# Navigating Uncertainty: A Framework for Benchmarking Decision Quality in Fuzzy Petri Nets

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**Abstract.** Fuzzy Petri nets (FPN) combine classical Petri net theory with fuzzy logic to model systems characterized by uncertainty and ambiguity, particularly in Decision Support Systems (DSS). This study evaluates the decision-making efficacy of a DSS model operating under uncertainty, considering the impact of three distinct FPN classes, four datasets with diverse statistical characteristics, and various performance metrics. The experimental phase utilized a train traffic control model and bespoke simulation software for automatic model analysis. The results address research challenges in FPN theory and applications as identified in the comprehensive review by K. Zhou and A. Zain (Artif. Intell. Rev., 2022).

**Keywords:** Fuzzy Petri net · Modeling · Simulation · Decision support system · Experimental evaluation.

## 1 Introduction

Reasoning under uncertainty and incompleteness is a core challenge in Artificial Intelligence. While Bayesian or neural networks, discussed by Ramirez [1] as well as Larranga and Moral [2], are widely used, they often struggle with systems requiring simultaneous modeling of concurrency and synchronization. FPNs, introduced by Looney [3], address these limitations. Since then, numerous variants with improved inference rules have emerged, such as those proposed by Shi and Liu [4] or Yu et al. [5].

This paper evaluates three FPN types: classical FPNs described by Chen et al. [6], generalized FPNs (GFP-nets) introduced by Suraj [7], and uninorm Petri nets (UP-nets) developed by Suraj [8]. Using a train traffic control system as a case study, we propose a methodology to compare these models across four large datasets. Since UP-nets generalize triangular norms through uninorm theory, this study aims to determine if this theoretical shift provides measurable added value for DSSs.

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### 1.1 Short review of related literature

The FPN landscape has evolved significantly. Zhou and Zain [9] classified FPN variants and highlighted the lack of standardized benchmarking - a gap this research addresses. A survey by Liu et al. [10] explored FPNs in decision-making, noting the transition from triangular norms to more expressive structures. Practical applications include interval-valued fuzzy sets in railway safety developed by Wang and Smith [11] and the integration of Petri nets with machine learning proposed by Zhang and Chen [12]. Finally, Yager and Rybalov [13] provide the formal justification for using uninorms to achieve superior data aggregation flexibility.

The paper is organized as follows: Section 2 recalls FPN theory; Section 3 describes the methodology; Section 4 presents the results; Section 5 concludes the study.

## 2 Auxiliary Concepts and Notation

This section recalls three FPN models. All share a common structure defined by the tuple $N = (P, T, I, O, M_0, S, \alpha, \beta, \gamma, Op, \delta)$, where $P, T$ are sets of places and transitions, $I, O$ are input/output functions, $M_0 : P \to [0, 1]$ is the initial marking, $S$ is a set of statements, $\alpha$ binds places to statements, while $\beta$ and $\gamma$ denote truth degrees and thresholds of transitions, respectively.

The models differ primarily in their operator sets $Op$ and binding functions $\delta$:

- FP-net [3]: $Op$ uses classical Zadeh and Goguen operators [14]. $\delta$ binds transitions to specific t-norms ($In, Trs$) and s-norms ($Out$).
- GFP-net [7]: $Op$ extends to the entire families of t-norms ($TN$) and s-norms ($SN$) [14].
- UP-net [8]: $Op$ utilizes conjunctive ($U_{\min}$) and disjunctive ($U_{\max}$) uninorms with a neutral element $e \in (0, 1)$ [13].

A marking $M : P \to [0, 1]$ represents the state. A transition $t$ with input places $\{p_{i1}, \ldots, p_{ik}\}$ is enabled if $In(M(p_{i1}), \ldots, M(p_{ik})) \geq \gamma(t) > 0$. Firing an enabled $t$ results in a successor marking $M'$ for each output place $p$:

$$M'(p) = Out(Trs(In(M(p_{i1}), \ldots, M(p_{ik})), \beta(t)), M(p)) \tag{1}$$

For non-output places, $M'(p) = M(p)$.

## 3 Methodology for Comparative Evaluation

**3.1. Methodological framework.** To assess how the choice of net architecture influences decision outcomes within an exemplary DSS, we established a systematic evaluation methodology (see Fig. 1).

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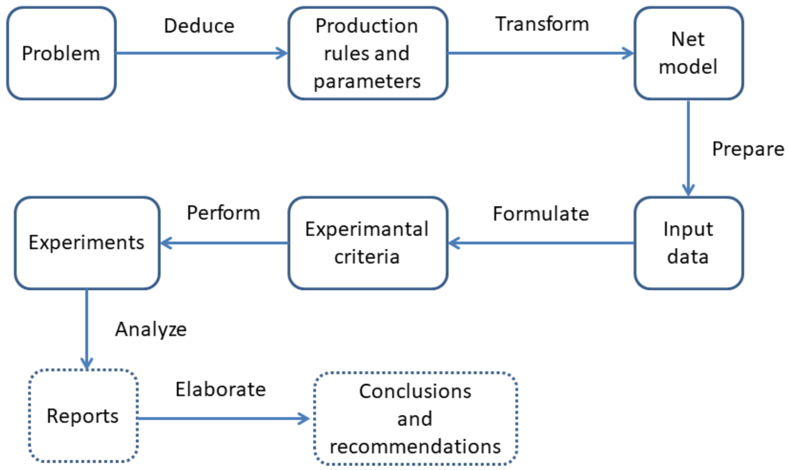


**Fig. 1.** Schematic overview of the research methodology framework.

**A. Exemplary DSS** (*Train Traffic Control*). A common operational conflict arises when train $B$ is scheduled to wait at a station for the arrival of train $A$ to facilitate passenger transfers, and train $A$ is delayed. This scenario presents three primary alternatives for resolution:

**(1)** Train $B$ waits for train $A$'s arrival and consequently departs late.
**(2)** Train $B$ departs on schedule, requiring passengers from train $A$ to wait for a subsequent connection.
**(3)** Train $B$ departs on time, and an additional train is deployed to accommodate train $A$'s passengers.

Selecting an optimal decision requires considering various internal conditions, such as the duration of the delay and the volume of transferring passengers. This analysis focuses solely on the modeling aspect; a broader discussion concerning the optimization of competing objectives - such as minimizing network - wide delays, ensuring customer satisfaction, and efficiently utilizing expensive resources - is beyond the scope of this discussion.

**B. Production rules and parameters.** The conflict is modeled via four rules:

**(r1)** IF $s2$ THEN $s6$ [$\beta$=.9, $\gamma$=.4];
**(r2)** IF $s3$ THEN $s6$ [$\beta$=.9, $\gamma$=.4];
**(r3)** IF $s1 \wedge s4 \wedge s6$ THEN $s7$ [$\beta$=.8, $\gamma$=.2];
**(r4)** IF $s4 \wedge s5$ THEN $s8$ [$\beta$=.8, $\gamma$=.2].

Linguistic labels $s1$–$s8$ represent: $s1$ : Train $B$ is the final train in this direction today; $s2$ : The delay on train $A$ is significant; $s3$ : There is an urgent need

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for the track currently occupied by train $B$; $s4$ : A large number of passengers intend to transfer to train $B$; $s5$ : The delay of train $A$ is minor; $s6$ : Let train $B$ depart according to the schedule; $s7$ : Provide an additional train (train C) in the same direction as train $B$; $s8$ : Let train $B$ wait for train $A$.

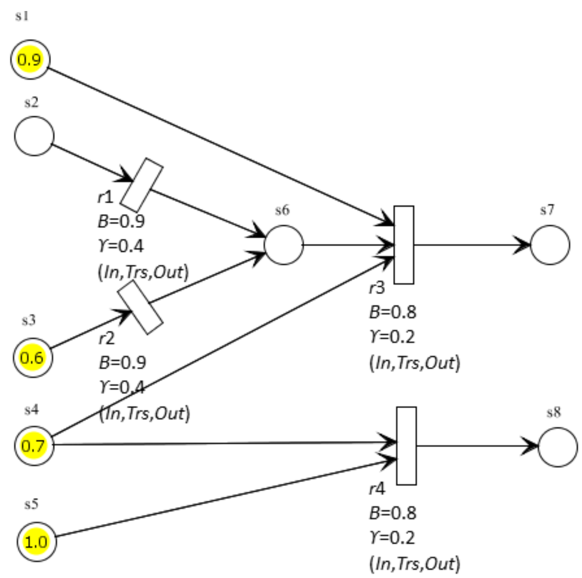


**Fig. 2.** General scheme of the Petri net model representing the train traffic conflict.

We compare three models based on Fig. 2:

**(a)** FP-net: $(In, Trs, Out) = (ZtN, GtN, ZsN)$ [3];

**(b)** GFP-net: $(In, Trs, Out) = (HtN, HtN, HsN)$ using Hamacher operators [7, 14];

**(c)** UP-net: $(In, Trs, Out) = (U, U, U')$ using uninorms $U, U'$ with $e \in (0, 1)$ [8, 13].

These archetypal models were selected from operator triplet lattices to ensure they are mutually incomparable [15, 16].

**C. Input data.** To conduct the experiments, four distinct numerical datasets (Test1-Test4) were generated using specialized software. The first and third sets consist of five-element sequences with values drawn from discrete sets: {0, 0.5, 1} for Test1 and {0, 0.25, 0.5, 0.75, 1} for Test3. In contrast, Test2 and Test4 maintain the same sequence lengths as their predecessors, but their values were randomly sampled from the continuous unit interval [0, 1].

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**Table 1.** Basic statistics for Test1–Test4 (Size, Median, S.D.).

| Par | Test1 | Test2 | Test3 | Test4 |
|---|---|---|---|---|
| Size | 243 | 243 | 3125 | 3125 |
| Me | (.5,.5,.5,.5,.5) | (.5,.5,.53,.5,.48) | (.5,.5,.5,.5,.5) | (.51,.5,.5,.51,.5) |
| S.D. | (.4,.4,.4,.4,.4) | (.29,.31,.28,.29,.28) | (.35,.35,.35,.35,.35) | (.29,.29,.29,.29,.29) |

## 4 Results and Discussion

We evaluated how net architecture and input data influence DSS decision quality.

**Experiment 1:** *Decision typology.* We tested two scenarios: *Case 1* (fixed threshold $Thr = 0.1$, $e = 0.5$) and *Case 2* (dynamic $Thr = |Me(s7) - Me(s8)|$, $e = 0.5$).

Performance was measured using:

**(1)** *Uam/Am*: unambiguous/ambiguous decisions;
**(2)** *Rel*: reliable decisions ($|v(d_1) - v(d_2)| > Thr$);
**(3)** *Rat1/2*: growth ratios vs. dataset scale;
**(4)** *AmN-Z/AmZ*: non-zero ambiguous/zero ambiguous decisions;
**(5)** *ET*: execution time [s].

Logical constraints: *Size*=*Uam*+*Am*, *Am*=*AmN-Z*+*AmZ*, and $Rel \leq Uam$.

**Table 2.** Impact of net architecture on decision typology (*Case 1*).

| Model | Data | *Uam* | *Am* | *Rel* | *Rat1* | *Rat2* | *AmN-Z* | *AmZ* | *ET* |
|---|---|---|---|---|---|---|---|---|---|
| FP-net | Test1 | 115 | 128 | 92 | 14.92 | 17.28 | 25 | 103 | .01 |
| GFP-net | Test1 | 140 | 103 | 110 | 15.47 | 15.81 | 0 | 103 | .01 |
| UP-net | Test1 | 95 | 148 | 89 | 13.01 | 12.27 | 45 | 103 | .02 |
| FP-net | Test2 | 141 | 102 | 110 | 13.96 | 13.82 | 33 | 69 | .02 |
| GFP-net | Test2 | 161 | 82 | 116 | 13.50 | 13.71 | 1 | 81 | .01 |
| UP-net | Test2 | 90 | 153 | 70 | 12.19 | 12.77 | 4 | 149 | .02 |
| FP-net | Test3 | 1716 | 1409 | 1590 | 14.92 | 17.28 | 620 | 789 | .13 |
| GFP-net | Test3 | 2166 | 959 | 1739 | 15.47 | 15.81 | 0 | 959 | .01 |
| UP-net | Test3 | 1236 | 1889 | 1092 | 13.01 | 12.27 | 267 | 1622 | .12 |
| FP-net | Test4 | 1968 | 1157 | 1528 | 13.96 | 13.82 | 375 | 782 | .14 |
| GFP-net | Test4 | 2174 | 951 | 1590 | 13.50 | 13.71 | 4 | 947 | .12 |
| UP-net | Test4 | 1097 | 2028 | 894 | 12.19 | 12.77 | 48 | 1980 | .14 |

**Analysis of** *Case 1*:

**(1)** GFP-net consistently yields the highest *Uam* and *Rel* values, outperforming other models.
**(2)** Growth rates for *Uam* and *Rel* (approx. 12.86) are exceeded by FP-net and GFP-net, while UP-net falls below.
**(3)** UP-net shows significantly higher *AmZ* values, indicating more non-activated transitions.

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**(4)** Computational efficiency ($ET$) is similar across models.

**Analysis of** *Case 2*:

Dynamic thresholds (averaging 0.033–0.055) are nearly 50% lower than in *Case 1*, leading to higher *Rel* counts across all models. Despite this shift, the performance hierarchy remains: GFP-net > FP-net > UP-net.

**Table 3.** Decision reliability with dynamic thresholds (*Case 2*).

| Model | Data | *Me(s7)* | *Me(s8)* | *Thr* | *Rel* |
|---|---|---|---|---|---|
| FP-net | Test1\|Test2 | .17\|.2 | .22\|.21 | .05\|.01 | 97\|136 |
| GFP-net | Test1\|Test2 | .17\|.16 | .22\|.21 | .05\|.05 | 118\|136 |
| UP-net | Test1\|Test2 | .4\|.2 | .42\|.24 | .02\|.04 | 95\|84 |
| FP-net | Test3\|Test4 | .19\|.19 | .24\|.24 | .05\|.05 | 1599\|1719 |
| GFP-net | Test3\|Test4 | .15\|.16 | .23\|.23 | .08\|.07 | 1781\|1726 |
| UP-net | Test3\|Test4 | .3\|.19 | .33\|.23 | .03\|.04 | 1092\|1033 |

The evaluation indicates that the GFP-net model demonstrates superior decision quality, higher reliability ($Rel$), and lower ambiguity ($Am$) compared to FP-net and UP-net architectures across tested datasets. While *Case 1* established a fixed threshold, transitioning to a dynamic, data-driven threshold in *Case 2* significantly increased the number of reliable decisions, consistently maintaining the performance hierarchy of GFP-net > FP-net > UP-net.

The findings are based on a comparative analysis of net architecture's impact on the DSS.

**Experiment 2:** *Sensitivity of UP-net neutral element (e).* We analyzed how $e$ affects decision typology in Test3 (regular) and Test4 (random) datasets with $Thr = 0.1$.

**Table 4.** Impact of $e$ on UP-net outcomes (Test3 | Test4).

| *e* | *Uam* | *Am* | *Rel* | *AmN-Z* | *AmZ* | *ET* |
|---|---|---|---|---|---|---|
| .1 | 1847\|2566 | 1278\|559 | 1307\|949 | 537\|213 | 741\|346 | .12\|.14 |
| .3 | 1236\|1866 | 1889\|1259 | 1092\|1140 | 267\|116 | 1622\|1143 | .12\|.13 |
| .5 | 1236\|1097 | 1889\|2028 | 1092\|894 | 267\|48 | 1622\|1980 | .12\|.12 |
| .7 | 603\|382 | 2522\|2743 | 589\|364 | 89\|10 | 2433\|2733 | .11\|.14 |
| .9 | 0\|0 | 3125\|3125 | 0\|0 | 0\|0 | 3125\|3125 | .13\|.12 |

**Conclusions:**

**(1)** Lower $e$ values correlate with higher $Uam$ and lower ambiguity ($Am$, $AmZ$) across all data types.

**(2)** Reducing $e$ generally enhances reliability ($Rel$), except for random data where it plateaus at very low $e$.

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**(3)** Execution time ($ET$) remains independent of $e$.

Overall, UP-net quality for regular data is inversely proportional to $e$, while higher $e$ values increase transition activation failures ($AmZ$) regardless of data structure.

## 5 Final Conclusions and Future Work

This paper introduces an original methodology for the comparative evaluation of three distinct FPN classes, specifically designed for modeling DSSs operating under conditions of uncertainty. Our research involved a rigorous analysis of the following models:

- FP-net: Utilizing standard, classical fuzzy logic operators.
- GFP-net: Offering increased flexibility by incorporating a broader range of t-norm and s-norm operators.
- UP-net: Representing the most generalized approach through the application of uninorm theory.

Experimental results, obtained from diverse numerical datasets and evaluated against standard statistical criteria, confirmed our primary hypothesis. The GFP-net model demonstrated superior efficacy, generating accurate and reliable decisions in a significantly higher number of cases compared to both FP-net and UP-net architectures. All experiments were conducted using our specialized software, developed for modeling and analyzing systems based on various Petri net formalisms, including fuzzy nets [17].

Future research will focus on several key areas to further validate these findings. Firstly, we intend to investigate whether this performance hierarchy persists when considering a wider spectrum of operator triplets, particularly extreme values, which - unlike the canonical "middle" operators used here - allow for more precise comparative analysis. Furthermore, we plan to examine the robustness of these FPN structures by applying various discretization and fuzzification techniques to the input data. Such investigations will determine if the observed model efficacy remains consistent across different data preprocessing scenarios, ultimately enhancing the reliability of FPNs in complex decision-making environments.

**Impact Statement:** The proposed methodology offers a comprehensive multi-criteria experimental evaluation framework with clearly defined parameters for assessing test data quality and decision accuracy. This approach is highly relevant to various domains of applied computer science, particularly for evaluating complex, uncertain DSSs where synchronization, communication, concurrency, and structural complexity are critical. Furthermore, it holds significant potential for advanced robotic control systems. Our experimental results validate the practical utility of this framework in selecting optimal modeling structures for high-complexity environments.

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