Natural convection in periodically heated porous-fluid systems under local thermal non-equilibrium conditions: a numerical study for enhanced thermal management

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Abstract. This numerical study investigates natural convection and heat transfer in a closed chamber with porous medium. The system combines a porous layer and a Newtonian fluid with temperature-dependent viscosity, subjected to time-dependent thermal excitation. Governing equations, formulated in dimensionless stream function and vorticity variables, integrate mass, momentum, and energy conservation using the Darcy-Brinkman model and Boussinesq approximation. The LTNE framework resolves thermal decoupling between the porous matrix and fluid, overcoming limitations of local thermal equilibrium assumptions. A finite difference numerical scheme is employed to solve the dimensionless equations, analyzing the interplay of LTNE parameters (interphase heat transfer, parameters of solid structure) and periodic heating (frequency, amplitude). Results demonstrate that LTNE conditions significantly alter thermal stratification, velocity asymmetry, and heat transfer rates (with the help of the Nusselt number). Elevated heating frequencies suppress convective instabilities, while variable viscosity amplifies thermal gradients. The porous-fluid conductivity ratio critically modulates thermal non-equilibrium, with lower ratios exacerbating temperature disparities. This work validates the necessity of LTNE models for systems involving rapid thermal transients, heterogeneous media, or variable properties. The findings provide critical insights for optimizing thermal management in energy storage, electronic cooling, and geothermal systems.

Keywords: Free convection, Closed rectangle, FDM, Periodically heating, Porous-fluid interaction, Local thermal non-equilibrium conditions.

1 Introduction

Natural convection in porous-fluid systems remains a critical area of research due to its applications in geothermal energy extraction, thermal storage, and advanced electronics cooling [1]. While classical studies often rely on local thermal equilibrium (LTE) assumptions, recent advances emphasize the necessity of local thermal non-equilibrium (LTNE) models to resolve thermal decoupling in dynamic systems. The investigation of natural convection in porous-fluid systems under local thermal non-equilibrium (LTNE) conditions and variable viscosity builds upon foundational and

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contemporary research. Rees and Pop [2] resolving thermal decoupling between fluid and solid phases in porous media, and demonstrated how interphase heat transfer parameters govern temperature disparities – an important problem for modeling systems with rapid thermal transients. Expanding on variable property effects, Khanafer and Vafai [3] revealed that temperature-dependent viscosity amplifies thermal gradients and flow asymmetry, underscoring the necessity of property variation models. Saeid and Pop [4] further explored periodic heating in porous enclosures, identifying frequency-dependent suppression of convective instabilities and synchronization between heating cycles and flow oscillations, aligning with the abstract's findings on γ-driven dynamics.

For energy storage applications, Baytas et al. [5] applied LTNE to solar-thermal storage systems, demonstrating thermal disequilibrium dominance under high-frequency excitation and optimizing conductivity ratios to mitigate temperature disparities. Replacing the original fifth reference, Dehghan et al. [6] advanced numerical methodologies by employing a multiscale approach to analyze LTNE in porous cavities with sinusoidal heating, validating computational frameworks for oscillatory thermal loads. Finally, Xiong et al. [7] integrated machine learning with LTNE models, achieving a 20% improvement in predicting interfacial heat transfer and supporting the need for advanced parameterization in dynamic systems.

This study bridges these gaps by analyzing natural convection in a porous-fluid cavity with variable viscosity under periodic heating, employing an LTNE model and dimensionless formulation. The work advances prior research by coupling LTNE with temperature-dependent viscosity, quantifying frequency-amplitude-phase interactions in thermal non-equilibrium. The findings address actual needs in next-generation thermal management, such as high-power electronics and phase-change materials, where rapid transients and material heterogeneity demand precise modeling.

2 The problem description

The schematic of convective heat transfer in the studied cavity is illustrated in Figure 1. The vertical walls are maintained at a constant cold temperature, while the horizontal boundaries are thermally insulated. A thermally active heat source with time-dependent heat generation $Q(t)=0.5Q_0\{1-\cos(ft)\}$ is embedded at the center of the lower boundary (f is volumetric thermal production frequency, t is dimensional time).

The flow is assumed laminar, with the working fluid exhibiting temperature-dependent viscosity $\{\mu=\exp(-\zeta\theta)\}$ with constant ζ [8]. The fluid is Newtonian, heat-conducting, and governed by the Boussinesq approximation. Contrary to traditional local thermal equilibrium (LTE) assumptions, this study employs a local thermal non-equilibrium (LTNE) model with fluid/solid matrix interface parameter ξ (Nield number), where different temperature fields are resolved for the fluid phase (T_f) and the porous solid matrix (T_s) within the porous part of the cavity. The porous matrix is isotropic, homogeneous, and permeable, allowing fluid penetration, while all external walls are impermeable. The governing equations adopt the transient Brinkman-

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extended Darcy model, coupled with separate energy conservation equations for the fluid and solid phases to account for interfacial thermal resistance and heat exchange.

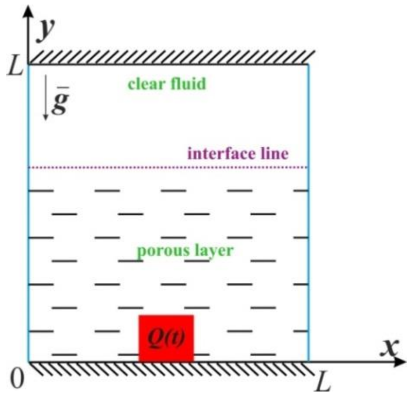

Fig. 1. The configuration of considered system.

2.1 The mathematical model

The mathematical model has developed based on the classical Navier-Stokes equations, transformed using dimensionless variables "stream function" and "vorticity" [8]:

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}}; \; \bar{\omega} = \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}}$$

The governing equations have been formulated and solved separately for each phase: the pure fluid domain, the porous layer, and the energy source. The constitutive equations in selected variables describing the process under consideration have been written in the same way as in the case of the problem of natural convection of a fluid of variable viscosity in a closed 2D cavity with a heat-generating energy source on the bottom wall of the cavity and a porous layer (in the LTNE framework) [8].

The system of equations for the part with clear fluid can be written as follow:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{1}$$

$$\begin{aligned} &\frac{\partial \omega}{\partial \tau} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \sqrt{\frac{Pr}{Ra}}\left(\frac{\partial^2(\mu\omega)}{\partial x^2} + \frac{\partial^2(\mu\omega)}{\partial y^2}\right) + \frac{\partial \theta}{\partial x} + \\ &+2\sqrt{\frac{Pr}{Ra}}\left[\frac{\partial^2 \mu}{\partial x^2}\frac{\partial u}{\partial y} - \frac{\partial^2 \mu}{\partial y^2}\frac{\partial v}{\partial x} + \frac{\partial^2 \mu}{\partial x \partial y}\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\right] \end{aligned} \tag{2}$$

$$\frac{\partial \theta}{\partial \tau} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{Ra \cdot Pr}}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{3}$$

The system of equations for the porous part of the cavity can be written as follow:

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$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\omega \tag{4}$$

$$\begin{gathered}\varepsilon\frac{\partial\omega}{\partial\tau}+u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\varepsilon\sqrt{\frac{Pr}{Ra}}\left(\frac{\partial^2(\mu\omega)}{\partial x^2}+\frac{\partial^2(\mu\omega)}{\partial y^2}-\varepsilon\frac{\mu\omega}{Da}\right)+\varepsilon^2\frac{\partial\theta_f}{\partial x}+\\+2\varepsilon\sqrt{\frac{Pr}{Ra}}\left[\frac{\varepsilon u}{2Da}\frac{\partial\mu}{\partial y}-\frac{\varepsilon v}{2Da}\frac{\partial\mu}{\partial x}+\frac{\partial^2\mu}{\partial x^2}\frac{\partial u}{\partial y}-\frac{\partial^2\mu}{\partial y^2}\frac{\partial v}{\partial x}+\frac{\partial^2\mu}{\partial x\partial y}\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\right]\end{gathered} \tag{5}$$

$$\varepsilon\frac{\partial\theta_f}{\partial\tau}+u\frac{\partial\theta_f}{\partial x}+v\frac{\partial\theta_f}{\partial y}=\frac{\varepsilon}{\sqrt{Ra\cdot Pr}}\left(\frac{\partial^2\theta_f}{\partial x^2}+\frac{\partial^2\theta_f}{\partial y^2}\right)+\frac{\xi}{\sqrt{Ra\cdot Pr}}\left(\theta_s-\theta_f\right) \tag{6}$$

$$(1-\varepsilon)\frac{\partial\theta_s}{\partial\tau}=\frac{(1-\varepsilon)\Lambda\gamma}{\sqrt{Ra\cdot Pr}}\left(\frac{\partial^2\theta_s}{\partial x^2}+\frac{\partial^2\theta_s}{\partial y^2}\right)+\frac{\xi\gamma}{\sqrt{Ra\cdot Pr}}\left(\theta_f-\theta_s\right) \tag{7}$$

The energy equation for the periodic heat release heater is written as follows:

$$\frac{\partial\theta}{\partial\tau}=\frac{\alpha_{hs}/\alpha_f}{\sqrt{Ra\cdot Pr}}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\lambda_f}{2\lambda_{hs}}\{1-\cos(\gamma\pi\tau)\}\right) \tag{8}$$

In this equation θ – dimensionless temperature; τ – dimensionless time; λ_f,λ_{hs} are thermal conductivity of working fluid and heater; α_f,α_{hs} are thermal diffusivity of working fluid and heater; γ – oscillation frequency of the volumetric heat generation; $\Lambda=\lambda_s/\lambda_f$ is the thermal conductivity ratio (between liquid and solid matrix); $Ra=\rho g\beta\Delta TL^3/(\alpha\mu_0)$ – Rayleigh number; $Pr=\mu_0/(\rho\alpha)$ – Prandtl number.

At the initial instant of time, the fluid in the cavity is stationary and all variables are equal zero. The boundary conditions correspond to the problem [8].

2.2 The solution methodology

The governing equations along with initial and boundary conditions have been solved numerically using a finite difference method on a uniform grid [8]. A second-order accurate discretization scheme has been applied to approximate convective and diffusive terms. Parabolic equations have been solved via Samarskii's locally one-dimensional scheme, while the resulting linear systems have been addressed using the Thomas algorithm. The stream function equations have discretized via a five-point stencil based on central differencing for second derivatives. To solve the resulting algebraic equations, the successive over-relaxation method has been employed, with the optimal relaxation parameter determined through numerical experimentation to ensure convergence efficiency. The final realization of the solution method has been made by development of home-made program code in C++ language.

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A grid sensitivity analysis has been conducted to assess the convergence behavior of the numerical solution. Five grid resolutions have been tested: 50×50, 100×100, 200×200 and 300×300 nodes. Figure 2 illustrates the temporal evolution of the average temperature within the periodically heated element for these grids under fixed dimensionless parameters (Pr=7.0, Ra=10^5, Da=10^{-4} (Darcy number), ε=0.8 (porosity of porous structure), ξ=100.0, ζ=1.0, γ=0.05, δ=0.5 (the dimensionless height of the porous layer)). The results exhibit periodic temperature oscillations synchronized with the heat source's time-dependent power law. Minimal deviations have been observed between the 200×200 grid and smaller grids 300×300, confirming sufficient spatial accuracy. Consequently, the 200×200 grid has been selected for subsequent simulations to balance computational efficiency and precision.

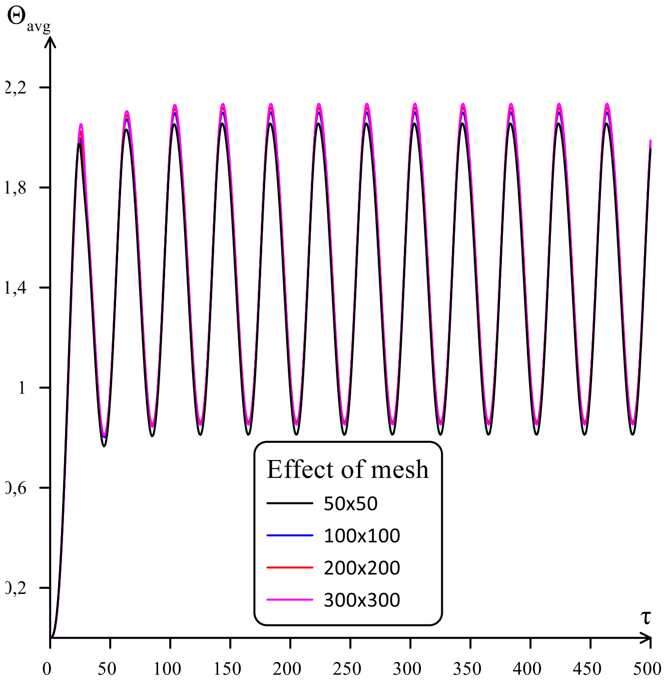

Fig. 2. Influence of grid parameters on time dependences of average source temperature.

It is worth noting, however, that the periodic heating from the source is directly reflected in the distribution of the average temperature at the source. The fluctuating values are directly correlated with the heating period of the source, and reflect the periodic heating load on the system.

Figure 3 shows the distributions of the stream function and temperature function for fixed values of Rayleigh, Darcy and Prandtl numbers. The isolines illustrate the formation of convective flow in the cavity. The convective flow is initiated by the action of the temperature difference between the vertical boundaries of the region. The data obtained on the basis of the developed numerical technique (Figure 3*b*) illustrate good agreement with the results of [9].

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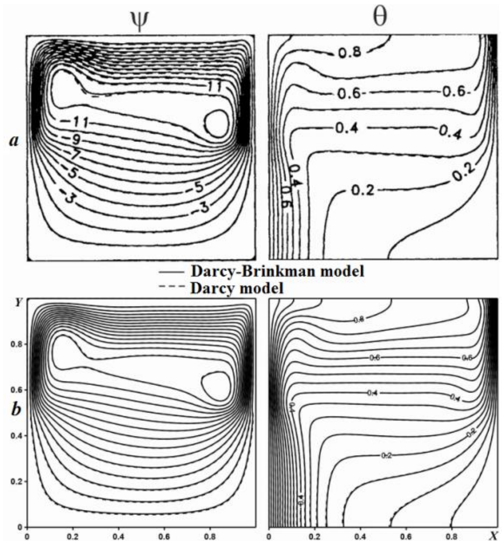

Fig. 3. Stream function and isotherms at Ra=10^6, Da = 10^{-5}, Pr = 0.71.

3 Results

Numerical calculations for the problem have been performed for different thermo-physical and geometrical properties of the porous skeleton, energy source, interfacial interaction, and for fluid with constant and variable viscosity. According to the obtained distributions of isolines of the stream function and temperature, as well as the integral characteristics of flow and heat transfer, the influence of the determining parameters on the operation of a thermal system with a periodic type of heating has been analyzed. The following ranges of determining dimensionless parameters have been considered: Pr=7.0, Ra=10^5, ε=0.8, Da=10^{-4}–10^{-3}, ξ=10.0–1000.0, ζ=0.0–1.0, γ=0.01–0.1, δ=0.0–1.0.

Figure 4 shows the time-dependences of the integral characteristics of heat transfer and flow (mean source temperature, maximum fluid flow rate in the cavity, mean number at the energy source surface) for different values of Darcy number (Da) and viscosity change parameter (ζ) for ξ=10.0, γ=0.1, δ=0.5. It should be noted that at ζ=0 we consider the case of a fluid of constant viscosity, and at ζ=1 we consider a fluid with variable viscosity (due to the chosen law of viscosity dependence). The obtained distributions confirm the obtained data in Figure 2. In this case the period of oscillation from the source γ=0.1, i.e. more in comparison with Figure 2, where γ=0.05. An increase in the parameter γ results in a reduction of both the oscillation period and amplitude. Notably, the maximum temperature and flow intensity are observed at the lowest γ value, while the minimum values occur at the highest γ. An increase in the permeability of the porous layer (Da) leads to a decrease in the temperature of the source by increasing the flow in the cavity and the intensity of fluid mixing in the cavity (Figure 3*b*).

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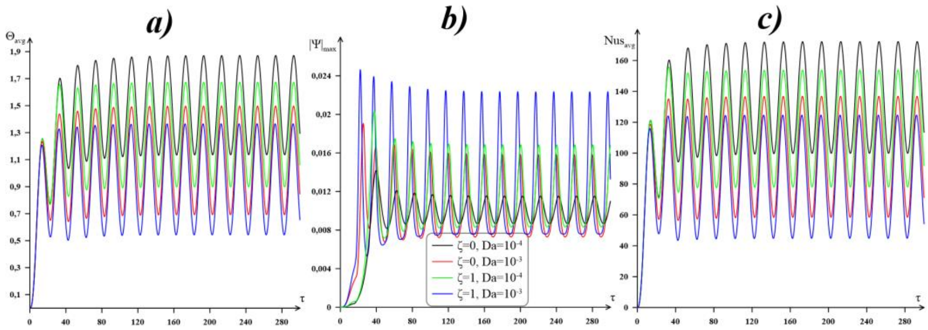

Fig. 4. Influence of Darcy number and viscosity change parameter on integral characteristics.

The minimum heat load is observed in the case of intermittent viscosity and maximum Darcy number. Taking into account the dependence of fluid viscosity on temperature shows more realistic results, since in thermal systems the thermophysical properties of the fluid strongly depend on the properties of the environment.

Figure 5 illustrates the evolution of fluid flow within the cavity over one oscillation cycle of the heater under fixed dimensionless parameters. The flow dynamics are analyzed for time step τ=275. This point in time is reflected in the maximum heat load of the system, according to the periodic heating law from the heating element on the bottom wall of the cavity. The case of variable viscosity (dashed lines) characterizes by more intensive flow and heat dissipation from the heater due to the reduction of viscous forces in the fluid due to the adopted law of viscosity change.

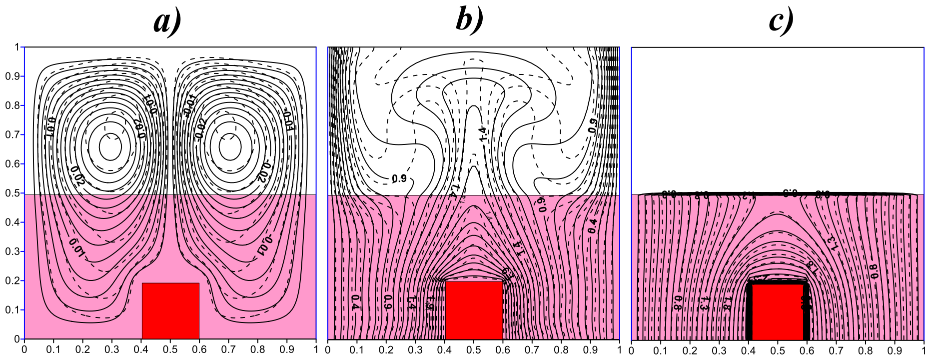

Fig. 5. Isolines of the stream function (*a*) and isolines of fluid (*b*) and solid structure (*c*) temperature for Ra=10^5, ξ=1000, γ=0.1, Da=10^{-3}, ζ=0 (solid lines), ζ=1 (dashed lines) for τ=275.

The results reveal the development of a time-dependent symmetric flow pattern, driven by the thermal gradient between the heating element and the cooled sidewalls. A peak in heat transfer enhancement occurs at the mid-period stage ($T_\tau/2$, τ=275), where fluid motion is most pronounced. Additionally, the porous structure within the cavity acts as a thermal energy accumulator, moderating heat distribution during the oscillatory process. This non-steady flow regime underscores the interplay between thermal forcing and energy storage in the system.

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4 Conclusion

In the present study numerical modeling of convective heat transfer processes in a closed two-dimensional partially porous cavity in the presence of a source of periodic heat release has been carried out. In addition, a local thermal non-equilibrium model has been used to describe the thermal interaction between the porous layer and the fluid. Key findings reveal that the oscillation frequency parameter γ exerts dominant control over the system's dynamic response: higher γ values suppress convective instabilities, reduce oscillation amplitudes, and diminish flow intensity, while lower γ amplifies thermal gradients and maximizes heat transfer rates. The Darcy number (Da) emerges as a pivotal factor governing porous-fluid interaction, with increased permeability enhancing fluid mixing and reducing source temperatures. Notably, variable viscosity (ζ) introduces secondary effects, elevating flow intensity but underscoring the importance of temperature-dependent property modeling for realistic thermal systems.

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Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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