Global Sensitivity Analysis for a Mathematical Model of the General Escape Theory of Suicide

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Abstract. This study explores a formalized dynamical systems model of the General Escape Theory of Suicide using Sobol and PAWN global sensitivity analyses. The findings highlight the importance of self-feedback loops, the effect of stressors on aversive internal states, and the interaction effects between aversive internal states and the urge to escape on suicidal ideation and non-suicidal escape behaviors. Time-dependent sensitivity analysis also reveals the long-term stability of parameter importance over time. These results hold potential for informing clinical interventions by identifying the most important influences for individual suicidal ideation.

Keywords: Sensitivity Analysis · Dynamical Systems · Uncertainty Quantification · Sobol Sensitivity Indices · PAWN Sensitivity Indices

1 Introduction

Suicide is a major global public health problem, with over 700.000 deaths a year [9]. Suicidal behavior is conceptualized as the result of the complex interaction between many different variables [2]. An influential review article stated that the prediction of suicidal ideation and behaviors has not improved over the last 50 years [3]. Within the field of suicide prevention, a novel route to better understand and study this complexity is by working with mathematical models. These formalized models force researchers and clinicians to make any assumptions about the relation of different factors explicit and allow them to test more vigorously what the theory proposes to predict.

In their paper "Mathematical and Computational Modeling of Suicide as a Complex Dynamical System," Wang et al. [15] developed a mathematical model of suicidal thoughts as a system of differential equations. Taking inspiration from applications of nonlinear dynamical systems theory, their model presents suicidal ideation as a system of interacting subcomponents, such as stressors, aversive internal states, suicidal thoughts, and escape behaviors. The advantage of such

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an approach is the ability to test predictions made by verbal theories, which have generally dominated suicide research in psychology. This model is built from the newly proposed General Escape Theory of Suicide by Millner et al. [8]. The formalization of this theory as a mathematical model then forces the developer to make choices about the type of interactions between subcomponents and the values of self-feedback loop and interaction parameters [15]. The simulated data provides insight into whether or not the mathematical model reflects the expected emergent behavior associated with the phenomena. In particular, the simulation provides evidence for the rapid-onset and high variability of suicidal thoughts in response to random stressors and heightened aversive internal states.

Methods for global sensitivity analysis are useful in understanding which parameters are influencing particular outputs from a model. It explores the effects of changes in all parameters across the entire parameter space, thereby also investigating parameter-interaction effects on the final outcome variable measured [10]. Conducting a sensitivity analysis for the suicide model by Wang et al. [15] is essential to understanding how the parameter choices affect the model outcomes, which may provide insights into how to improve model parsimony or where the mathematical and verbal model may not align (i.e. if the verbal model posits that a particular parameter is extremely important but has little effect on the outcome in the simulation). In this project, Sobol sensitivity and PAWN sensitivity analyses were conducted to explore these questions.

1.1 The Model

The model by Wang et al. [15] defines stressors (S_t) with a Brownian motion equation, including both the deterministic drift parameter μ, the stochastic volatility parameter σ, and the regulating effect of externally-focused strategies, modulated by parameter f_1. Next, the change in a patient's aversive internal state (A) is increased by the stressor according to another parameter, a, and a logistic growth term with a carrying capacity of K_2. We also assume that the aversive internal state is reduced by suicidal thoughts and escape behaviors according to parameters d_2 and e_5, since these can have a functional purpose as a reprieve from the aversive feelings. Finally, aversive internal state is improved by the effect of internally-focused strategies according to parameter g_2. The change in the urge to escape (U) is governed by a negative self-feedback loop and positive influence of the aversive internal state, controlled by parameter b_3. They define the change in suicidal thoughts (T) as a sigmoidal function which has a negative self-feedback loop and whose structure is dictated by two parameters, c_{41} and c_{42}, which determines the steepness and midpoint of the sigmoidal curve respectively. The change in escape behavior (X) is defined nearly identically, with parameters c_{51} and c_{52}. The change in externally-focused strategies (E) and internally-focused strategies (I) are also governed by a logistic equation, plus a positive impact of aversive internal states (regulated by parameters b_6 and b_7, respectively)and a negative impact of the urge to escape (with parameters c_6 and c_7 for externally and internally focused strategies). The equations governing the model can be found below.

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$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t - f_1 E}, \tag{1}$$

$$\frac{dA}{dt} = b_2 A\left(K_2 - A\right) + a_2 S - d_2 T - e_2 X - g_2 I, \tag{2}$$

$$\frac{dU}{dt} = -c_3 U + b_3 A, \tag{3}$$

$$\frac{dT}{dt} = -d_4 T + \frac{1}{1 + e^{-c_{41}(U - c_{42})}}, \tag{4}$$

$$\frac{dX}{dt} = -e_5 X + \frac{1}{1 + e^{-c_{51}(U - c_{52})}}, \tag{5}$$

$$\frac{dE}{dt} = f_6 E\left(K_6 - E\right) + b_6 A - c_6 U, \tag{6}$$

$$\frac{dI}{dt} = g_7 I\left(K_7 - I\right) + b_7 A - c_7 U \tag{7}$$

1.2 Global Sensitivity Analysis

While local sensitivity analyses investigate the effect of changes in a single variable on the model output, global sensitivity analysis, all parameters are varied across the defined parameter space at the same time to investigate both parameters' single and interaction based effects [16]. In this paper, we will focus on two prominent methods: Sobol sensitivity analysis, a variance-based method, and PAWN sensitivity analysis, a moment-independent method.

Sobol Sensitivity Analysis. Sobol sensitivity analysis decomposes the model output variance into sensitivity indices for each parameter, allowing for an interpretation of how much a particular output's variability can be attributed to a particular parameter or parameter interaction [13]. First, we choose reasonable parameter ranges which are then sampled using a Sobol sequence, a quasi-random sequence that more efficiently covers the parameter space than completely random methods such as Monte Carlo. The model is then run for each combination of parameter values. From here, first-order and higher-order sensitivity indices are calculated for each parameter.

If we define $X = (X_1, ..., X_n)$ as an input vector with n parameters, and $f(X) = Y$ as the model output, then the variance of the average of that model output for all possible parameter values provides a useful measure of model variability as a result of that parameter set. We are particularly interested in the variance of the average across all possible values of X_i to avoid any dependence on the value of X_i in the parameter space. For ease of interpretation, we can define $\mu_{X_i} = E(Y|X_i)$ as the conditional expectation when the parameter X_i is fixed and $\mu_{X_{\sim i}} = E(Y|X_{\sim i})$ as the expectation when all other parameters are held constant except X_i, as per standard convention.

The first-order sensitivity index is then described in Sobol [13] as the proportion of the total variance that can be explained by variation in the parameter set X_i, i.e.

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$$S_i = \frac{V(\mu_{X_i}(Y))}{V(Y)}, \, 0 \leq S_i \leq 1. \tag{8}$$

This provides the main effect of the particular parameter X_i on the output variability of the model. If this value is high, we can assume a strong, direct effect of this parameter on the variability in the model output. We can similarly extend this to second-order indices, where we explore the variance in model output as a result of all other variables while two particular parameters are fixed. This can be depicted with the following equation, adapted from Sobol [13],

$$S_{i,j} = \frac{V(\mu_{X_{\sim i,j}}(Y)) - V(\mu_{X_i}(Y)) - V(\mu_{X_j}(Y))}{V(Y)}. \tag{9}$$

If the second-order sensitivity index for a particular parameter combination is high, we can assume a strong interaction effect between these two parameters on the output. Given that there are 24 parameters to investigate, looking at all possible interaction levels would be unrealistic and computationally demanding. Therefore, we can use the total-order sensitivity index as a metric for these higher-order interactions. The total-order sensitivity index describes the proportion of variance caused by that particular parameter while all other parameters are fixed. From the Law of Total Variance, we know that

$$\frac{E_{X_{\sim i}}(V_{X_i}(Y|X_{\sim i}))}{V(Y)} + \frac{V_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))}{V(Y)} = 1. \tag{10}$$

Hence, the first term in the above expression encapsulates total-order sensitivity, as it averages across all other parameters, the conditional variance caused by the parameter of interest. Therefore, the total-order sensitivity index of a particular parameter, X_i, can be described in the following equation (once again adapted from Sobol [13]),

$$S_{T_i} = 1 - \frac{V(\mu_{X_{\sim i}}(Y))}{V(Y)}. \tag{11}$$

A high total-order index for a parameter implies it has a strong overall effect, which includes its interaction and main effects. This would imply that if first or second-order indices for a parameter were low, but total-order indices were high, then the model output was likely due to higher-order interactions between multiple parameters and the parameter of interest.

PAWN Sensitivity Analysis. In situations where output distributions are highly skewed or bimodal, variance decomposition techniques may not provide

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the most reliable estimates of sensitivity indices. Another approach is PAWN, which is moment independent [10]. Rather than measuring changes in variability, it measures distributional changes in the output. After sampling for parameter combinations, the model is evaluated and its overall cumulative distribution function (CDF) is calculated without any parameter held constant, representing the baseline distribution of the output. Then, each parameter is sampled for a particular conditioning value, X_i. The model is then evaluated for random samples of all other parameters for each conditioned value of X_i. The output CDF for each conditioned value of X_i is compared to the original, overall output CDF using the Kolmogorov-Smirnov statistic. This statistic is then used to calculate the sensitivity index for the overall effect of that parameter on the output, akin to the Sobol total-order sensitivity index [11]. However, this calculation cannot capture first or second-order variance decomposition. As a result, we will use PAWN to verify the robustness of our result from the Sobol sensitivity analysis.

2 Methods Implementation

2.1 Model Implementation

The model implementation is the same as the version published by Wang et al. [15], which can be found at `https://github.com/ShirleyBWang/math_model_suicide`. As in the original paper, the simulation length was two weeks, calculated as $(15 \cdot 1440)$ minutes in 15 days, with increments of 0.01. The only adjustment made is the use of the Python compiler library, Numba, which uses a different random number generator [6]. Given the numerical nature of the model itself, Numba lends itself well to speeding the model evaluations, reducing a single model run from 0.3 seconds to 0.002 seconds.

Additionally, for the time-dependent sensitivity analysis, the seed for the model was set to compare sensitivity for different output values for a particular run. This output was chosen for its medial variability in output values, lending itself to interesting interpretations whilst not being overly complex. A visualization of this output can be found in figure 1.

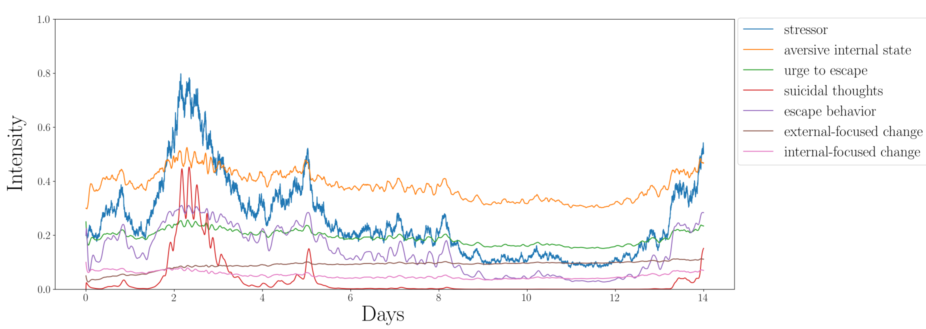

Fig. 1. Simulation output for Numba seed 504 from the model provided in Wang et al.[15].

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2.2 Sensitivity Analysis Implementation

The sensitivity analysis was implemented using SALib (version 1.5.0) [5], a Python library designed for the task. All documentation for library can be found at `https://salib.readthedocs.io/en/latest/index.html`. The `saltelli()` function from the `sample` package was used to sample the parameter space according to the Saltelli algorithm, which reduces error associated with the Sobol sequence used for quasi-random sampling of parameter space [12]. The analyze() functions from packages `sobol` and `pawn` were utilized to estimate the sensitivity indices. The number of conditioning intervals used for the PAWN analysis was dependent on the sample size, with the number of conditioning intervals being roughly 1% of the sample size for reliable convergence [1]. Additionally, the NumPy library (version 1.26.4) [4] and Pandas library (version 2.2.2) [7] were employed for data manipulation and analysis. Finally, MatPlotLib (version 3.8.0) [14] was imported for visualizations.

3 Results and Discussion

3.1 Maximum Model Output Sensitivity Analysis

We first plotted the output distributions for runs with 2^{10} parameter space samples to determine the appropriateness of the sensitivity analysis method. From figure 2, we notice that the output distributions for the maximum of suicidal thoughts and the maximum escape behavior are highly right-skewed, meaning that comparison of variance-based sensitivity results with moment-independent, PAWN sensitivity indices is useful in confirming results. The distribution of aversive internal state model outputs is relatively evenly distributed, indicating that we can rely on the results of variance-based, Sobol analysis results for this output.

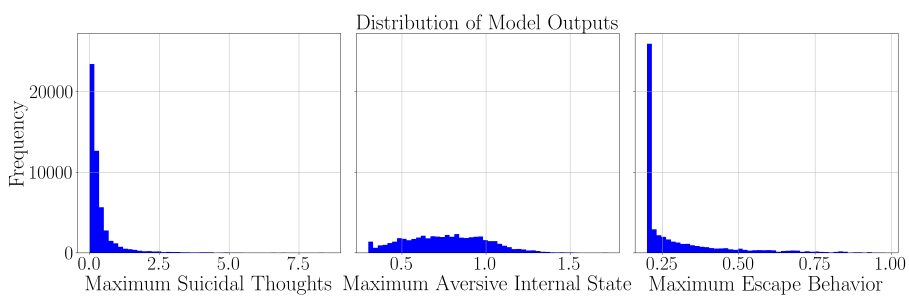

Fig. 2. Output distributions for maximum each output variable from model runs (suicidal thoughts, aversive internal state, and escape behavior) with 2^{10} parameter space samples.

We notice similar results reflected in convergence graphs for both Sobol total-order indices and PAWN indices (seen in figure 3 and figure 4 respectively). Increasing the number of parameter space samples for aversive internal state

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model outputs does not significantly improve the convergence of sensitivity indices. However, overall, we notice that the total-order sensitivity indices for any of the three outputs does not substantially change for more than 2^{16} samples, with only a minor reduction in the confidence interval. For PAWN indices, little information is gained past 2^{10} parameter space samples, since values have already converged. Hence, further analyses for maxima of model outputs were conducted with 2^{17} parameter samples for Sobol indices and 2^{14} parameter samples for PAWN indices.

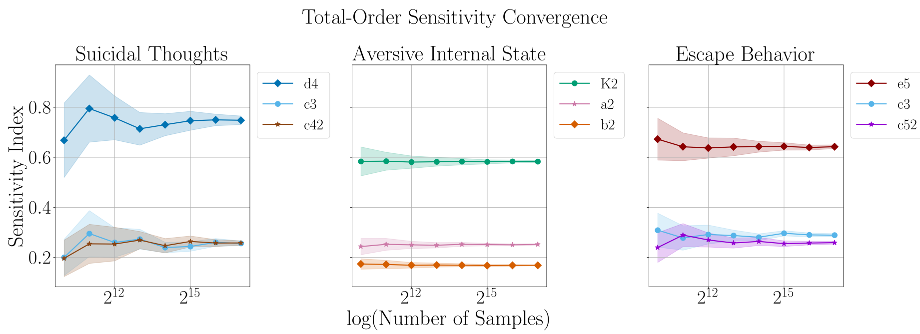

Fig. 3. Sobol total-order index values for the three highest sensitivity indices for the maximum of each model output (suicidal thoughts, aversive internal state, and escape behavior) for an increasing number of parameter space samples with 95% confidence.

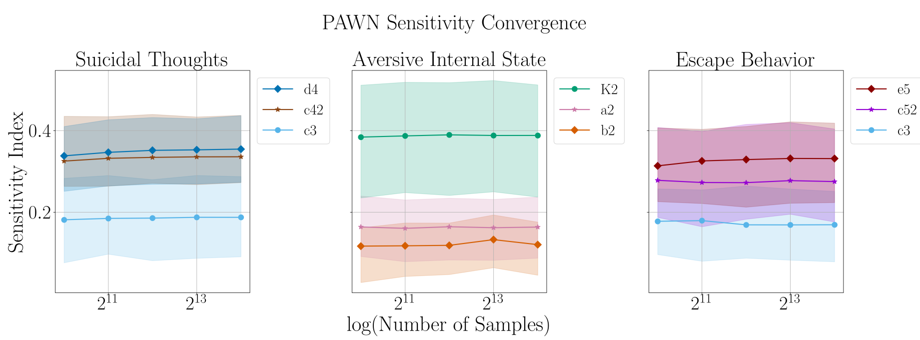

Fig. 4. PAWN sensitivity index values for the three highest sensitivity indices for the maximum of each model output (suicidal thoughts, aversive internal state, and escape behavior) for an increasing number of parameter space samples with the coefficient of variation as error.

The PAWN approach to sensitivity indices captures a more general effect of each parameter on the model output distribution, which may include some higher-order interactions with other parameters. We can then compare the PAWN sensitivity indices and the total-order Sobol sensitivity indices to evaluate the robustness of the results, found in figure 5. As illustrated in both the convergence plots and the side-by-side total-order and PAWN sensitivity plots, we notice that the only disagreement between the two methods is between the relative

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importance of the self-feedback loop parameters and c_{42} and c_{52}, which control the horizontal placement of the midpoint of the sigmoidal curves for suicidal thoughts and escape behaviors respectively. The two sensitivity analysis methods cannot be compared exactly, since they rely on two distinct measurements. However, the disagreement between relative importance of the midpoints of the sigmoidal curves indicates that these parameters have a greater impact on the asymmetric distributional changes on the model output than the overall variance in the model output. Regardless, the methods agree on which parameters remain the most important to the model outputs, regardless of their original distribution.

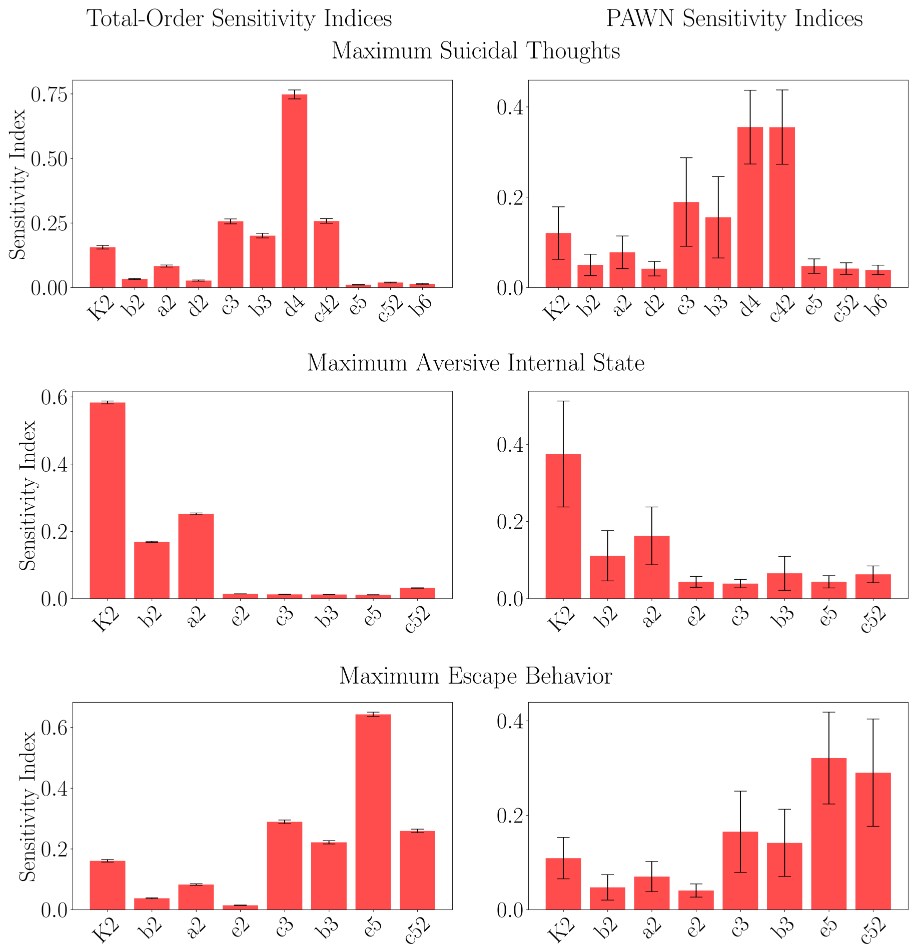

Fig. 5. Total-order Sobol sensitivity indices (left) and PAWN sensitivity indices (right) for the maximum of the specified model output. Error for Sobol indices is measured as a 95% confidence interval and the coefficient of variation as error for PAWN indices. Only parameters with an index greater than 0.01 are visualized for ease of reading. Sobol sensitivity indices were calculated with 2^{17} samples, compared with PAWN sensitivity indices calculated with 2^{14} samples.

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As mentioned in section 1.2, the main effects of each parameter can be captured with first-order Sobol sensitivity indices, shown in figure 6. These agree with the highest total-order sensitivity indices for each model output. The most relevant parameters that contribute to variance in the maximum of suicidal thoughts are d_4 (the self-feedback loop of suicidal thoughts), c_{42} (the midpoint of the sigmoidal curve for suicidal thoughts), b_3 (the effect of the aversive internal state on the change in the urge to escape), c_3 (the self-feedback loop on the urge to escape) and K_2 (the carrying capacity of the aversive internal state) with the self-feedback loop having the greatest impact. For aversive internal state, the most influential parameters are overwhelmingly K_2 (the carrying capacity for aversive internal state), b_2 (the self-feedback loop on aversive internal states), and a_2 (the effect of stressors on the change in aversive internal states). For escape behavior, the most important parameters identified are e_5 (the self-feedback loop of escape behaviors), c_{52} (the midpoint of the sigmoidal curve for escape behavior), and once again c_3, b_3, and K_2.

The high relevance of the self-feedback loops associated with each maximum for the outcome variable relates to the nonlinear effect of feedback on the system. All of the self-feedback loops have negative effects on their respective output, dampening the state's proportional effect on the rate of change of its output. Therefore, as these values decrease, the exponential effect of the model output state is strengthened, and that particular variable can maximize at a higher peak. This can lead to greater variability in model outputs as a result of changes to any of their respective self-feedback loops.

In comparison to the total-order Sobol indices seen in figure 5, the same parameters of importance are highlighted for all model outputs in figure 6. However, for the maximum of suicidal thoughts and escape behavior, the first-order sensitivity indices generally have much lower indices than their total-order counterparts. This indicates a high amount of second-order or higher-order interactions for each of these variables. However, the values of the sensitivity indices for maximum of aversive internal state are almost identical to their total-order sensitivity indices, indicating that the vast majority of variance in model output is due to the individual effect of each of these parameters with minimal reliance on higher-order interactions. This begs the question as to which particular interactions are relevant for the model outputs. As expected, the only second-order parameter interaction with a sensitivity index greater than 0.01 is between b_2 and a_2, since a_2 is the effect of stressors on the change in aversive internal state, which in turn directly effects the self-feedback loop parameter b_2.

For maximum escape behavior and the maximum of suicidal thoughts, there are more interesting second-order interaction dynamics, particularly between parameters governing self-feedback loops. The results of the second-order Sobol sensitivity analyses are found in figure 7.

We notice that all of the most relevant parameter interactions for a particular model output involve the parameter associated with its self-feedback loop. For the maximum of suicidal thoughts and escape behavior, this is d_4 and e_5 respectively. We also notice the high relevance of K_2, a_2, b_3, and particularly c_3 (the

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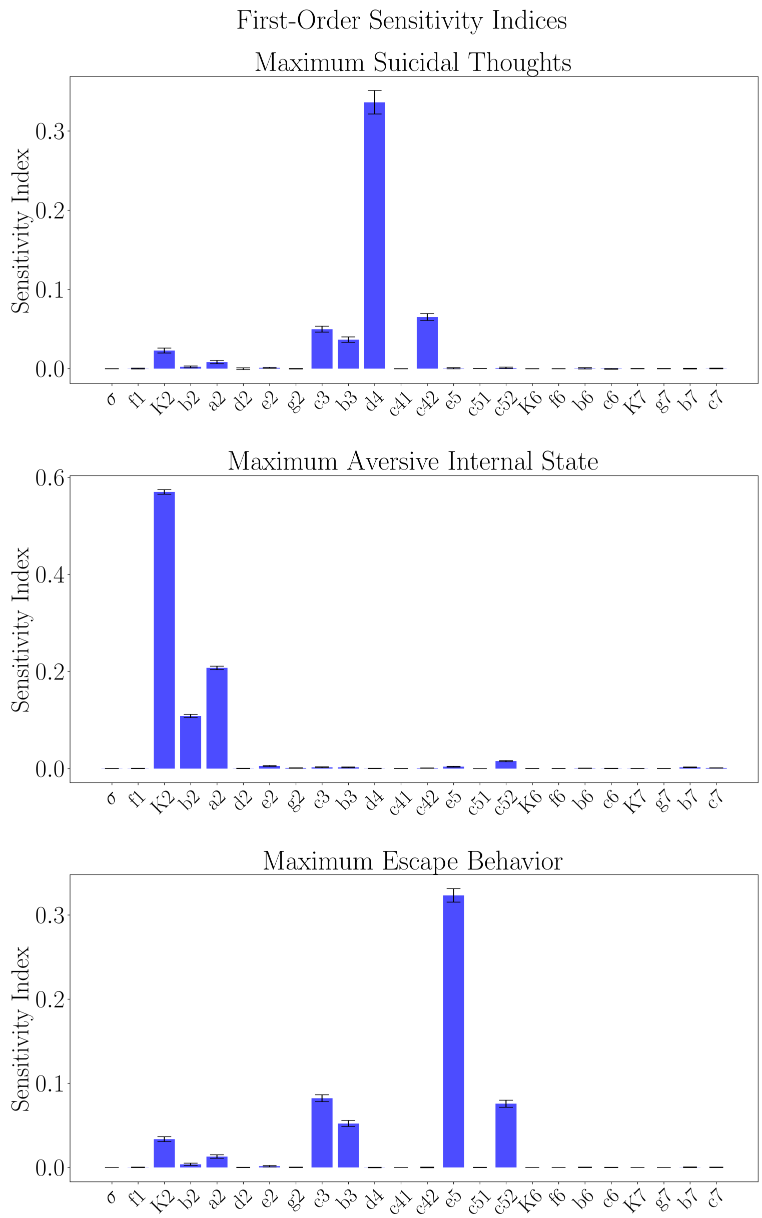

Fig. 6. First-order Sobol sensitivity indices for all parameters for the maximum of each model output.

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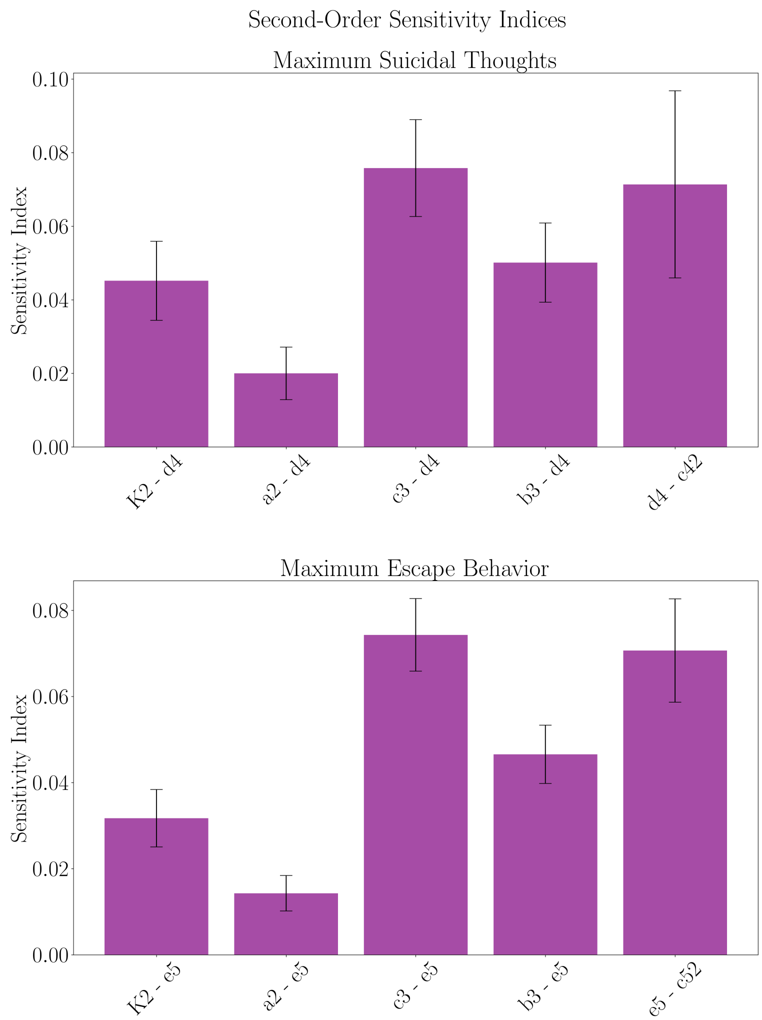

Fig. 7. Second-order Sobol sensitivity indices for parameters with indices greater than 0.01 for the maximum of suicidal thoughts and escape behaviors.

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parameter associated with the self-feedback loop for the urge to escape). This highlights, once again, the impact of exponential growth for self-feedback loops between coupled states. Since the urge to escape has a positive, nonlinear effect on both suicidal thoughts and escape behaviors, any exponential growth that results from lowering c_3 will greatly impact the results of both escape behaviors and suicidal thoughts. The cascading effect of variables is also highlighted, as K_2 is the carrying capacity for aversive internal state, a_2 is the positive effect of stressors on aversive internal state, and b_3 is the positive effect aversive internal state on the urge to escape. This in turn feeds both escape behaviors and suicidal thoughts, whose state will dictate their growth rate as a result of their self-feedback loops. We again notice the high importance of interactions with c_{42} and c_{52}: since these parameters control the placement of these sigmoidal curves, changes in these parameters will affect the threshold of urge to escape at which either escape behaviors or suicidal thoughts will sharply increase. Small perturbations in the parameters introduce nonlinearities that, when combined with their own self-feedback loops, will starkly affect the model output.

3.2 Time-Dependent Sensitivity Analysis

In order to assess any changes in parameter influence over the two-week simulation, a sensitivity analysis for all three model outputs was conducted each day of the simulation output found in figure 1. This particular realization of the model was chosen for its sufficiently fluctuating dynamics, particularly its peak in stressors between the second and third day of the simulation and trough at day 11. In figure 8, we see the results for the first-order and total-order sensitivity indices for the five highest sensitivity indices for each model output with 2^{16} samples. For both types of sensitivity indices, we notice a general trend of stability in the value of sensitivity indices over the course of the simulation. The general stability of sensitivity indices over the course of the simulation, despite fluctuations in variable outputs, indicates that the choice of simulation realization seen in figure 1 does not influence the sensitivity analysis output. The individual influence of d_4 (self-feedback loop of suicidal thoughts) decreases over the course of the simulation, while its interactions remain highly relevant throughout for suicidal thoughts. This indicates that while d_4 may have a direct importance in the trajectory of suicidal thoughts initially, its interactions with parameters governing aversive internal state or urge to escape maintain the overall influence of the self-feedback loop on the suicidal thoughts output. The self-feedback loop governed by d_4 may also diminish in individual importance as the system stabilizes and suicidal thoughts decrease.

For aversive internal state, higher-order interactions involving b_6 (effect of aversive internal states on external-focused strategies), c_3 (self-feedback loop on urge to escape), b_3 (effect of aversive internal states on change in urge to escape), and c_6 (effect of urge to escape on external-focused strategies) all increase slightly on the second day of the simulation when stressors inflate, before remaining relatively constant for the remainder of the simulation. As stressors spike in the early part of the simulation, aversive internal state and the urge

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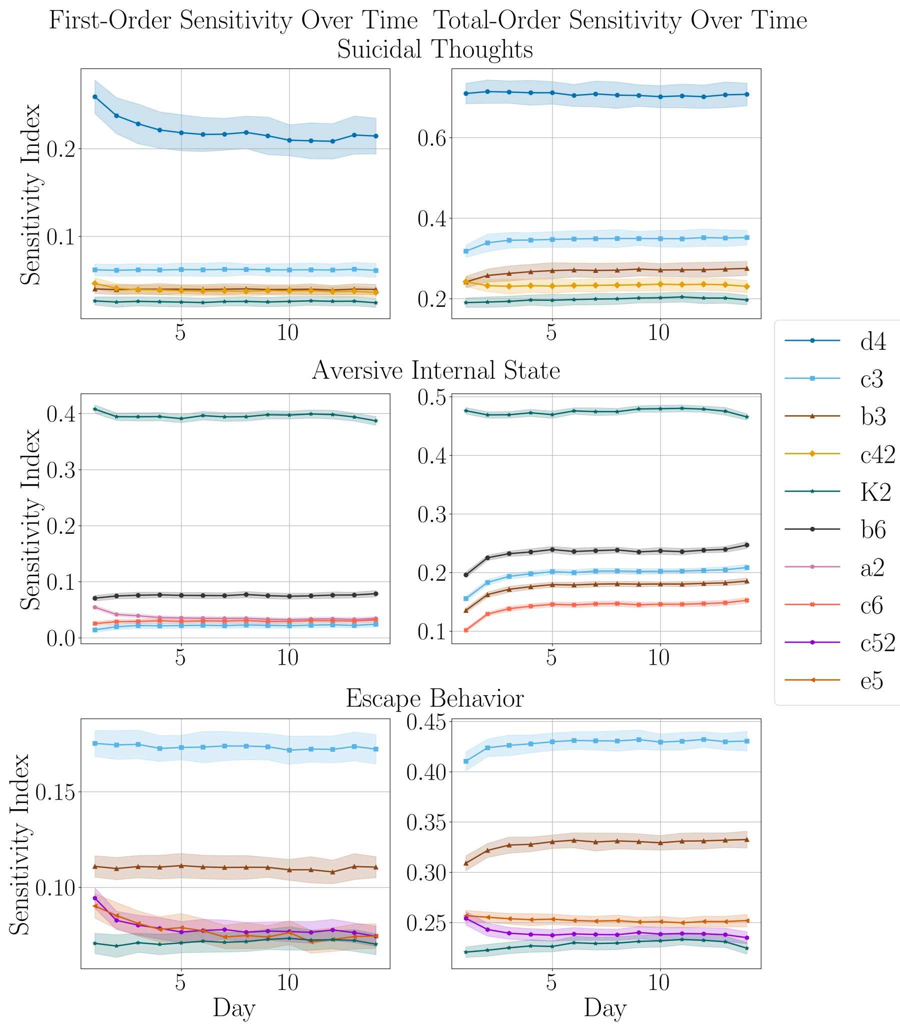

Fig. 8. The five highest first and total-order sensitivity indices for each model output on each day of the simulation.

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to escape temporarily amplify each other. This highlights the interaction between mediating parameters like b_3 and c_3 that capture the reinforcing loop where higher aversive states drive a greater urge to escape. This sharp increase in stressors also drives the activation of externally based strategies as a result of increased aversive internal state and the urge to escape, which underscores the importance of parameter interactions involving b_6 and c_6. We also notice that a_2 (the positive effect of aversive internal states on change in urge to escape) has a modest individual influence on aversive internal state. In this way, it acts as a bridge between aversive internal state and the urge to escape, but does not amplify the reinforcing loop between the two states.

We see a similar trend for higher-order interactions with c_3 and b_3 for escape behavior, and a slight decline in interaction importance for c_{52} following the first day of the simulation and a modest increase in interaction importance for K_2. This is likely due to the fact that c_{52} (the midpoint of the sigmoidal curve for escape behavior) strongly influences the sharp increase of escape behaviors by interacting with the variable's self-feedback loop when stressors are high in the early days of the simulation. Interactions with the carrying capacity for aversive internal state (K_2) increase in relevance as the simulation proceeds, indicating greater reliance on K_2 as a regulating mechanism for aversive internal state.

4 Conclusion and Future Work

Results from a sensitivity analysis of the General Escape model of suicidal thoughts by Wang et al. [15] highlight the importance of parameters governing self-feedback loops, carrying capacities for aversive internal states, and the downregulation of cascading effects of stressors on escape behaviors and suicidal thoughts. The results also indicate that the relative influence of these parameters are stable throughout the simulation. Interaction effects between aversive internal state and the urge to escape were also found to greatly impact the model output. Future research should validate these model findings with clinical data and explore individual differences to refine the model's applicability to intervention strategies.

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