Attribute Importance in Conflict Models: Using Clustering Metrics and Bi-Coalitions for Issue Evaluation

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Abstract. This paper introduces a new approach to assessing attribute importance in conflict situations to better support negotiation processes. Building on Pawlak's conflict model, it proposes a bi-coalition-based measure that identifies issues enabling unanimous agreement and examines their impact on coalition formation. This qualitative perspective is contrasted with a quantitative method based on clustering metrics, revealing complementary dimensions of conflict dynamics. Applied to the Russia-Ukraine case, the framework highlights territorial integrity as the central dividing issue and shows how the bi-coalition-based measure helps pinpoint factors that hinder or facilitate consensus, offering practical insights for dispute mediation.

Keywords: Conflict analysis, Bi-coalitions, Attribute importance, Consensus, Multi-Agent Decision-Making

1 Introduction

Understanding contemporary geopolitical conflicts requires analytical tools capable of representing divergent interests, asymmetric dependencies, and multidimensional issue interactions. A foundational approach in this domain is Pawlak's conflict model, which encodes disagreements through ternary evaluations and rough-set-based relational structures [6]. Its subsequent developments – covering extended distance functions, Boolean reasoning, and negotiation-oriented perspectives – demonstrate how structured opinion data can be used to derive coalition patterns, conflict intensity, and consensus conditions [2, 7, 9]. The tri-valued representation of actor positions is further supported by the theory of three-way decisions, which generalizes classical rough-set partitions into positive, boundary, and negative regions and offers a decision-theoretic semantics for interpreting support, neutrality, and opposition [12, 13]. This framework provides a principled foundation for the threshold-controlled aggregations employed

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in our analysis. Complementary insights arise from quantitative measures used to evaluate structural groupings. Cluster validity indices – the Dunn [5], Silhouette [4], and Davies-Bouldin [11] measures – originally designed to assess cluster compactness and separability, have recently been applied to coalition analysis, where relational structures between actors resemble distance-based clustering [8]. Parallel research in network science highlights how densely connected actors tend to form coherent communities, whereas game-theoretic approaches illuminate strategic incentives, externalities, and stability properties underlying coalition formation [1, 10]. These perspectives jointly broaden the conceptual background for analysing coalitional structures within formal conflict-modelling frameworks. Building on these methodological foundations, this study introduces an integrated approach that: (i) employs clustering-inspired quality indices to quantify coalition cohesion and separability; (ii) advances attribute-importance analysis through a dual perspective combining removal-based coalition-quality degradation with a qualitative evaluation using Pareto-optimal bi-coalitions; and (iii) applies this framework to analyse the Russia-Ukraine conflict. The results consistently identify the restoration of Ukraine's territorial integrity as the dominant axis of disagreement shaping coalition stability, while distinguishing between issues that restrict coalition expansion and those that enable broader consensus. Overall, the proposed methodology contributes both a refined theoretical extension of conflict-modelling techniques and a practical toolset for interpreting complex international disputes.

2 Conflict model - preliminaries

The present paper builds on Pawlak's conflict model [6], in which a conflict between agents is represented through discrepancies in their ternary evaluations of a set of issues. Each agent assigns to every issue a value from $V = \{-1, 0, +1\}$, corresponding to opposition, neutrality, and support, forming an information table with agents as rows and issues as columns.

Definition 1 (Conflict model). *Let $X = \{x_1, \ldots, x_m\}$ be a finite set of agents and $I = \{i_1, \ldots, i_n\}$ a finite set of issues. A conflict situation is a quadruple (X, I, V, r), where $V = \{-1, 0, +1\}$ and $r : X \times I \to V$ assigns a ternary rating to each agent–issue pair.*

For each issue i, the value space induces the classes $X_i^{+1} = \{x \mid r(x,i) = 1\}$, $X_i^0 = \{x \mid r(x,i) = 0\}$, $X_i^{-1} = \{x \mid r(x,i) = -1\}$, representing support, neutrality, and opposition. These partitions serve as the basic components for multi-issue aggregation.

For any subset $B \subseteq I$, aggregated opinions are defined by $r(x,B) = \frac{\sum_{i \in B} r(x,i)}{|B|}$, which induces the three-way split (with thresholds $-1 \le l_1 < 0 < h_1 \le 1$):

$$X_B^+ = \{x \mid r(x,B) \ge h_1\}, X_B^0 = \{x \mid l_1 < r(x,B) < h_1\}, X_B^- = \{x \mid r(x,B) \le l_1\}.$$

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Disagreement between agents x_j and x_k is quantified by the distance

$$d(x_j, x_k) = \frac{\sum_{i \in I} \varphi_i(x_j, x_k)}{|I|},$$

where

$$\varphi_i(x_j, x_k) = \begin{cases} 0, & r(x_j, i) = r(x_k, i), \\ 0.5, & r(x_j, i) \cdot r(x_k, i) = 0 \text{ and } r(x_j, i) \neq r(x_k, i), \\ 1, & r(x_j, i) = -r(x_k, i). \end{cases}$$

Using thresholds $0 \leq l_2 < h_2 \leq 1$, agent pairs are classified as $R^{=} = \{(x_j, x_k) \mid d(x_j, x_k) \leq l_2\}$, $R^{\approx} = \{(x_j, x_k) \mid l_2 < d(x_j, x_k) < h_2\}$, $R^{\neq} = \{(x_j, x_k) \mid d(x_j, x_k) \geq h_2\}$.

This relational tri-partition distinguishes aligned, neutral, and conflicting pairs. By symmetry, analogous partitions may be defined for subsets of agents to determine the issues they predominantly support, oppose, or treat neutrally.

3 Coalition strength recognition through clustering-inspired quality measures

Coalition strength reflects how cohesive a group of agents is and how clearly it stands apart from other groups. In Pawlak's conflict model, alliances are determined through pairwise agreement thresholds, but the model itself does not evaluate how compact or well-separated these coalitions are. To address this limitation, we extend the framework with clustering-based quality measures that assess coalition cohesion and distinctness, following the approach in [8]. We employ three widely used indices: the Dunn index (separation vs. compactness), the Silhouette index (individual fit within coalitions), and the Davies–Bouldin index (average compactness relative to inter-group distance).

Coalitions are defined as maximal subsets $C \subseteq X$ such that for all $x, y \in C$ the allied condition $d(x, y) < 0.5$ holds. Since some agents may be close to multiple groups, coalitions may overlap. Let $\mathcal{C} = \{C_1, \ldots, C_K\}$ denote the resulting structure.

Dunn index $D = \dfrac{\min\limits_{p \neq q} \min\limits_{x \in C_p,\ y \in C_q} d(x, y)}{\max\limits_{k} \max\limits_{x, y \in C_k} d(x, y)}$.

Silhouette index Let $a(x)$ be the average distance between x and members of its coalition, and $b(x)$ the minimum average distance to another coalition.

$$s(x) = \frac{b(x) - a(x)}{\max\{a(x), b(x)\}}, S = \frac{1}{|X|} \sum_{x \in X} s(x).$$

Davies–Bouldin index Let S_k denote the mean intra-coalition distance in C_k, and M_{kj} the mean pairwise distance between C_k and C_j:

$$DB = \frac{1}{K} \sum_{k=1}^{K} \max_{j \neq k} \frac{S_k + S_j}{M_{kj}}.$$

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Together, these metrics provide a complementary evaluation: Dunn emphasizes global separation, Silhouette measures how well individual agents fit their coalitions, and Davies–Bouldin captures average compactness relative to inter-coalition similarity. High-quality coalition structures thus exhibit high D and S, and low DB. To summarize these criteria, we define a normalized combined quality measure $Q = w_D\, D^{norm} + w_S\, S^{norm} - w_{DB}\, DB^{norm}$, where each index is normalized to $[0, 1]$ based on its minimum and maximum values obtained across all attribute-removal variants. Since distances between agents are calculated directly from issue evaluations, each issue contributes differently to coalition formation and stability. Attribute-importance analysis therefore examines how the coalition structure and quality change when an issue is removed.

Definition 2. *For each issue $a \in I$, let Q_a denote the quality of the coalition structure obtained from the reduced system $IS_a = (X, I \setminus \{a\})$. A higher value of Q_a indicates that removing issue a increases coalition coherence and separation, meaning that a plays a critical role in shaping the original conflict. The most influential issue is identified by* $\arg\max_{a\in I} Q_a$.

This measure provides a practical tool for analysing conflict drivers: issues whose removal improves coalition quality act as major sources of disagreement, while those whose removal weakens coalition clarity support internal cohesion or maintain boundaries between groups. The attribute-ranking procedure follows the hierarchical evaluation scheme introduced in [8], where successive removals reveal the relative strategic importance of each issue.

4 Qualitative conflict analysis

We analyse qualitative agreement structures using the concept of bi–coalitions, introduced in [3], which extends Pawlak's conflict model by identifying groups of agents who unanimously agree on a subset of issues. A bi–coalition is a pair (Y, B) with $Y \subseteq X$ and $B \subseteq I$ such that all agents in Y assign identical values to all issues in B. Formally, $\sigma_B(x_j, x_k) = \{\, i \in B \mid r(x_j, i) = r(x_k, i) \,\}$, and (Y, B) is a bi–coalition if

$$B = \bigcap_{x_j, x_k \in Y} \sigma_B(x_j, x_k) \neq \emptyset.$$

Its strength may be evaluated by

$$\mathrm{str}_1(Y, B) = \frac{|Y|}{|X|} \cdot \frac{|B|}{|I|} \quad \text{or} \quad \mathrm{str}_2(Y, B) = \frac{|Y| + |B|}{|X| + |I|},$$

which measure the extent of consensus. A relaxed version that treats neutrality as compatible replaces σ_B with $\sigma^*_B(x_j, x_k) = \{\, i \in B \mid r(x_j, i) = r(x_k, i) \;\vee\; r(x_j, i) = 0 \;\vee\; r(x_k, i) = 0 \,\}$.

We focus on Pareto–optimal bi–coalitions (denoted $BIC^{\max}$), defined as those (Y, B) for which no enlargement of Y or B is possible without violating unanimity: $(Y', B') \succ (Y, B) \;\Rightarrow\; (Y, B)$ not Pareto–optimal.

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Bi–coalitions provide a direct constraint-based view of agreement formation and enable issue-level importance assessment. Since each (Y, B) is characterised by the issues enabling unanimous agreement, the presence and strength of bi–coalitions reveal which issues support broad consensus and which restrict coalition expansion.

For each issue $i \in I$, we define its importance based on Pareto-optimal bi–coalitions as

$$Imp_{BIC}(i) = \sum_{(Y,B)\in BIC^{\max},\ i\in B} \mathrm{str}(Y, B),$$

where str denotes any chosen strength measure. A high value indicates that many agents can jointly agree on i, meaning that the issue is non-obstructive in coalition building. A low value reflects that only small groups can reach unanimity on i, identifying it as a significant barrier to consensus.

To align the bi–coalition framework with the removal-based approach used for distance-based coalitions, we define a complementary importance measure. For each issue i, consider the reduced system $IS_{-i} = (X, I \setminus \{i\}, V, r|_{I\setminus\{i\}})$, recompute its set of Pareto-optimal bi–coalitions $BIC^{\max}_{-i}$, and evaluate their total quality

$$Q = \sum_{(Y,B)\in BIC^{\max}} \mathrm{str}(Y, B), \qquad Q_{-i} = \sum_{(Y,B)\in BIC^{\max}_{-i}} \mathrm{str}(Y, B).$$

The importance of issue i is defined by $Imp_{rem}(i) = Q - Q_{-i}$.

A larger value of $Imp_{rem}(i)$ means that the removal of issue i significantly weakens the structure of unanimous-agreement groups, identifying it as essential for maintaining stable agreement patterns. Issues with low values play a minor role and do not substantially influence the qualitative coalition landscape.

5 Russia–Ukraine conflict

To illustrate the proposed methodology, we analyse the Russia–Ukraine conflict, one of the most consequential and persistent geopolitical crises of recent years. The conflict is represented by six key agents (Ukraine, Russia, United States, European Union, NATO, China) and seven issues covering territorial, military, political, and economic dimensions. Belarus is excluded, as its evaluations coincide with Russia's. The resulting conflict situation is shown in Table 1. The issues encode: i_1 territorial restoration; i_2 NATO membership; i_3 Western military aid; i_4 recognition of Russian annexations; i_5 ceasefire on current lines; i_6 non-NATO security guarantees; i_7 Western sanctions on Russia.

Using the distance measure from Section 3, the agents form two coalitions: $C_1 = \{x_1, x_3, x_4, x_5\}, C_2 = \{x_2, x_6\}$, corresponding respectively to a Western-aligned group and a Russia-aligned bloc.

Applying the Dunn, Silhouette and Davies–Bouldin indices produces: $D = 2.0$, $S = 0.728$, $DB = 0.553$, indicating that these coalitions are internally coherent and clearly separated.

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Table 1. Agents $\times$ Issues: Russia–Ukraine conflict

A\I	i_1	i_2	i_3	i_4	i_5	i_6	i_7
Ukraine (x_1)	+1	+1	+1	-1	-1	+1	+1
Russia (x_2)	-1	-1	-1	+1	+1	-1	-1
United States (x_3)	-1	0	+1	-1	0	+1	+1
European Union (x_4)	0	+1	+1	-1	0	0	+1
NATO (x_5)	0	0	+1	-1	0	+1	+1
China (x_6)	0	-1	-1	0	+1	0	-1

To evaluate issue importance, each issue is removed in turn, coalition structures are recomputed, and the resulting quality indices are normalized to obtain: $Q_a = \frac{1}{3}D_a^{norm} + \frac{1}{3}S_a^{norm} - \frac{1}{3}DB_a^{norm}$. The values Q_a (summarized in Table 2) show that issue i_1 (territorial integrity) is the strongest driver of the coalition split. Issues such as i_3 (military aid) and i_7 (sanctions) have low or negative Q_a, meaning that their removal does not substantially alter the coalition structure and they play a secondary role.

Table 2. Quality indices and issue quality Q_a after removing issue i_a.

Issue removed	D_a	S_a	DB_a	D_a^{norm}	S_a^{norm}	DB_a^{norm}	Q_a
i_1	4	0.81	0.39	1	1	0	0.67
i_2	2	0.73	0.57	0.2	0.47	0.6	0.02
i_3	1.5	0.66	0.69	0	0	1	-0.33
i_4	1.75	0.72	0.53	0.1	0.4	0.47	0.01
i_5	2.33	0.73	0.57	0.33	0.47	0.6	0.07
i_6	1.75	0.77	0.46	0.1	0.73	0.23	0.2
i_7	1.5	0.66	0.69	0	0	1	-0.33

We now illustrate the two previously defined importance measures: the participation based $Imp_{BIC}(i)$ and the removal-based $Imp_{rem}(i)$. In this example, we use the str_1 strength measure. Using the full conflict table, we compute all Pareto-optimal bi–coalitions (Y, B); their sets and strengths are listed in Table 3. Based on these values, the participation-based importance measure $Imp_{BIC}(i)$ is reported in Table 5. Issues i_3 and i_7 obtain the highest scores, indicating frequent participation in strong unanimity groups. Issues i_4 and i_5 play a moderate role, while i_1, i_2, and i_6 appear mostly in small coalitions and therefore show low importance.

The total bi–coalition quality for the full system equals $Q = 2.9762$. To compute the removal-based measure, we remove each issue in turn, recompute all Pareto-optimal bi–coalitions, and obtain $Imp_{rem}(i) = Q - Q_{-i}$, where Q_{-i} is the total strength in the reduced system. These values are summarized in Table 5. Removing issue i_1 produces the largest drop (0.4762), confirming that it is the most restrictive and structurally significant dimension. Issues i_3, i_5, i_6, and i_7 show moderate influence, while issue i_2 has only marginal effect. Both

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Table 3. Pareto-optimal bi-coalitions in the full system.

Bi–coalition (Y,B), str_1	Bi–coalition (Y,B), str_1
$(\{x_1\}, \{i_1, \ldots, i_7\})$, 0.1667	$(\{x_5\}, \{i_1, \ldots, i_7\})$, 0.1667
$(\{x_2\}, \{i_1, \ldots, i_7\})$, 0.1667	$(\{x_6\}, \{i_1, \ldots, i_7\})$, 0.1667
$(\{x_3\}, \{i_1, \ldots, i_7\})$, 0.1667	$(\{x_1, x_4\}, \{i_2, i_3, i_4, i_7\})$, 0.1905
$(\{x_4\}, \{i_1, \ldots, i_7\})$, 0.1667	$(\{x_2, x_3\}, \{i_1\})$, 0.0476
$(\{x_2, x_6\}, \{i_2, i_3, i_5, i_7\})$, 0.1905	$(\{x_4, x_6\}, \{i_1, i_6\})$, 0.0952
$(\{x_3, x_5\}, \{i_2, i_3, i_4, i_5, i_6, i_7\})$, 0.2857	$(\{x_1, x_3, x_5\}, \{i_3, i_4, i_6, i_7\})$, 0.2857
$(\{x_4, x_5\}, \{i_1, i_3, i_4, i_5, i_7\})$, 0.2381	$(\{x_3, x_4, x_5\}, \{i_3, i_4, i_5, i_7\})$, 0.2857
$(\{x_4, x_5, x_6\}, \{i_1\})$, 0.0714	$(\{x_1, x_3, x_4, x_5\}, \{i_3, i_4, i_7\})$, 0.2857

Table 4. Issue importance $Imp_{BiC}(i)$.

Issue i	i_1	i_2	i_3	i_4	i_5	i_6	i_7
$\mathrm{Imp}_{\mathrm{BiC}}(i)$	1.4524	1.6667	2.7619	2.5714	2.0000	1.6667	2.7619

measures consistently indicate that i_1 is the key point of disagreement, while i_3, i_7, and partially i_4 correspond to issues with relatively broad agreement and lower restrictiveness.

6 Discussion and Conclusion

This study presents an integrated framework for analysing coalition structures and assessing attribute importance in conflict situations, combining clustering-based metrics with a qualitative bi-coalition approach. Building on Pawlak's conflict model, we introduced a unanimity-focused measure and compared it with a quantitative distance-based assessment. The Russia–Ukraine case study showed that both measures consistently identify territorial integrity as the principal factor shaping coalition divisions, while other issues display varying levels of flexibility. The bi-coalition measure reveals negotiation-relevant agreement patterns, complementing the structural perspective provided by clustering indices.

The proposed dual-perspective approach offers concrete analytical support for negotiation and mediation processes. The clustering-based component highlights structural divisions and identifies issues that most strongly shape coalition boundaries, while the bi–coalition analysis pinpoints where unanimous agreement is feasible and where it is structurally constrained. Together, these insights help practitioners prioritise negotiation topics and estimate the room for possible consensus. While the current study focuses on a representative medium-sized conflict scenario, the methodological design allows for straightforward application to larger multi-agent or multi-issue environments. As the number of potential bi–coalitions may grow with system complexity, future work may explore efficient pruning strategies or approximate evaluation schemes to enhance scalability without reducing interpretability. Thresholds and weights used in the

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Table 5. Values of Q_{-i} and removal-based issue importance $Imp_{rem}(i)$.

Issue i	i_1	i_2	i_3	i_4	i_5	i_6	i_7
Q_{-i}	2.5	2.9722	2.8056	2.8611	2.8056	2.8056	2.8056
$\mathrm{Imp_{rem}}(i)$	0.4762	0.004	0.1706	0.1151	0.1706	0.1706	0.1706

clustering-based measure follow conventions from earlier conflict-modelling studies and standard clustering literature; however, their robustness and sensitivity would benefit from a separate parametrical analysis. This suggests an additional avenue for future research.

Overall, the results demonstrate that analysing attribute importance is essential for understanding conflict dynamics and supporting negotiation processes.

References

1. Amirkhani, A., Barshooi, A.H.: Consensus in multi-agent systems: a review. Artificial Intelligence Review **55**(5), 3897–3935 (2022)
2. Deja, R.: Conflict analysis. Int. J. Intell. Syst. **17**(2), 235–253 (2002)
3. Deja, R., Przybyła-Kasperek, M.: Bi-coalitions analysis in the rough sets conflict model. Information Sciences p. 122746 (2025)
4. Dudek, A.: Silhouette index as clustering evaluation tool. In: Classification and Data Analysis: Theory and Applications 28. pp. 19–33. Springer (2020)
5. Gupta, T., Panda, S.P.: Clustering validation of clara and k-means using silhouette & dunn measures on iris dataset. In: 2019 COMITCon. pp. 10–13. IEEE (2019)
6. Pawlak, Z.: Some remarks on conflict analysis. Eur. J. Oper. Res. **166**(3), 649–654 (2005)
7. Przybyła-Kasperek, M.: Coalitions' weights in a dispersed system with pawlak conflict model. Group Decision and Negotiation **29**(3), 549–591 (2020)
8. Przybyła-Kasperek, M., Deja, R., Wakulicz-Deja, A.: Hierarchical system in conflict scenarios constructed based on cluster analysis-inspired method for attribute significance determination. Applied Soft Computing **167**, 112304 (2024)
9. Skowron, A., Ramanna, S., Peters, J.F.: Conflict analysis and information systems: A rough set approach. In: Rough Sets and Knowledge Technology: First International Conference, RSKT 2006, Chongquing, China, July 24-26, 2006. Proceedings 1. pp. 233–240. Springer (2006)
10. Tran, T.N.T., Felfernig, A., Le, V.M.: An overview of consensus models for group decision-making and group recommender systems. User Modeling and User-Adapted Interaction (2023)
11. Xiao, J., Lu, J., Li, X.: Davies bouldin index based hierarchical initialization k-means. Intelligent Data Analysis **21**(6), 1327–1338 (2017)
12. Yao, J., Medina, J., Zhang, Y., Ślęzak, D.: Formal concept analysis, rough sets, and three-way decisions. Int. J. Approx. Reason. **140**, 1–6 (2022)
13. Yao, Y.: Three-way decisions with probabilistic rough sets. Inf. Sci. **180**(3), 341–353 (2010)

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