Local Markovian Consensus for Ranking Aggregation: A Novel Approach to Weak Ordinal Dominance

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Abstract. This paper introduces LMC-WOD (Local Markovian Consensus with Weak Ordinal Dominance), an extension of the LMC method designed to aggregate rankings that include ties and inconsistent pairwise preferences. The approach builds a local Markov chain based on direct dominance relations and computes a stationary distribution to represent the collective consensus. Unlike global methods, LMC-WOD relies on local interactions and supports partial and weak orders. It handles transitivity violations, reflects local consensus, and remains robust to small perturbations. Two case studies illustrate its behavior under consistent and conflicting inputs, with and without ties. A comparison with PageRank shows how each method propagates influence differently across alternatives. Results confirm that LMC-WOD can capture both strong and weak ordinal patterns and highlight its potential for applications where standard ranking assumptions do not hold. Its sensitivity to highly diverse rankings and the use of a fixed damping factor highlights areas for further investigation.

Keywords: Consensus Ranking · Weak Ordinal Dominance · Markov Stationary State · MCDA

1 Introduction

Ranking aggregation is a fundamental task in fields where decisions must be derived from multiple, often conflicting, preferences, such as expert assessments, survey data, elections, and peer evaluations [10]. In many practical scenarios, input rankings may contain ties, partial orders, or inconsistencies that challenge traditional aggregation methods. These issues arise in diverse domains, including recommendation systems, group decision-making, and multi-criteria evaluations.

To address these challenges, this paper proposes a local stochastic approach to ranking aggregation that preserves ordinal structure while accommodating weak and conflicting assessments.

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DOI: 10.1007/978-3-031-97567-7_29

1.1 Research Gap and Motivation

Existing aggregation methods are generally categorized into majority-based and distance-based approaches. Majority-based methods seek consensus through frequent pairwise preferences, such as Kemeny–Young [1, 5, 7]. Distance-based methods, including VIKOR [12], minimize aggregate deviations between rankings. Fuzzy aggregation techniques extend this landscape by incorporating uncertainty [8, 13].

However, these global approaches often overlook local structures within rankings. Majority-based methods assume disagreement resolution via frequency, potentially missing dependency patterns among alternatives. Distance-based methods treat alternatives as isolated units in metric space, neglecting relational influence. Strict ranking assumptions artificially separate alternatives that may be equally preferred. This highlights a need for methods that support local consensus and allow ties to be made naturally.

1.2 Proposed Approach and Contribution

This study introduces a novel extension to the Local Markovian Consensus (LMC) method, referred to as LMC-WOD, which incorporates weak ordinal dominance to support rankings with ties. The algorithm evaluates how often an alternative is locally outranked across rankings, without requiring global majority or distance measures.

Unlike the previous version of LMC [9], the proposed method accounts for relational dependencies while preserving the natural ambiguity in real-world preference data. This is especially relevant when evaluators assign equal standing to alternatives, such as performance assessments, competitions, or peer reviews.

Through case studies and comparative analysis with the PageRank algorithm, the paper demonstrates how incorporating local dominance and stochastic influence yields more nuanced rankings. These insights underscore the limitations of conventional aggregation strategies and validate the proposed method's adaptability in contexts where preference uncertainty and dependency structures are prominent.

2 Related Work

Rank aggregation has been extensively studied, with numerous methods leveraging Markovian frameworks to derive consensus rankings from multiple sources. A seminal approach by Dwork et al. [6] introduced several transition models to generate consensus rankings. Subsequent models, such as the Uniform Transition Model (MC1), Pairwise Preference Model (MC2), Prior-Based Model (MC3), and Weighted Rank Model (MC4), have refined the definition of transition probabilities to capture different forms of preference relationships.

PageRank-inspired methods have also been employed, treating rankings as directed graphs and computing stationary distributions to aggregate input preferences. For example, Rank Centrality [11] applies pairwise comparisons within a

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DOI: 10.1007/978-3-031-97567-7_29

Markov chain framework to infer consensus rankings under noise. Wu et al. [14] proposed a method based on absorbing Markov chains, where transitions are influenced by preferences derived from multiple input rankings.

More recent advances have introduced novel approaches to address the limitations of classical methods. FairMC [2] proposes a fairness-aware Markov chain-based rank aggregation method to ensure equitable representation in the final consensus across the groups. Graph-based rank aggregation methods [15] introduce the ratio of out- and in-degrees concept to deal with partial rankings. Link prediction-based aggregation [4] has also emerged as a promising paradigm for settings with limited or partial ranking information.

3 Preliminaries

This section introduces the notation and concepts related to ranking sequences, ordinal dominance, Markov chains, and their stationary states.

Let A denote the set of alternatives (e.g., options, strategies, or candidates), with cardinality $|A| = m$.

Definition 1. *A full ranking* $\mathbf{r}$ *is a vector representing the order of* m *alternatives:*

$$\mathbf{r} = [r(a_1), r(a_2), ..., r(a_m)]; \quad where \quad r(x) \in \{1, \ldots, z\} \wedge a_i \in A.$$

Each $r(a_i)$ *is an integer indicating the rank of alternative* a_i*, where a rank of* 1 *denotes the highest preference, and* $z = \max\{r(a_i)\}$ *is the lowest (worst) rank. The length of the ranking vector is* $|\mathbf{r}| = |A| = m$*.*

Assume an ordinal preference over the alternatives is given for each $r(a_i)$. If $r(a_i) < r(a_j)$, then the alternative a_i is strictly preferred to a_j or equivalently, strictly ordinally dominates a_j. If $r(a_i) \leq r(a_j)$, then a_i is at least as preferred as a_j or weakly ordinally dominates a_j.

Definition 2. *A set of* n *rankings over a fixed set of* m *alternatives* $A = \{a_1, a_2, \ldots, a_m\}$ *is defined as:*

$$\mathbf{R} = \{\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n}\},$$

where each $\mathbf{r_i}$ *is a full ranking given by the vector:*

$$\mathbf{r_i} = [r_i(a_1), r_i(a_2), \ldots, r_i(a_m)],$$

and $r_i(a_j)$ *denotes the rank assigned to alternative* a_j *in the* i*-th ranking.*

The set $\mathbf{R}$ can be equivalently represented as an $n \times m$ ranking matrix:

$$\mathbf{R} = \begin{array}{c} \\ \mathbf{r_1} \\ \mathbf{r_2} \\ \ldots \\ \mathbf{r_n} \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & \ldots & a_m \end{array} \\ \begin{bmatrix} r_1(a_1) & r_1(a_2) & \ldots & r_1(a_m) \\ r_2(a_1) & r_2(a_2) & \ldots & r_2(a_m) \\ \ldots & \ldots & \ldots & \ldots \\ r_n(a_1) & r_n(a_2) & \ldots & r_n(a_m) \end{bmatrix} \end{array} \tag{1}$$

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Definition 3. *A Markov chain is a discrete-time stochastic process* $\{X_t\}_{t=0}^{\infty}$ *defined on a finite state space* $A = \{a_1, a_2, \ldots, a_m\}$*, representing a set of alternatives. It satisfies the Markov property:*

$$\Pr(X_{t+1} = a_j \mid X_1 = a_{i_1}, X_2 = a_{i_2}, \ldots, X_t = a_i) = \Pr(X_{t+1} = a_j \mid X_t = a_i),$$

for all $t \geq 0$ *and* $a_i, a_j \in A$*. The probability of transitioning to the next alternative depends only on the current one.*

The transitions between alternatives are encoded in the *transition probability matrix* $\mathbf{P} = (p_{ij}) \in \mathbb{R}^{m \times m}$, where each entry p_{ij} is defined as:

$$p_{ij} = \Pr(X_{t+1} = a_j \mid X_t = a_i),$$

and the matrix satisfies the standard stochastic conditions:

1. $p_{ij} \geq 0$ for all i, j,
2. $\sum_{j=1}^{m} p_{ij} = 1$ for all i.

Each alternative a_i is interpreted as a node in the Markov process in ranking applications. In *PageRank*, a transition from a_i to a_j corresponds to a hyperlink from one web page to another. In contrast, in *Local Markovian Consensus (LMC)*, the transition reflects a local ordinal dominance between alternatives a_i and a_j, derived from a collection of individual rankings.

Thus, the entry p_{ij} may represent a probability of preference, influence, or dominance from alternative a_i to alternative a_j, depending on the specific construction of the matrix P.

Definition 4. *A stationary distribution* $\pi = [\pi_1, \ldots, \pi_m]$ *is a probability vector satisfying*

$$\pi \mathbf{P} = \pi, \quad \sum_{i=1}^{m} \pi_i = 1, \quad \pi_i \geq 0 \; \forall i.$$

The stationary distribution π represents the long-term proportion of time the Markov chain spends in each state. In ranking applications, this vector induces a global ranking of the alternatives. In the PageRank model, higher values of π_i indicate greater importance of the corresponding alternative a_i. In contrast, in the LMC, smaller values of π_i indicate greater importance, as they correspond to a lower probability of being dominated by other alternatives.

The existence and uniqueness of the stationary distribution π are guaranteed under the assumption that the Markov chain is *ergodic*; that is, it is irreducible, aperiodic, and positive recurrent.

4 Local Markovian Consensus (LMC-WOD) for Weak Ordinal Dominance

This section presents LMC-WOD, an enhanced version of the Local Markovian Consensus method [9], extended to handle full rankings with ties under weak ordinal dominance. The method comprises two key steps: (1) constructing the transition matrix $\mathbf{P}$, and (2) computing its stationary distribution to obtain the consensus ranking.

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4.1 Transition Matrix and Aggregation in LMC-WOD

The transition matrix is derived from input rankings by counting how often one alternative locally dominates another, based on their relative positions. Only local transitions—between adjacent or tied items—contribute to the matrix. This design ensures the Markov process captures local ordinal relationships, not global shifts.

Algorithm 1 Generate Transition Matrix $\mathbf{P}$ in LMC-WOD

Input: Rankings matrix $\mathbf{R}$ of size $n \times m$
Output: Transition matrix $\mathbf{P}$
Initialize $\mathbf{G}$ (transition frequency matrix) as a zero matrix of size $m \times m$
for each ranking $\mathbf{r}$ in $\mathbf{R}$ **do**
    Extract the sorted list of unique rank values $\mathbf{Z} = [z_1, z_2, \ldots, z_k]$ from $\mathbf{r}$
    for $i = 0$ to $|\mathbf{Z}| - 1$ **do**
        Identify indices $index_1$ where $\mathbf{r} == \mathbf{Z}[i]$
        if $i + 1 < |Z|$ **then**
            Identify indices $index_2$ where $\mathbf{r} == \mathbf{Z}[i + 1]$
        else
            $index_2 \leftarrow \emptyset$
        end if
        for each i_1 in $index_1$ **do**
            if $index_2 \neq \emptyset$ **then**
                for each i_2 in $index_2$ **do**
                    $\mathbf{G}[i_1, i_2] \leftarrow \mathbf{G}[i_1, i_2] + 1$
                end for
            else
                $\mathbf{G}[i_1, i_1] \leftarrow \mathbf{G}[i_1, i_1] + 1$
            end if
        end for
    end for
end for
Normalize $\mathbf{G}$ row-wise to obtain the transition matrix $\mathbf{P}$
return $\mathbf{P}$

The core of Algorithm 1 is to count how often one alternative is locally dominated by another across the input rankings. Only comparisons between consecutive rank levels are recorded, emphasizing that small positional differences carry more influence than distant ones.

For each ranking:

- Alternatives sharing the same rank (including ties) are grouped.
- Each rank level is compared to the one immediately below.
- Counts are incremented for each pair where a higher-ranked item dominates one below it.
- Self-loops are added when no lower-ranked alternatives exist, ensuring stochasticity.

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After processing all rankings, the frequency matrix is normalized row-wise to obtain the transition matrix $\mathbf{P}$, used to compute the stationary distribution.

Example 1: Consider three alternatives: $A = \{a_1, a_2, a_3\}$, with two rankings allowing ties:

$$\mathbf{R} = \begin{array}{c} \\ \mathbf{r_1} \\ \mathbf{r_2} \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix} \end{array}$$

The algorithm updates the dominance frequency matrix $\mathbf{G}$ as follows:

1. From $\mathbf{r_1}$: a_1 dominates a_2 and a_3: $\mathbf{G}[1,2] \leftarrow 1,\ \mathbf{G}[1,3] \leftarrow 1$
2. From $\mathbf{r_2}$: a_1 and a_2 dominate a_3: $\mathbf{G}[1,3] \leftarrow 2,\ \mathbf{G}[2,3] \leftarrow 1$
3. Self-loops are added for the lowest-ranked elements to ensure stochasticity: $\mathbf{G}[2,2] \leftarrow 1,\ \mathbf{G}[3,3] \leftarrow 2$

The resulting matrices are:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(indices: } a_1,\ a_2,\ a_3\text{)}$$

After constructing the transition matrix $\mathbf{P}$, the next step is to compute the consensus ranking using a stationary distribution, as detailed below.

Once the transition matrix $\mathbf{P}$ is computed, the algorithm iteratively updates a domination vector π to reflect overall dominance via Markovian convergence. The process:

1. Initializes π as a probability distribution over alternatives.
2. Applies weighted updates combining π and $\mathbf{P}$
3. Balances the influence of observed local transitions against a uniform dominance distribution, which serves as a universal prior representing complete indifference or the absence of any ordinal dominance by a damping factor $d \in (0, 1)$.
4. Stops when changes in π fall below a threshold $\epsilon > 0$.

The final ranking is based on the stationary distribution π, where lower values imply stronger alternatives, less likely to be dominated.

Example 1 cont. Starting with domination vector $\pi = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$ and $d = 0.95$, the iteration yields:

$$\pi = [0.0167,\ 0.0418,\ 0.9415]$$

According to the LMC-WOD principle, the consensus ranking is $\mathbf{c} = [1\ 2\ 3]$, reflecting persistent dominance of a_1 and weak standing of a_3.

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4.2 LMC-WOD Algorithm

The iteration converges to the domination vector π, combining local structure from $\mathbf{P}$ with a uniform influence via the damping factor (Algorithm 2). This allows propagation even with ties or disconnected dominance, enabling LMC-WOD to capture strong and weak ordinal relations in the final ranking.

Algorithm 2 LMC-WOD: Consensus from Weak Ordinal Dominance

1: **Input**: Damping factor $d \in (0, 1)$, tolerance $\epsilon > 0$
2: **Output**: Consensus ranking $\mathbf{c}$
3: Compute transition matrix $\mathbf{P}$ using Algorithm 1
4: Initialize domination vector: $\pi \leftarrow [\frac{1}{m}, \ldots, \frac{1}{m}]$
5: **repeat**
6: $\quad \pi_{\text{current}} \leftarrow \pi$
7: $\quad$ Update: $\pi \leftarrow \left((1-d) \cdot \frac{1}{m} \cdot \mathbf{1} + d \cdot P^T\right) \pi_{\text{current}}$
8: **until** $|\pi - \pi_{\text{current}}| < \epsilon$
9: Convert π to ranking $\mathbf{c}$
10: **return** $\mathbf{c}$

Here, $\mathbf{1}$ (line 8) is an $m \times m$ matrix with all entries equal to 1.

4.3 Case Studies Demonstrating LMC-WOD

This subsection presents two case studies demonstrating how LMC-WOD produces a consensus ranking for both strict and weakly ordered inputs (i.e., with ties), using five rankings over ten alternatives in each case.

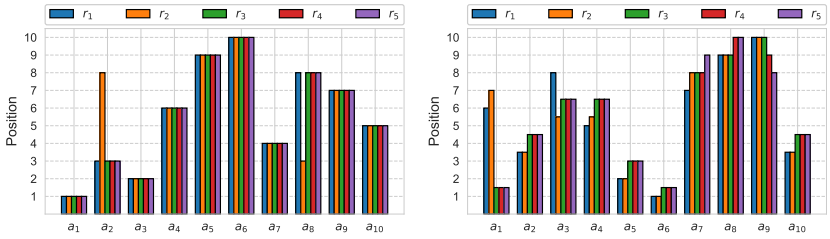

Fig. 1. Rankings used in both case studies. Left: strict rankings with a large positional shift. Right: rankings with ties and non-consecutive values.

In Case Study 1, all rankings are strict. One ranking introduces a positional swap between a_2 and a_8, shifting them by five places (Fig. 1, left). This local inconsistency alters the consensus, which deviates from the dominant pattern in rankings r_1, r_3, r_4, and r_5: $[1, 3, 2, 6, 9, 10, 4, 8, 7, 5]$. This illustrates LMC-WOD's sensitivity to local disruptions.

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In Case Study 2, all rankings include ties and exhibit several non-consecutive rank levels. For instance, in rankings r_3, r_4, and r_5, alternatives a_1 and a_6 are consistently tied for first place (rank 1.5), while a_3 and a_4 share the same mid-level rank (6.5). Additionally, a_{10} is tied with a_2 at rank 3.5 in r_1 and r_2, and similar patterns appear across other rankings. These structures reflect a weak ordinal setting with consistent local agreements but reduced positional resolution. The rankings used in this scenario are visualized in Fig. 1 (right).

From the rankings in Fig. 1, the transition matrices $\mathbf{P}$ were derived using Algorithm 1. Fig. 2 shows these matrices as heatmaps: left for Case Study 1 (no ties), right for Case Study 2 (with ties).

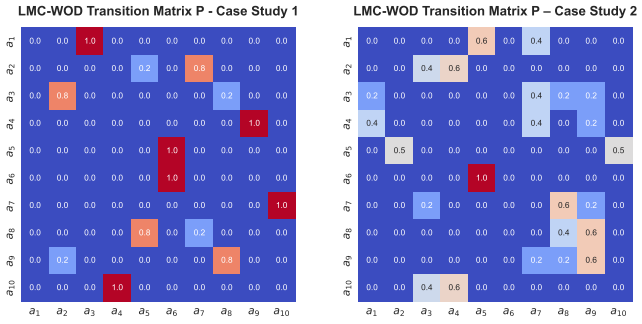

Fig. 2. Heatmap representation of the transition matrices $\mathbf{P}$: (Left) Case Study 1 — without ties, (Right) Case Study 2 — with ties.

The second phase of LMC-WOD was applied using a damping factor $d = 0.95$. Fig. 3 shows the convergence of dominance values, capturing how influence propagates across alternatives.

At each iteration, the dominance vector is updated via the transition matrix $\mathbf{P}$, gradually refining the ranking. This amplifies strong ordinal signals while retaining local structure.

Dominance trajectories for all alternatives are plotted for both cases. Fig. 3 (left) shows the case without ties; the right panel illustrates the dynamics with ties. Differences in convergence reflect the method's sensitivity to tie patterns and ranking inconsistencies.

The following sections will present a detailed analysis of how dominance is strengthened or weakened during iterations.

Case Study 1: Iterative Influence Propagation Without Ties (Fig. 3, Left) At iteration $k = 1$, the top-ranked alternative is a_1 and the lowest is a_6. By the second iteration, a_3 begins to lose dominance due to its link with a_1, which already holds a low dominance value. At this point, a_{10} remains inactive.

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DOI: 10.1007/978-3-031-97567-7_29

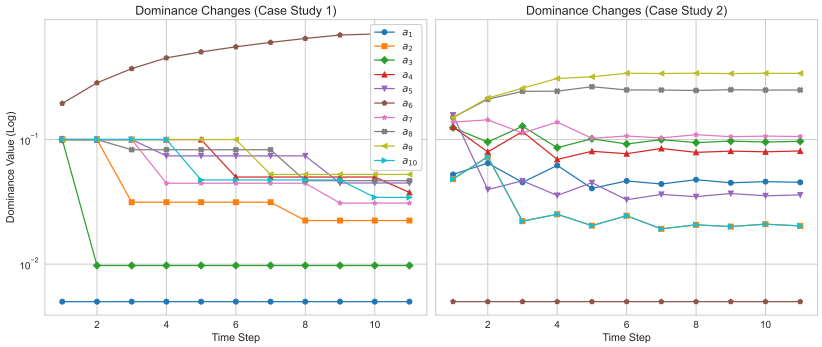

Fig. 3. Evolution of dominance values over iterations for all alternatives. (Left) Case Study 1 — without ties, (Right) Case Study 2 — with ties.

In iteration three, a_2 and a_8 are affected through a_3. While a_2 co-occurs with a_3 in four rankings, a_8 appears just once. In the next step, a_2 influences a_7, and a_8 affects a_5. They also impact each other's neighbors: $a_2 \to a_5$, $a_8 \to a_7$. The decrease in dominance is distributed proportionally across these connections, with a_2 exerting a stronger influence overall.

By iteration five, a_{10} is pulled by a_7, and a_5 slows a_6's rise. In the final step, a_6 passes influence to a_4, which then affects a_9. This step-by-step process captures key dominance pathways.

The elevation of a_8 in the final ranking reflects the algorithm's sensitivity to local propagation. Strong connections and initial positions jointly shape the outcome, distinguishing LMC-WOD from majority voting or distance-based methods.

The consensus ranking for Case Study 1 is:

$$\mathbf{c_{LMC-WOD:Case2}} = [1,\ 3,\ 2,\ 6,\ 8,\ 10,\ 4,\ 7,\ 9,\ 5]. \tag{2}$$

Although a_2 remains highly ranked, this may seem counterintuitive given its low placement in one ranking. LMC-WOD underweights such outliers when a strong majority supports a high position. This reveals both a limitation—insensitivity to sharp local deviations, and a strength—robustness to manipulative rankings.

Case Study 2: Influence Propagation with Ties (Fig. 3, Right) At iteration $k = 1$, a_6 is the least dominated alternative. a_1, a_2, and a_{10} show similar, low dominance levels. A second group: a_3, a_4, a_5, a_7, a_8, and a_9, emerges with higher dominance scores.

In the next step, a_5's score drops due to its link with weakly dominated a_6. Mutual influence keeps a_8 and a_9 highly dominated early on. As their neighbors decline, their scores slowly increase throughout the process.

Due to many interdependencies and shared ranks, convergence is gradual. The final ranking includes a tie between a_2 and a_{10}, reflecting their nearly identical dominance values.

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DOI: 10.1007/978-3-031-97567-7_29

$$\mathbf{c_{LMC-WOD:Case2}} = [5,\ 2.5,\ 7,\ 5,\ 6,\ 1,\ 8,\ 9,\ 10,\ 2.5] \tag{3}$$

Though a_5 often ranks near the top in input data, it drops in the consensus due to ties and frequent co-occurrence with a_1 and a_6. These relations diffuse their dominance, reducing their final rank. This outcome highlights LMC-WOD's sensitivity to local structures of dominance relations, where shared rank positions and ambiguous relative orderings may dilute the impact of a highly placed alternative in the aggregate result.

Both cases show how LMC-WOD incorporates local dominance into a consistent consensus.

5 Comparative Analysis of PageRank and LMC-WOD

PageRank builds a transition matrix from the hyperlink structure of web pages. Using a Markov process, it computes a stationary distribution that reflects page importance. In the random surfer model, transitions follow links, with a damping factor d controlling the chance of random jumps [3].

For ranking aggregation, PageRank is adapted by linking each higher-ranked item to all lower-ranked ones (equivalent to the MC2 algorithm described [6]). PageRank adds a transition whenever an item is outranked by another, producing a dense matrix that enables global influence propagation.

LMC-WOD, though inspired by PageRank, defines its transition matrix differently. Instead of all pairwise links, it counts how often an alternative is directly dominated by a neighboring (adjacent or tied) alternative in input rankings. LMC-WOD restricts movement to local structures, emphasizing proximity in ordinal position.

Table 1 outlines key differences in how both methods construct and interpret the matrix $\mathbf{P}$.

Table 1. Key Differences Between PageRank and LMC-WOD in the Construction and Interpretation of the Transition Matrix $\mathbf{P}$

Aspect	PageRank	LMC-WOD
Definition of P	Encodes all pairwise dominance relationships	Reflects only local (adjacent or tied) dominance
Transition Dynamics	Allows transitions between all alternatives	Permits only local movements within rankings
Influence Scope	Global, enabling long-range propagation	Local, based on immediate ordinal structure
Self-Transitions	Retained only for the top-ranked node	Retained when no successors are available
Treatment of Ties	Not explicitly addressed	Explicitly supports and encodes ties
Damping Factor Role	Introduces global randomness	Balances locality and structural gaps

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5.1 Differences Between PageRank and LMC-WOD in Ranking Aggregation

This section compares PageRank and LMC-WOD using the two case studies from Section 4.3: Case 1 (no ties) and Case 2 (with ties), both with a damping factor $d = 0.95$.

The comparison focuses on key mathematical properties of the transition matrix $\mathbf{P}$, summarized in Table 2.

Table 2. Mathematical Properties Used for Comparison

Property	Definition
Eigenvalues	Scalars λ_i satisfying $\mathbf{Pv} = \lambda_i \mathbf{v}$, where $\mathbf{v}$ is the eigenvector. Limited to λ_1 and λ_2.
Mixing Time	Estimated convergence time to stationary state: $T_{\text{mix}} = \frac{1}{1-\lambda_2}$, where λ_2 is the second-largest eigenvalue.
Steady-State Distribution	A probability vector π such that $\pi\mathbf{P} = \pi$, representing the long-run state of the process.
Periodicity	A chain is aperiodic if it does not return to a state in regular cycles. Aperiodicity ensures convergence to a unique steady state.
Graph Representation	Directed graph with nodes as alternatives and edges as transition probabilities from $\mathbf{P}$.

The heatmaps are shown for the PageRank-based matrices in Fig. 4 to illustrate the structure of the transition matrix $\mathbf{P}$. The differences from LMC-WOD highlight the distinct design principles behind each method.

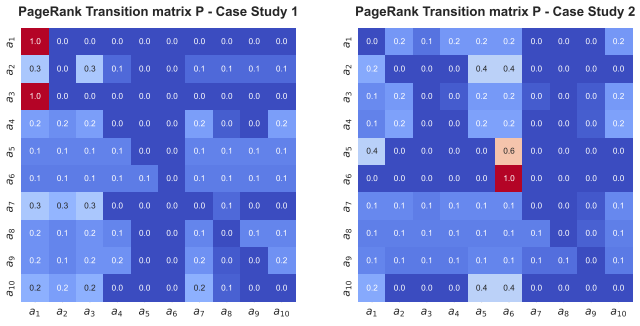

Fig. 4. Heatmaps of transition matrices $\mathbf{P}$ from PageRank: (Left) Case Study 1 — no ties; (Right) Case Study 2 — with ties.

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Case Study 1: Comparing LMC-WOD and PageRank Without Ties In LMC-WOD, the transition matrix $\mathbf{P}$ includes an absorbing state (Table 3, Fig. 5): once the process enters a_6, it cannot leave. The steady-state distribution concentrates entirely there. A mixing time of 4.91 steps indicates slower convergence, as the Markov chain requires more iterations to reach equilibrium.

PageRank, in contrast, avoids absorbing states. Probability is spread across all alternatives, enabling broader transitions. Its lower mixing time of 1.82 steps signals faster stabilization from any initial state.

Table 3. Transition Matrix Comparison — Case Study 1

Property	LMC-WOD	PageRank
Largest Eigenvalue	$\lambda_1 = 1$	$\lambda_1 = 1$
Second Eigenvalue	$\lambda_2 = 0.796214$	$\lambda_2 = 0.451643$
Mixing Time	4.91 steps	1.82 steps
Steady State	Absorbing state at a_6	Spread across states
Periodicity	Possible (complex eigenvalues)	Possible, less pronounced

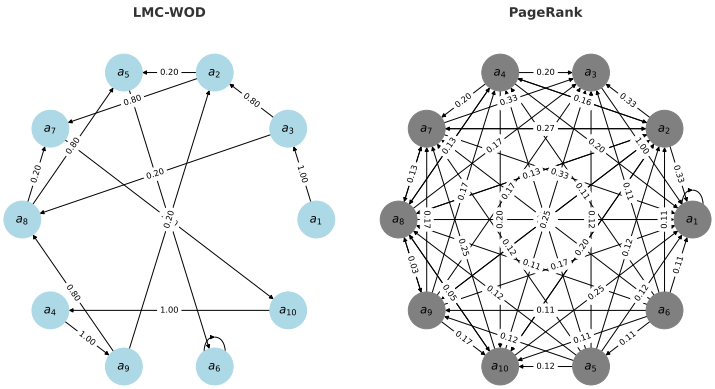

Fig. 5. Graphs from transition matrices for Case Study 1 (no ties): LMC-WOD (left), PageRank (right).

Fig. 5 shows the graph structures from $\mathbf{P}$. LMC-WOD yields sparse connections, reflecting its local design. PageRank connects each higher-ranked item to all lower-ranked ones, creating a denser graph. Both methods retain minimal probability for connections not explicitly observed in the input rankings, preserving ergodicity.

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PageRank produces the consensus ranking:

$$\mathbf{c}_{\mathbf{PageRank:Case1}} = [1,\ 3,\ 2,\ 6,\ 9,\ 10,\ 4,\ 7,\ 8,\ 5]. \tag{4}$$

Both methods agree on 8 out of 10 positions (cf. Eq. 2). The difference lies in a_5 and a_9: LMC-WOD ranks a_5 higher due to its sensitivity to local preferences in the input.

Case Study 2: Comparing LMC-WOD and PageRank with Ties Table 4 summarizes results for Case Study 2, where input rankings include ties. As in Case 1, LMC-WOD keeps transitions local, while PageRank spreads influence more broadly.

Table 4. Transition Matrix Comparison — Case Study 2

Property	LMC-WOD	PageRank
Largest Eigenvalue	$\lambda_1 = 1$	$\lambda_1 = 1$
Second Eigenvalue	$\lambda_2 = 0.668547$	$\lambda_2 = 0.634557$
Mixing Time	3.02 steps	2.74 steps
Steady State	Concentrated in a few states	More evenly spread
Periodicity	Possible (complex eigenvalues)	Transitions more stable

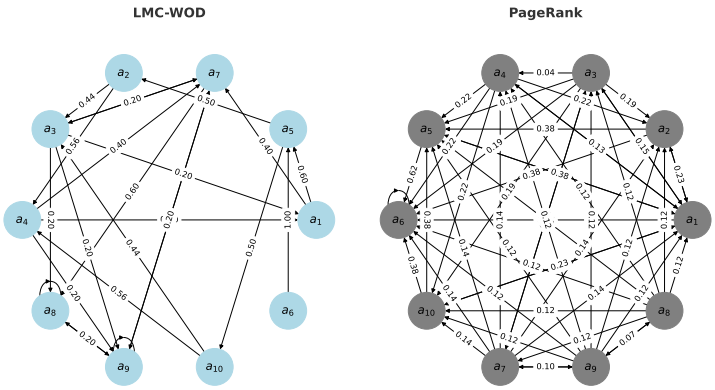

Fig. 6. Graphs from transition matrices for Case Study 2 (with ties): LMC-WOD (left), PageRank (right).

Fig. 6 compares graph structures. LMC-WOD remains sparse, reflecting its local scope. PageRank forms denser links, connecting each higher-ranked item to all lower-ranked ones.

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DOI: 10.1007/978-3-031-97567-7_29

PageRank produces the consensus:

$$\mathbf{c}_{\mathbf{PageRank:Case2}} = [3,\ 4.5,\ 7,\ 6,\ 2,\ 1,\ 8,\ 10,\ 9,\ 4.5] \tag{5}$$

The methods diverge on several items, especially a_2, a_5, and a_{10}. Both agree on a_6, and align closely on a_3 and a_7. LMC-WOD boosts a_2 and a_{10} due to strong local dominance, while PageRank favors a_5 through broader link influence.

Generally, for relatively consistent sets of input rankings, both methods produce similar results. However, in the presence of substantial conflicts or ties among the rankings, the differences between LMC-WOD and PageRank become more pronounced, especially in how ranking influence is propagated across alternatives.

6 Conclusions

This paper introduced LMC-WOD (Local Markovian Consensus with Weak Ordinal Dominance), an extension of the LMC method for aggregating rankings with non-strict preferences.

Two case studies illustrated its operation under contrasting conditions: when rankings were consistent versus diverse, and when aggregation occurred in settings that excluded or permitted ties. This design exposed the model's behavior across structured and ambiguous inputs.

A comparative analysis with PageRank, based on transition matrix eigenstructure and stationary distributions, highlighted key differences in influence propagation and ranking mechanisms.

The main methodological observations are:

- LMC-WOD can accommodate pairwise comparisons violating strict transitivity, but the interpretation of its stationary distributions is not always intuitive in systems with many alternatives and no dominant options.
- The method's output may be sensitive to diversity in the input set, which may be challenging in highly heterogeneous contexts.
- In contrast, it shows strong robustness when input rankings are aligned mainly, so small perturbations do not significantly affect the result.
- Its convergence behavior has not yet been thoroughly tested on large or randomly structured input sets, limiting the generalizability of current results.
- The damping factor $d = 0.95$, fixed in all experiments, was not evaluated for sensitivity, leaving its effect on stability and convergence open.
- A practical issue arises when an alternative receives many incoming transitions, which may disproportionately reduce its rank if those transitions come from highly ranked items. This effect suggests a need for weighting mechanisms that account for the influence or credibility of the source.

These limitations point to future directions: large-scale simulations, experiments with controlled inconsistency, real-world data, and adaptive strategies for parameter tuning could all help refine the method, improve its practical use, and make its behavior more straightforward to understand. Overall, all findings support LMC-WOD as a promising tool for weakly ordered data.

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DOI: 10.1007/978-3-031-97567-7_29

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DOI: 10.1007/978-3-031-97567-7_29