i,j)} = \Pi_{\mathcal{W}}(\mathbf{w}_0^{(i)} + \delta \mathbf{e}_j). \quad (26)$$

Because the projection operator  $\Pi_{\mathcal{W}}$  is nonlinear, the resulting sensitivity measures correspond to finite differences of the composite mapping

$$F_i \circ \Pi_{\mathcal{W}}, \quad (27)$$

rather than to classical gradients in  $\mathbb{R}^k$ .

Accordingly, the fragility index

$$FI_i = \sqrt{\sum_{j=1}^k CSI_{i,j}^2} \quad (28)$$

should be interpreted as the Euclidean norm of the local simplex-projected sensitivity vector, quantifying structural instability under admissible weight redistributions.

### 3.10 Role of $\delta$ and $\epsilon$

The parameters  $\delta$  and  $\epsilon$  control, respectively, the magnitude of admissible perturbations and the size of the local sampling neighborhood.

Since sensitivities are computed as simplex-projected finite differences,  $\delta$  must be sufficiently small to approximate local behavior while avoiding dominance of nonlinear projection effects. If  $\delta$  is too large, estimates reflect curvature and redistribution effects rather than local responsiveness. If  $\delta$  is too small, numerical noise and projection artifacts may dominate.

Similarly, if  $\epsilon$  is too large, the analysis ceases to be local; if too small, sampling variability leads to unstable estimates.

A systematic calibration of  $(\delta, \epsilon)$  is therefore required. In this paper, we set  $\delta = 0.02$  and  $\epsilon = 0.02$ , chosen to preserve locality while maintaining stable numerical estimates.

Overall, the proposed analysis bridges global robustness and local sensitivity by (i) providing criterion-level explainability, (ii) revealing instabilities that may be obscured by aggregate robustness measures, and (iii) positioning KDE-based EORS as an interpretable framework for weight-space decision analysis.

## 4 Study case

This section presents a theoretical study case used to illustrate the proposed analysis. We generate a random decision matrix by sampling each performance value independently from a continuous uniform distribution on the half-open interval  $[0.0, 1.0)$ . For simplicity, all criteria are assumed to be of the profit type, and the resulting matrix is presented in Table 1. As the underlying MCDM method within EORS, we employ TOPSIS [6], which is widely used and well studied in the literature.

**Table 1.** Decision matrix of the theoretical decision-making problem.

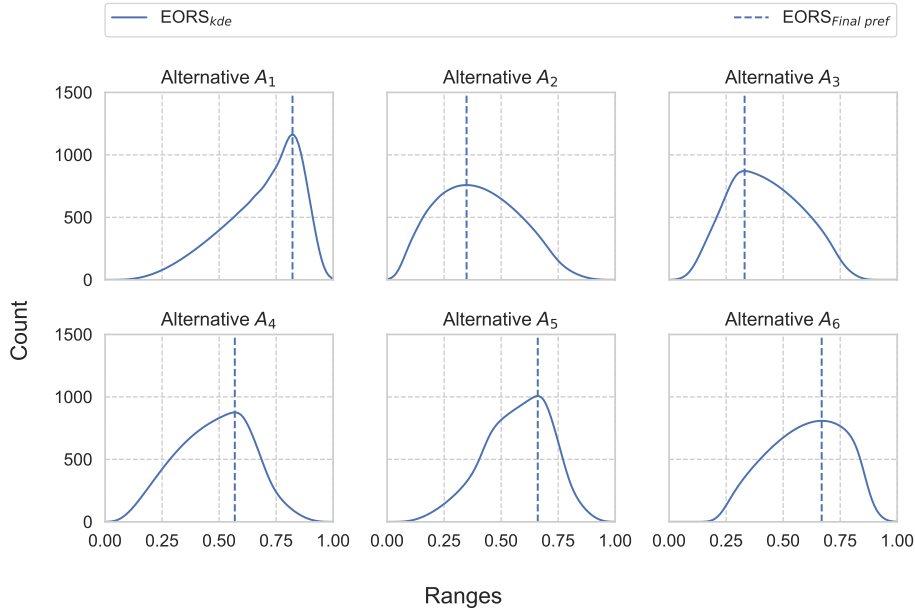
$A_i$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	0.5496	0.7418	0.8094	0.7423	0.8859
$A_2$	0.6290	0.0564	0.0118	0.7930	0.4387
$A_3$	0.3167	0.9139	0.5930	0.2710	0.3636
$A_4$	0.6205	0.1898	0.5624	0.1648	0.4158
$A_5$	0.8486	0.7923	0.8370	0.1300	0.4117
$A_6$	0.9984	0.2117	0.5740	0.4393	0.3821
Criteria type	<i>Profit</i>	<i>Profit</i>	<i>Profit</i>	<i>Profit</i>	<i>Profit</i>

For transparency, Table 2 reports the final EORS preference values and the resulting ranking for the considered set of alternatives. These values summarize the global EORS aggregation over the sampled weight space and serve as the baseline outcome for the local sensitivity analysis conducted in the remainder of this section.

**Table 2.** Results of EORS for the theoretical decision-making problem.

$A_i$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$
Preference	0.8216	0.3484	0.3309	0.5687	0.6605	0.6688
Ranking	1	5	6	4	3	2

For additional insight, Fig. 1 visualizes the KDE-estimated preference distributions for all alternatives, with the corresponding final preference values indicated by dashed lines. This visualization provides an intuitive summary of the global EORS output and supports the subsequent discussion of local sensitivity. In particular, steeper and more concentrated densities suggest greater stability of the representative outcome, whereas flatter or multimodal shapes may indicate higher susceptibility to weight perturbations.

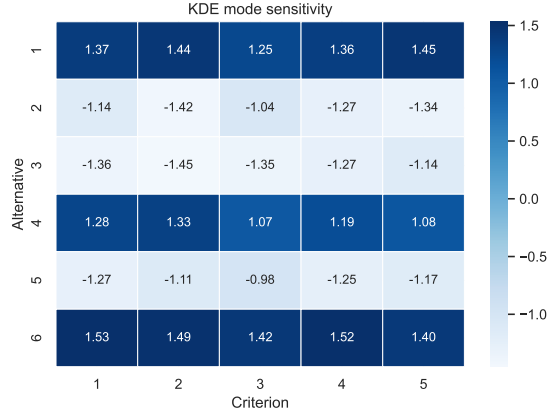


**Fig. 1.** Distribution of preferences and final preference value for original and improved EORS method.

We next compute the local KDE mode sensitivity described in Section 3.4. Fig. 2 summarizes the resulting sensitivity matrix  $S^{(P)}$  as a heatmap, where rows correspond to alternatives and columns correspond to criteria. Cell intensity reflects the magnitude of the estimated sensitivity over  $\delta$ , while the sign indicates whether increasing the weight of criterion  $j$  locally increases or decreases the modal preference of alternative  $i$ . Concentrated high-magnitude regions therefore identify criteria that drive local changes in the representative outcome, as well as alternatives that are most susceptible to marginal reweighting.

Because  $S^{(P)}$  is not standardized, comparisons should be made only within a given decision problem. In particular, the metric should not be interpreted on a ratio scale (i.e., one entry should not be read as “ $x$  times larger” than another). For example, for alternative  $A_4$ , the heatmap indicates that criterion  $C_2$  exhibits the highest local susceptibility, whereas  $C_3$  exhibits the lowest. This

suggests that, locally around the reference weight vector, the modal preference of  $A_4$  is more sensitive to changes in  $w_2$  than to changes in  $w_3$ .



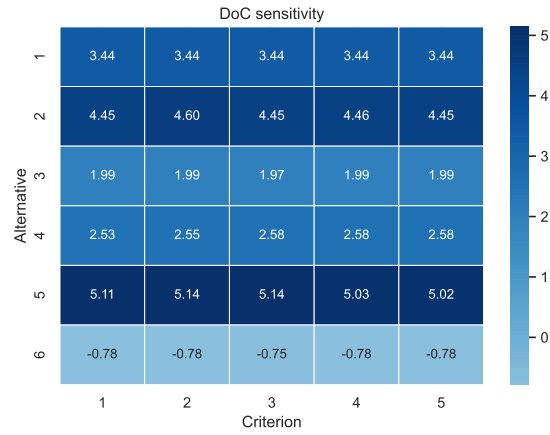
**Fig. 2.** Heatmap of local KDE mode sensitivity  $S^{(P)}$  for the study case.

We next compute the local degree of confidence sensitivity described in Section 3.5. Fig. 3 reports the resulting sensitivity matrix  $S^{(DoC)}$  as a heatmap, again with alternatives on rows and criteria on columns. Cell intensity reflects the magnitude of the estimated sensitivity over  $\delta$ , while the sign indicates whether increasing the weight of criterion  $j$  locally increases or decreases the confidence associated with alternative  $i$ . High-magnitude regions therefore indicate criteria whose marginal reweighting substantially changes local confidence, even when the preference mode remains comparatively stable. As with  $S^{(P)}$ , these values are intended for within-problem comparison rather than ratio-scale interpretation.

In this study case, alternative  $A_5$  exhibits the largest overall DoC sensitivity across criteria, indicating that its confidence is most affected by local weight perturbations. Additionally, the differences in  $S^{(DoC)}$  across criteria reveal finer-grained effects that may influence the final decision outcome.

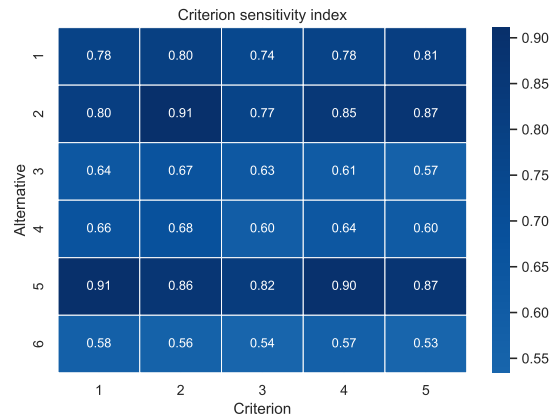
Next, we compute the criterion sensitivity index introduced in Section 3.7. Fig. 4 visualizes the CSI values as a heatmap that integrates preference-mode sensitivity and DoC sensitivity into a single criterion-level diagnostic measure. As before, rows correspond to alternatives and columns to criteria. Because CSI is computed using max-absolute normalization, it does not preserve the direction of change and should be interpreted only in terms of magnitude. High-magnitude CSI entries indicate criteria that exert the strongest combined influence on both the representative preference outcome and its associated confidence, thereby highlighting the main drivers of local stability around the reference weights.

This representation enables interpretation at the criterion level and provides an overview of which criteria most strongly affect each alternative. Such informa-



**Fig. 3.** Heatmap of local degree of confidence sensitivity  $S^{(\text{DoC})}$  for the study case.

tion may be useful for decision-making, for example, by indicating where more precise weighting or robustness checks should be prioritized.

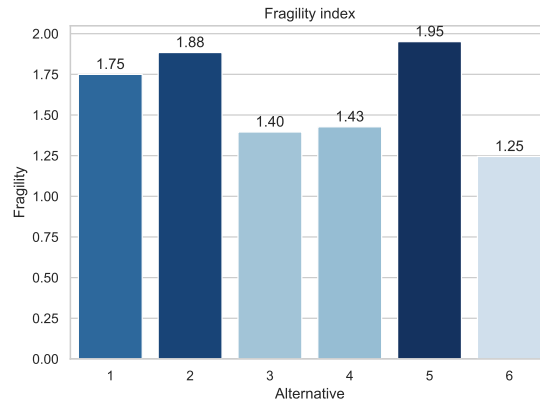


**Fig. 4.** Heatmap of the criterion sensitivity index (CSI) for the study case.

Finally, Fig. 5 presents the fragility index (FI) values for all alternatives, obtained by aggregating the CSI entries across criteria into a single stability score per alternative. Higher FI values indicate alternatives whose local outcomes are more sensitive to marginal changes in the weight vector (i.e., greater structural fragility), whereas lower values indicate alternatives that remain comparatively stable under local reweighting. In this way, the FI provides a concise, ranking-

independent summary of local stability that complements the criterion-resolved heatmaps.

For this study case, the results indicate that alternative  $A_6$  is the least fragile with respect to local weight changes, whereas alternative  $A_5$  is the most fragile. Such conclusions help assess whether the final preference values are robust and decision-relevant. Notably, the proposed local sensitivity analysis is not intended to explain the ranking alone; instead, it focuses on the underlying preference values, which retain information about how much more (or less) preferable one alternative is relative to another.



**Fig. 5.** Fragility index (FI) of alternatives for the study case.

## 5 Conclusions

This paper extended KDE-based EORS with a local sensitivity analysis that complements the global, distribution-based ranking by explicitly quantifying neighborhood-level responsiveness to marginal weight perturbations. In the study case, the global EORS outcome (Table 2) was enriched by local measures that revealed which criteria drive changes in modal preferences, how confidence measures vary under reweighting, and which alternatives are structurally stable.

The local KDE mode sensitivity  $S^{(P)}$  (Fig. 2) provided criterion-resolved information about how the representative preference value responds to perturbations around the reference weights, enabling the identification of alternatives and criteria with high local susceptibility (e.g., the sensitivity pattern of  $A_4$  with respect to  $C_2$  versus  $C_3$ ). The DoC sensitivity  $S^{(\text{DoC})}$  (Fig. 3) further showed that confidence can be locally fragile even when preference modes appear comparatively stable; in particular,  $A_5$  exhibited the strongest overall DoC sensitivity in the study case. By integrating both sources of information, the criterion sensitiv-

ity index (CSI) (Fig. 4) offered a compact criterion-level diagnostic highlighting the structural robustness that jointly affect preference and confidence.

Finally, the fragility index (FI) (Fig. 5) summarized local stability at the alternative level, providing a ranking-independent stability profile that supports decision communication and follow-up weight refinement. In the study case, FI indicated that  $A_6$  is comparatively robust under local reweighting, whereas  $A_5$  is most fragile, suggesting where additional attention or stakeholder input on weights may be most valuable.

Future work should focus on systematic calibration of the locality parameters  $(\delta, \epsilon)$ , exploring alternative neighborhood definitions and projections, and validating the proposed indicators on real-world decision problems and different MCDM methods.

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