Modelling Heat Conduction between Two Contacting Particles in Vacuum Insulation Panels Made with Granular Porous Media

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Abstract. Solid thermal conduction is the dominant heat transfer mechanism for Vacuum Insulation Panels (VIPs) made with granular porous media, especially under vacuum conditions. Researchers tend to analyse heat transfer using numerical methods for granular porous media with complicated structures. However, the minimal contacts between spherical particles introduce significant complexity to numerical methods to solve heat transfer problems. To improve computational efficiency, this study quantitatively investigates the relationship between heat conduction and the contact radius between two contacting particles. A more accurate closed-form relation is proposed, and a scaling model is built to address the limitations of existing works that suffer from systematic errors and limited applicability. Compared to oversimplified relations in existing studies, numerical validation demonstrates that the proposed scaling model in this study achieves a maximum error of less than 1% in the range of 1 / 1000 to 1, significantly and fundamentally improving the predictive accuracy. This work provides a theoretical foundation for computing conductive heat transfer effectively in future studies in multiparticle systems.

Keywords: Effective thermal conductivity · Granular porous media · Vacuum Insulation Panels · Solid conduction.

1 Introduction

Vacuum Insulation Panels (VIPs) are high-performance thermal insulators with a porous core sealed in a gas-impermeable barrier, minimizing heat transfer via vacuum [1]. Due to their low thermal conductivity and structural adaptability, granular porous media, such as fumed silica [2, 3], aerogels [4, 5], and glass beads [6–8], are key core materials.

For granular porous media, the contact radius between particles is typically minimal, often 1% of the particle radius or less [6]. These numerous, minuscule contacts pose significant challenges for numerical analysis, as they complicate the

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mesh and drastically increase computational complexity. If a quantitative relationship between the contact radius ratio and the conductive heat flux in chain-like structures can be established, researchers could apply numerical methods to an equivalent physical structure with a larger contact radius and subsequently scale the computed results to obtain the actual conductive heat flux.

However, existing studies on this quantitative relationship exhibit notable limitations. Based on Batchelor's [9] and Zhu's [10] work, this study investigates the relationship between the radius of contact of the sphere and the effective thermal conductivity (ETC) of the system in a two-touching-sphere structure. This study proposes a new scaling model that enables efficient and accurate computation of conductive heat transfer between two touching spheres. Using the scaling model proposed in this study, researchers can input a simplified structure with a larger contact radius into the finite element analysis and subsequently apply the scaling model to obtain the actual thermal conductivity of conduction.

2 Existing work and limitations

For a chain structure formed by two or more overlapping spheres, assuming one-dimensional steady-state heat conduction exists, as illustrated in Figure 1, one can intuitively infer a positive correlation between the overlap radius of the particles and the effective thermal conductivity λ_e of the structure: the larger the overlap radius, the higher the effective thermal conductivity, and vice versa.

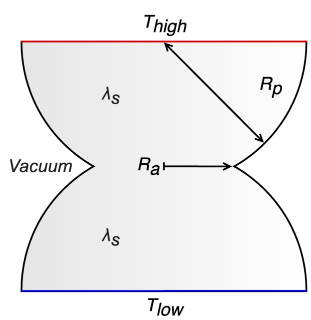

Fig. 1. Definition sketch of Batchelor's models and Zhu's model

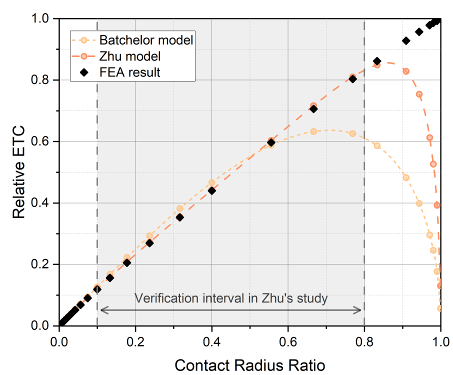

Fig. 2. Relative ETC of numerical model (FEA result), Batchelor's model and Zhu's model

Firstly, in 1976, Batchelor [9] proposed that the effective thermal resistance R_e of the system can be estimated as

$$R_e \approx \frac{1}{2\lambda_s r_a} \tag{1}$$

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where λ_s is the thermal conductivity of solid phase and r_a is the contact radius. Then, the thermal conductivity can be derived as:

$$\lambda = \frac{\Delta x}{A \cdot R} \tag{2}$$

where Δx is the thickness of the system (measured on a path parallel to the heat flow), and A is the cross-sectional area perpendicular to the path of heat flow. Using Equation 1 and Equation 2, λ_e, the effective thermal conductivity can be calculated as:

$$\lambda_e \approx \frac{2\sqrt{r_p^2 - r_a^2}}{R_e \cdot \pi r_p^2} = \frac{2\sqrt{r_p^2 - r_a^2}}{\frac{1}{2\lambda_s r_a} \cdot \pi r_p^2} \tag{3}$$

where r_p is the particle radius. However, no error analysis was performed for the approximation in his study.

In 2021, Zhu [10] improved the calculation of effective thermal conductivity λ_e from Batchelor's study. Zhu found that Equation 3 only shows good agreement with the numerical verification results when the contact radius ratio $k_r = \frac{r_a}{r_p}$ lies in the range between 0.1 to 0.5. When $k_r > 0.5$, Batchelor's model significantly underestimated the effective thermal conductivity.

Based on this observation, Zhu deduced that this error is caused by the ignorance of the thermal resistance of two touching hemispheres at large k_r. Zhu then proposed a correlation function to adjust the total thermal resistance:

$$R_{zhu} \approx \frac{1}{1 + 0.605k_r - 1.1713k_r^2} \cdot R_e \tag{4}$$

and derived the effective thermal conductivity as:

$$\lambda_{zhu} \approx \frac{2\sqrt{r_p^2 - r_a^2}}{R_{zhu} \cdot \pi r_p^2} = \frac{2\sqrt{r_p^2 - r_a^2}}{\frac{1}{1+0.605k_r - 1.1713k_r^2} \cdot \frac{1}{2\lambda_s r_a} \cdot \pi r_p^2} \tag{5}$$

Equation 5 shows good agreement between the predictive value and the numerical results when k_r lies in the range between 0.1 to 0.8. Unfortunately, Zhu did not detail the line of thought, nor did he validate his correlation function when $k_r > 0.8$ and $k_r < 0.1$. If Zhu had conducted such validations, he would found that his correlation function does not correct the systematic errors in Batchelor's model (as illustrated in Figure 2). The effective thermal conductivity predicted by Zhu's model still significantly underestimates the value after $k_r > 0.8$ due to the inherent defects.

In Figure 2, the vertical axis represents the relative value λ^* of each result relative to the solid phase thermal conductivity λ_s, given by:

$$\lambda^* = \frac{\lambda_e}{\lambda_s} \tag{6}$$

In conclusion, Batchelor's model provides reasonably good results for the two-touching-spheres structure when the contact radius ratio k_r is small. As the

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contact radius ratio increases, Batchelor's model exhibits a significant systematic error, notably underestimating the effective thermal conductivity of the system. Zhu tried but did not address this systematic error. Furthermore, when the contact radius ratio is small, Batchelor's model demonstrates an approximately 9% error, which remains around 2% after the improvements proposed by Zhu.

3 Proposed scaling model

To investigate the relationship between effective thermal conductivity and radius contact ratio, this study employs an axisymmetric three-dimensional periodic structure to calculate the numerical results of effective thermal conductivity λ_e. The cross-section of the three-dimensional axisymmetric model is shown in Figure 3, where R_p is the particle radius, R_a is the contact radius, and $2d$ is the distance from the hot plate to the cold plate.

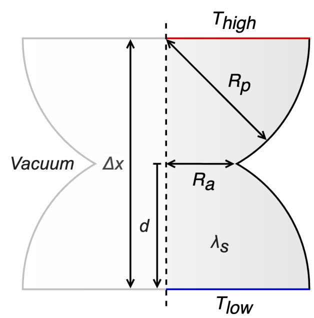

Fig. 3. Definition sketch of the present model

Assuming the two-touching-spheres structure is at the centre of a panel with a thickness of $D = 2$ cm, an initial temperature of $T = 25°$C, and the temperature difference between the hot side and the cold side of the panel is $T_D = 2°$C.

The temperature of the hot side of the two-touching-spheres structure is

$$T_{high} = T + \frac{d}{D} \cdot T_D \tag{7}$$

The temperature of the cold side of the two-touching-spheres structure is

$$T_{low} = T - \frac{d}{D} \cdot T_D \tag{8}$$

The effective thermal conductivity is:

$$\lambda_e = \frac{\mathcal{H} \cdot 2d}{\Delta T} \tag{9}$$

where $\mathcal{H}$ is the normal heat flux and

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$$\Delta T = T_{high} - T_{low} \tag{10}$$

By reviewing Batchelor's study, this study found that Batchelor assumed that two spheres were in slight contact. In this case, Δx, the thickness of the system (the length parallel to the heat flow):

$$\Delta x = 2d = 2\sqrt{r_p^2 - r_a^2} \tag{11}$$

remains almost unchanged and can be approximately regarded as a constant.

Under this assumption, the effective thermal resistance only depends on the contact radius ratio k_r. However, when the contact radius ratio k_r increases to 0.5 or higher, the structure's thickness decreases rapidly, and Δx cannot be considered a constant anymore. Therefore, the effective thermal conductivity of the system is no longer solely determined by the contact radius ratio k_r, it is also affected by the changing Δx.

Therefore, this study modifies Equation 1 to:

$$R_e' = \frac{1}{2\lambda_s r_a} \cdot \frac{2d}{r_p} = \frac{1}{2\lambda_s r_a} \cdot \frac{2\sqrt{r_p^2 - r_a^2}}{r_p} \tag{12}$$

Then, Equation 3 is updated to:

$$\lambda_e' = \frac{4}{R_e \cdot \pi r_p} = \frac{4}{\pi} k_r \lambda_s \tag{13}$$

The above equation no longer predicts a significantly decreasing thermal conductivity value when k_r is large. However, it overestimates the thermal conductivity of the structure at large k_r. For example, when $k_r = 1$, the thermal conductivity of the system should be equivalent to the thermal conductivity of the solid phase λ_s, but the result given by the formula is:

$$\lambda_e' = \frac{4}{\pi} \cdot \lambda_s \approx 1.27 \cdot \lambda_s \tag{14}$$

This is obviously inconsistent with reality. Observing λ_e' and λ_{exp}, this study finds that they have an approximate proportional magnification relationship of $\frac{\pi}{4}$. Therefore, the correction approach is to first multiply λ_e' by $\frac{\pi}{4}$ to obtain λ_e'' (as shown in Equation 15) to correct the predicted value of thermal conductivity when k_r is large.

$$\lambda_e'' = \frac{\pi}{4} \cdot \lambda_e' \tag{15}$$

Observing λ_e'' and λ_{exp}, this study finds that λ_{exp} is not a strictly linear function. To improve accuracy, λ_e'' is further corrected to obtain the final result of the current model, λ_e''', given by:

$$\lambda_e''' = \left(\left(1 - \sqrt{k_r} \right) \cdot \left(\frac{4}{\pi} - 1 \right) + 1 \right) \cdot \lambda_e'' \tag{16}$$

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which can be rewritten as:

$$\lambda_e''' = f(k_r) \cdot \lambda_s = \left(\left(1 - \sqrt{k_r}\right) \cdot \left(\frac{4}{\pi} - 1\right) + 1 \right) \cdot \lambda_e \tag{17}$$

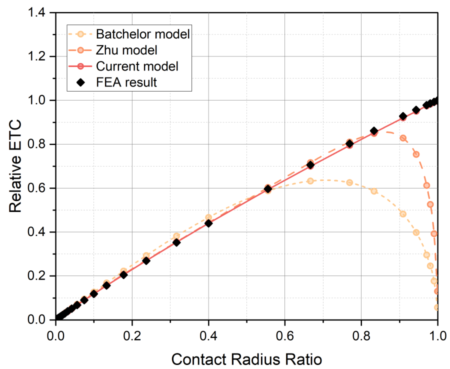

Fig. 4. Relative ETC results of Batchelor's model, Zhu's model, current model and numerical model (FEA result)

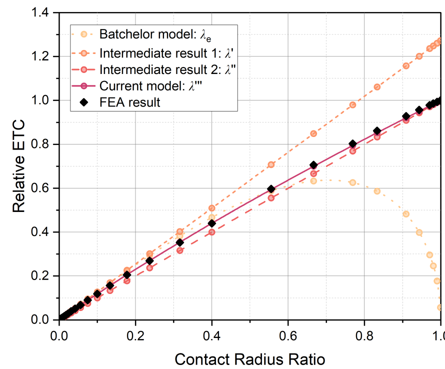

Fig. 5. Relative ETC results of intermediate steps and numerical model (FEA result)

The numerical results of the thermal conductivity of the two-touching-sphere structure, the Batchelor model result, the Zhu model result, and the current model result are shown in Figure 4. The comparison between the results of the intermediate steps and the numerical result is presented in Figure 5.

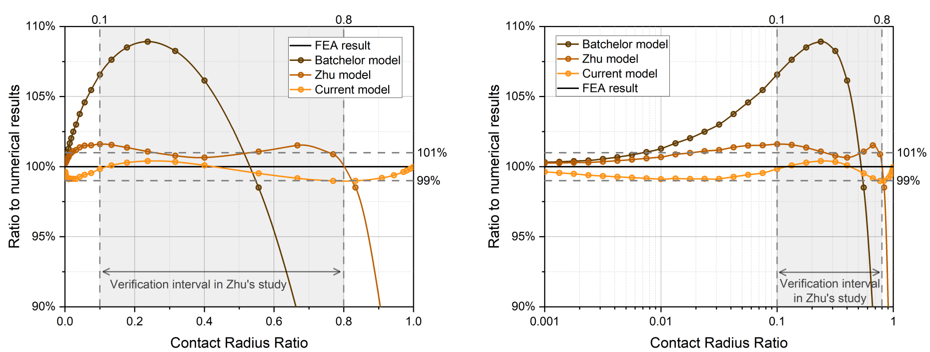

Fig. 6. Error analysis between numerical model (FEA results) and current model, Batchelor's model and Zhu's model in (left) linear scale (right) logarithmic scale

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4 Results

This study performed an error analysis of Batchelor's model, Zhu's model, and the current proposed model, with the results presented in Figure 6. The model proposed in this study effectively addresses the systematic error existing in Batchelor's model and Zhu's model when k_r is large.

In further evaluations, the proposed scaling model was tested across a wide span of contact radius ratios, from extremely small partial overlaps ($k_r < 0.01$) to complete overlap ($k_r = 1$). Through multiple finite element simulations, the proposed model achieves a prediction error of less than 1% across the entire range of k_r, representing a significant advancement over previous studies and highlighting its robustness. This accuracy persisted under different boundary conditions and material properties.

5 Application

Taking advantage of the non-linear relationship proposed in this study (Equation 17), suppose there is a granular porous media with the actual contact radius ratio k_r, and let k_r' denote the contact radius ratio applied to the numerical model, which is greater than k_r. Equation 18 and Equation 19 can be derived:

$$\lambda(k_r) = f(k_r) \cdot \lambda_s \tag{18}$$

$$\lambda(k_r') = f(k_r') \cdot \lambda_s \tag{19}$$

where $f(k)$ is the scaling factor in Equation 17, and λ_s is the thermal conductivity of the solid phase. Then, Equation 20 can be derived:

$$\lambda(k_r) = \frac{f(k_r)}{f(k_r')} \cdot \lambda(k_r') \tag{20}$$

Equation 20 indicates that one can obtain $\lambda(k_r)$, the effective thermal conductivity of a structure with the actual contact radius ratio k_r, by multiplying $\lambda(k_r')$, the effective thermal conductivity for a model with a larger contact radius ratio k_r', by the scaling factor $\frac{f(k_r)}{f(k_r')}$.

6 Conclusion

This study proposes an innovative scaling model that accurately captures heat conduction between two contacting spheres in vacuum insulation panels composed of granular porous media. By eliminating the systematic errors observed in previous analytical models, this study reduces the maximum prediction discrepancy to below 1%. The refined approach significantly improves computational efficiency, as a simplified geometry with larger contact surfaces can be simulated and then rescaled to represent actual microscopic contacts. This innovation is

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particularly crucial for heat transfer modelling of granular porous media, where minimal particle contacts drive complex thermal pathways, posing substantial challenges for conventional finite element analyses. Compared to earlier methodologies, our model consistently demonstrates superior predictive accuracy and robustness over a wide range of contact radius ratios. This study offers a powerful tool for researchers and engineers, facilitating the design of advanced VIP cores and enhancing the reliability of heat transfer predictions in multiparticle systems.

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