

The Role of Preference Reidentification in MCDA: Comparing Weight-Based, Normalization, and Reference-Object Approaches

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Abstract. Reidentification of decision-maker preferences is a crucial aspect of Multi-Criteria Decision Analysis (MCDA), as it enables a structured evaluation of decision-making methodologies. This study presents a comparative assessment of three stochastic reidentification techniques: Stochastic Identification of Weights (SITW), Stochastic Fuzzy Normalization (STFN), and Stochastic Identification of Models (SITCOM). Each method models decision-maker preferences based on different paradigms: weight-based aggregation, normalization, and reference-object-based evaluation. To systematically analyze their effectiveness, we employ three benchmark preference functions: a monotonic function representing linear preference structures, a non-monotonic function with a single extremum reflecting a decision-maker with a specific optimal point, and a non-monotonic function with multiple extrema modeling complex preference structures with multiple local optima.

Our findings indicate that SITW is most effective for monotonic preferences, STFN provides superior performance in single-extremum cases, and SITCOM excels in handling multiple-extrema scenarios. The comparative analysis highlights the limitations of weight-based approaches in complex decision problems, demonstrating that reference-object-based models are better suited for non-trivial preference structures. The study contributes to the understanding of how different MCDA reidentification techniques perform under varying decision-making conditions, offering practical insights into the selection of appropriate methods. Future research should focus on integrating hybrid methodologies to enhance reidentification accuracy and applying these techniques in real-world decision-making contexts.

Keywords: Multi-Criteria Decision Analysis, Preference Reidentification, Stochastic Optimization, Decision-Maker Modeling, MCDA Methods

1 Introduction

In an era of rapid technological advancements and increasing globalization, decision-making processes have become more complex than ever before. The

challenges associated with modern decision-making stem not only from the vast amounts of data that need to be processed but also from the presence of multiple, often conflicting criteria that must be simultaneously considered. The high dimensionality of information further exacerbates the difficulty of selecting optimal solutions, particularly when numerous alternative options exist. In such cases, traditional decision-making approaches often prove inadequate, necessitating the use of more sophisticated analytical tools.

To address these challenges, Multi-Criteria Decision Analysis (MCDA) has emerged as a powerful methodological framework designed to support decision-makers in structuring, evaluating, and comparing alternative solutions. Unlike traditional decision-making approaches, MCDA enables the systematic consideration of multiple factors, incorporating both quantitative and qualitative criteria to enhance the decision-making process. These methods are particularly valuable in scenarios characterized by uncertainty, conflicting stakeholder interests, or the need for trade-offs between different objectives.

Due to their flexibility and robustness, MCDA techniques have been widely adopted across various domains, including transportation, energy, logistics, and healthcare. By providing structured decision-making frameworks, these methods contribute to improved efficiency, increased transparency, and more informed choices, ultimately enhancing the quality of decisions made in complex environments. As a result, MCDA has become an indispensable tool for supporting analytical decision-making in both academic and practical applications.

Given the diversity of decision-making problems, a wide range of MCDA methodologies have been developed to accommodate different analytical needs. As highlighted in [3], MCDA has been extensively applied in the development of Decision Support Systems (DSS), particularly in the environmental sector. The study explores the conceptual foundations of MCDA and discusses some of the most frequently used approaches for solving multi-attribute decision problems, emphasizing their significance in addressing real-world decision-making challenges.

A more comprehensive review of MCDA methods was conducted by Wang et al. in [7], where the authors proposed a systematic classification of preference modeling techniques. Their analysis identified several fundamental approaches, including goal-based modeling, weight assignment, reference vectors, preference relations, utility functions, outranking methods, and implicit preferences. Each of these approaches has distinct characteristics that influence its applicability in specific decision-making contexts, making it crucial for decision-makers to carefully select the most suitable method based on the nature of the problem at hand.

However, selecting the appropriate MCDA method remains a significant challenge due to the vast number of available techniques and the varying nature of decision-making scenarios. To address this issue, Wątróbski et al. [8] introduced a generalized framework for multi-criteria method selection. This framework provides a structured approach to choosing MCDA techniques based on predefined criteria, improving the reliability and consistency of the selection process.

Building upon this work, subsequent research has focused on further systematizing MCDA methodologies and improving their applicability in practical decision-making contexts. In [1, 2], a comprehensive taxonomy of MCDA methods was introduced to aid in the recommendation of the most appropriate techniques based on problem-specific characteristics. This taxonomy provides a more refined structure for categorizing decision analysis methods, enabling decision-makers to make informed choices while minimizing the risk of misapplying a given method.

Despite the existence of various MCDA methods and classification frameworks, there remains a gap in understanding how different modeling parameters influence decision-making outcomes. In this study, we aim to analyze the key parameters used in modeling decision-makers' preferences within the MCDA framework. Our objective is to examine how these parameters shape the decision-making process and to evaluate the strengths and limitations of different methodological approaches.

Given the multitude of MCDA methods and existing taxonomies, we conduct a comparative analysis of selected techniques to identify their key characteristics and assess their practical applicability. This comparison allows us to determine which methods are best suited for specific decision-making scenarios, providing valuable insights for both researchers and practitioners. Ultimately, our study seeks to equip decision-makers with the knowledge required to select the most appropriate MCDA methods for modeling their preferences, thereby improving the overall efficiency and effectiveness of decision-making processes across various application domains.

To provide a structured analysis of the reidentification methods in MCDA, this paper is organized as follows. Section 2 presents the methodological framework, detailing the selected benchmark preference functions and the reidentification techniques applied in this study. Section 3 focuses on benchmarking studies, where the effectiveness of SITW, STFN, and SITCOM is systematically evaluated using three distinct preference functions—monotonic, non-monotonic with a single extremum, and non-monotonic with multiple extrema. The results are analyzed through visualizations and statistical comparisons to highlight the strengths and limitations of each method. Finally, Section 4 provides concluding remarks and directions for future research, emphasizing potential advancements in hybrid reidentification techniques and their application in real-world decision problems.

2 Methodology

This study focuses on evaluating the effectiveness of different MCDA (Multi-Criteria Decision Analysis) re-identification methodologies in reconstructing decision-maker preferences across various decision-making scenarios. To establish a structured evaluation framework, three types of benchmark functions were selected, each representing a distinct preference structure. These functions serve as math-

emational models that simulate decision-making behavior, allowing for a comparative analysis of different re-identification approaches.

The first category includes monotonic functions, which represent decision-makers whose preferences increase or decrease consistently with criterion values. This structure is characteristic of decision-making scenarios where alternatives are evaluated in a strictly hierarchical manner. The second category consists of non-monotonic functions with a single extremum, representing decision-makers who have a clearly defined optimal point rather than following a simple increasing or decreasing trend. Such preferences occur in situations where balance or a specific target value is prioritized over extreme alternatives. The third category includes non-monotonic functions with multiple extrema, modeling decision-makers who assign high preference values to multiple distinct regions rather than a single optimal point. This preference structure is relevant in cases where different alternative solutions provide similarly desirable outcomes depending on contextual factors.

Each benchmark function is used to generate a structured preference representation, which serves as a basis for evaluating different re-identification methodologies. The preference structure derived from these functions provides an analytical foundation for comparing the ability of various methods to reconstruct decision-maker preferences effectively.

To perform the re-identification process, three different methodologies were employed, each representing a distinct paradigm of preference modeling. The first method, Stochastic Identification of Weights (SITW), estimates the relative importance of each criterion in decision-making by optimizing weight distributions. The second method, Stochastic Fuzzy Normalization (STFN), utilizes fuzzy logic to approximate preference distributions through triangular membership functions, allowing for a more flexible modeling approach. The third method, Stochastic Identification of Models (SITCOM), reconstructs decision-maker preferences using reference objects, relying on characteristic alternatives to infer preference relationships.

The evaluation of the re-identification process is conducted by analyzing the reconstructed preference structures generated by each methodology. The results are assessed in terms of their ability to replicate the original preference functions, allowing for a comparative analysis of the strengths and limitations of each approach. The methodological framework, summarized in Figure 1, illustrates the key stages of data generation, preference modeling, and re-identification. This structured approach ensures a comprehensive comparison of MCDA re-identification methodologies in terms of their adaptability to different decision-making scenarios.

2.1 Methods

This section presents the re-identification methods used in this study, which serve as mechanisms for simulating decision-maker or expert preferences. These methods are designed to reconstruct preference structures using stochastic optimization techniques and are depicted in Figure 2.

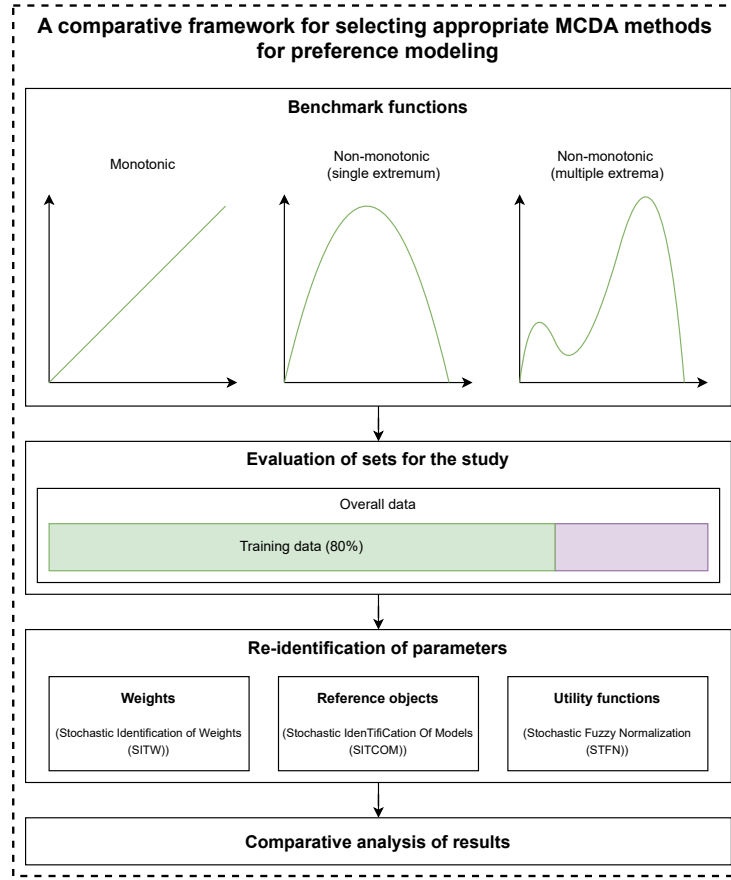


Fig. 1: Overview of the methodological framework used in this study.

The first method is Stochastic Identification of Weights (SITW), initially proposed in [6]. This approach focuses on the re-identification of criterion weights for a given MCDA method using stochastic optimization. The fitness function in this optimization procedure is structured so that the identified weights are integrated into the selected MCDA method alongside the decision matrix. The resulting ranking is then compared to the reference ranking using a weighted Spearman's rank correlation coefficient. The optimization algorithm aims to maximize this correlation coefficient, meaning that the more closely the generated ranking aligns with the reference ranking, the better the identified weights reflect the decision-maker's preference structure. As a result, this method provides a set of optimized weights that can be used to model decision-maker preferences in multi-criteria decision-making problems.

The second method is Stochastic Fuzzy Normalization (STFN), introduced in [5]. This method is designed for the re-identification of the cores of triangu-

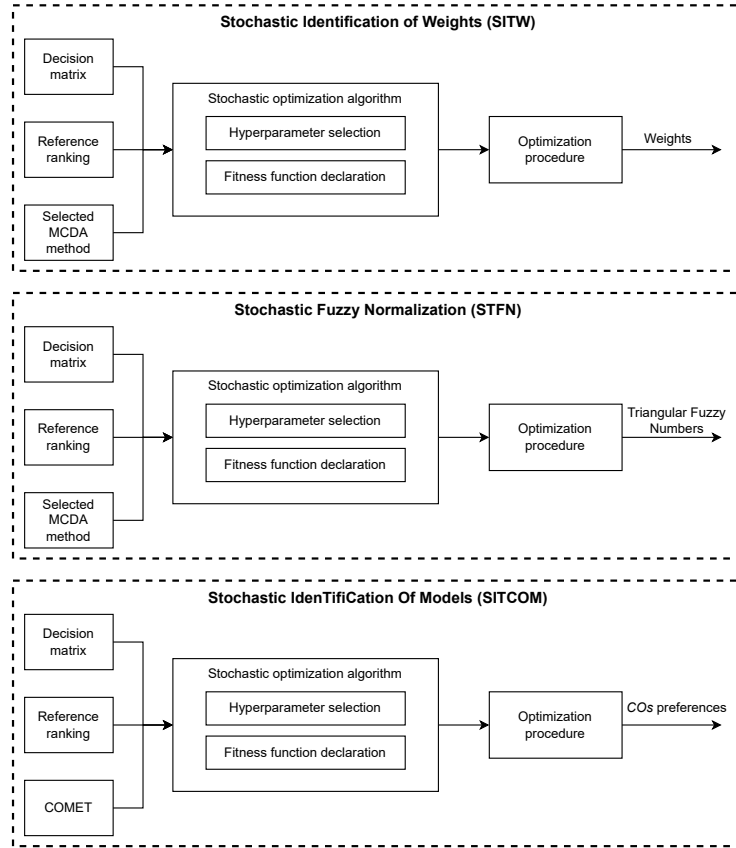


Fig. 2: Re-identification methods used in this study.

lar fuzzy numbers (TFNs), which serve as normalization parameters in MCDA methods. The range of TFNs can either be pre-defined by the decision-maker or inferred from the boundary values of the decision matrix for each criterion. The fuzzy numbers are then used for normalizing decision matrix values within the selected MCDA method. The cores of the TFNs are determined through stochastic optimization, where the fitness function is designed to integrate the identified cores into the pre-defined fuzzy numbers. These values are subsequently incorporated into the MCDA method along with the decision matrix to generate a ranking, which is then compared to the reference ranking using a weighted Spearman's correlation coefficient. The optimization algorithm maximizes this coefficient, ensuring that the identified fuzzy number cores closely align with the decision-maker's original preference structure. The final output of this method is a set of optimized triangular fuzzy numbers, which enhance the normalization process in MCDA applications.

The third method is Stochastic Identification Of Models (SITCOM), as introduced in [4]. This approach focuses on the re-identification of characteristic object (reference object) preferences within the COMET (Characteristic Object METHod) framework using stochastic optimization. In this approach, a temporary COMET model is constructed, where characteristic values are initially provided to define the decision space. However, at this stage, the characteristic objects do not yet have assigned preferences. These preferences are subsequently determined through a stochastic optimization process. The fitness function is designed such that the optimized preferences for characteristic objects are integrated into the COMET method alongside the decision matrix, generating a ranking that is compared to the reference ranking using a weighted Spearman's correlation coefficient. The objective of the optimization process is to maximize the correlation coefficient, ensuring that the assigned characteristic object preferences reflect the decision-maker's underlying preference structure. The final output of this method is a set of optimized preference values for characteristic objects,

3 Benchmarking studies

This section presents a detailed analysis of reference functions used to evaluate reidentification methods within the framework of Multi-Criteria Decision Analysis (MCDA). To create a reliable comparison of the SITW, STFV, and SITCOM methods, three different classes of reference functions were selected, reflecting different decision-making approaches and preference structures. Each of these functions represents distinct decision-maker models that vary in their decision rationale, sensitivity to criteria, and level of complexity.

The selected benchmark functions aim to test the adaptability and effectiveness of reidentification methods in capturing different preference patterns. These functions are mathematically defined and simulate the decision-making behavior of experts or decision-makers.

To systematically assess the effectiveness of MCDA reidentification methods, the following benchmark functions were chosen:

- **Monotonic function** – Represents a decision-making model where preferences increase or decrease consistently with the values of the decision criteria. This function simulates a decision-maker whose choices are straightforwardly guided by the nature of the criteria (e.g., minimizing cost or maximizing profit). The function is defined as:

$$f_{\text{monotonic}}(x, y) = x + y \quad (1)$$

- **Non-monotonic function with a single extremum** – Models a decision-maker who prefers a specific optimal value rather than simply maximizing or minimizing the criteria. Such behavior is typical in cases where decision-makers aim for a balanced trade-off rather than extreme values. For example, in environmental management, a moderate level of resource utilization might

be optimal rather than complete depletion or conservation. The function is given by:

$$f_{\text{extremum}}(x, y) = -((x - 50)^2) - ((y - 50)^2) + 10 \quad (2)$$

- **Non-monotonic function with multiple extrema** – Reflects a more complex decision-making behavior where multiple local optima exist. This function represents cases where decision-makers might have multiple preferred states or decision thresholds, such as in financial portfolio optimization, where different combinations of investments could yield similar levels of satisfaction. The function is defined as:

$$f_{\text{extrema}}(x, y) = -\sin\left(\frac{x}{15}\right) + 0.5 \sin\left(\frac{2y}{15}\right) \quad (3)$$

To ensure fair comparisons among different reidentification methods and to maintain consistency across different decision-making scenarios, the functions are normalized. Normalization ensures that all preference values are mapped to a standardized range, typically $[0, 1]$, preventing discrepancies caused by varying scales of input data.

The normalization function is defined as:

$$\text{normalize}(z) = \frac{z - \min(z)}{\max(z) - \min(z)} \quad (4)$$

where $\min(z)$ and $\max(z)$ are the minimum and maximum values of z , ensuring that all outputs are within the range $[0, 1]$. This transformation allows reidentification methods to be evaluated consistently without being affected by differences in function magnitude.

3.1 Monotonic

The monotonic benchmark function represents a decision-making scenario in which preferences are directly proportional to the values of decision criteria. This means that as the values of criteria increase, the preference value also increases, and conversely, as they decrease, the preference value also decreases. This behavior is commonly observed in real-world decision-making situations, such as cost minimization or profit maximization, where a straightforward relationship exists between the criteria and the preference score.

This function creates a smooth and predictable preference surface, as shown in Figure 3, where higher values of x and y result in higher preference values.

To evaluate the effectiveness of different reidentification methods in reconstructing decision-maker preferences, the SITW, STFNN and SITCOM models were applied to the monotonic function. Figure 4 presents the reconstructed preference surfaces for each of the three models. The reconstructed preference surfaces demonstrate that both SITW-TOPSIS and STFNN-TOPSIS closely follow the structure of the original monotonic function. This indicates that these

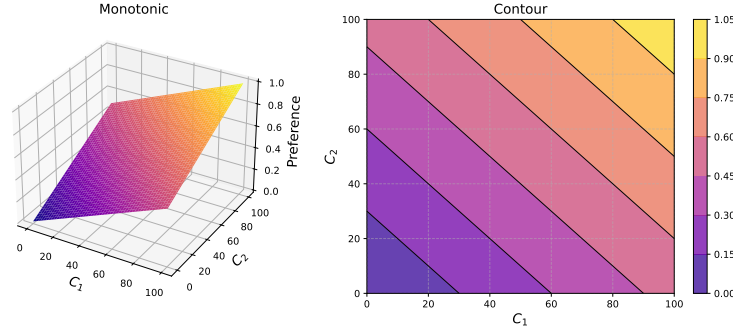


Fig. 3: Graphical representation of the monotonic benchmark function.

methods effectively capture the linear preference trend by adjusting criterion weights and fuzzy normalization, respectively. However, the SITCOM model shows a slight deviation, particularly in areas where preference values change gradually. This suggests that the reference object-based approach may struggle with purely monotonic structures, as it relies on characteristic objects rather than directly learning preference gradients.

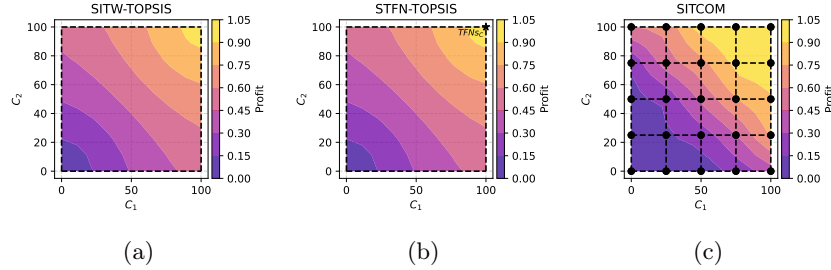


Fig. 4: Comparison of reconstructed preference surfaces for different reidentification methods applied to the monotonic function.

To evaluate the accuracy of the reconstructed preference models, we analyze the absolute differences between the reconstructed surfaces and the original monotonic function. Figure 5 presents the differences for each method. SITW-TOPSIS and STFNTOPSIS show minimal errors, with differences uniformly distributed across the surface, reinforcing their robustness for monotonic preferences. In contrast, SITCOM exhibits higher deviations, especially in regions where the function value changes smoothly. This suggests that the reference object approach struggles with maintaining consistency in strictly monotonic decision-making environments, making it less suitable for such cases.

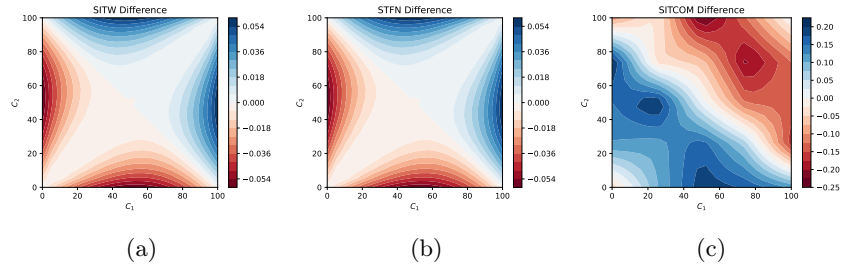


Fig. 5: Visualization of differences between the original monotonic function and reconstructed models.

The statistical results in Table 1 confirm the observations from Figures 4 and 5. Both SITW-TOPSIS and STFV-TOPSIS achieve a mean difference of zero, indicating an exact reconstruction of the monotonic preference structure. Additionally, their standard deviation is significantly lower than that of SITCOM, reinforcing their stability. SITCOM, however, shows a higher standard deviation and greater min/max deviations, demonstrating its difficulty in approximating purely monotonic preferences. This highlights the importance of selecting reidentification methods based on the nature of the decision problem—where strictly monotonic preferences favor weight-based or fuzzy normalization approaches over reference object-based methods.

Table 1: Statistical evaluation of reidentification accuracy for the monotonic function.

| | SITW | STFV | SITCOM |
|------|-----------|-----------|-----------|
| Min | -0.059175 | -0.059175 | -0.226951 |
| Mean | 0.000000 | 0.000000 | 0.010621 |
| Max | 0.059175 | 0.059175 | 0.203255 |
| Std | 0.024897 | 0.024897 | 0.119327 |

3.2 Nonmonotonic (single extremum)

The nonmonotonic function with a single extremum represents a decision-making scenario where the preference does not continuously increase or decrease but instead has a single optimal peak. Unlike the monotonic case, where preference grows or declines uniformly, this function simulates situations in which a specific range of values is preferred over extremes. Such decision-making behavior is common in real-world problems where an optimal balance exists, such as in healthcare (e.g., maintaining an optimal medication dosage) or logistics (e.g., selecting a delivery time that balances cost and efficiency). Figure 9 illustrates

the shape of the function, with a clear peak at (50,50), showing that preference decreases symmetrically as values deviate from this optimal region.

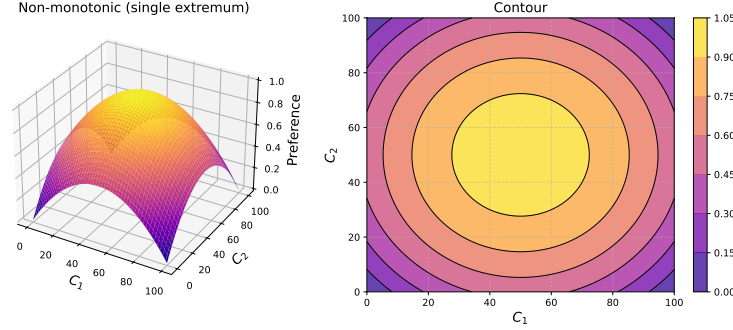


Fig. 6: Graphical representation of the nonmonotonic function with a single extremum.

The same reidentification methods as in the monotonic case were applied to reconstruct decision-maker preferences for this scenario. These include SITW-TOPSIS, STFN-TOPSIS, and SITCOM. The primary goal was to evaluate how well these methods adapt to a nonmonotonic preference structure with a single optimal peak, which introduces additional complexity compared to strictly increasing or decreasing preferences. Figure 7 presents the reconstructed preference surfaces for each model.

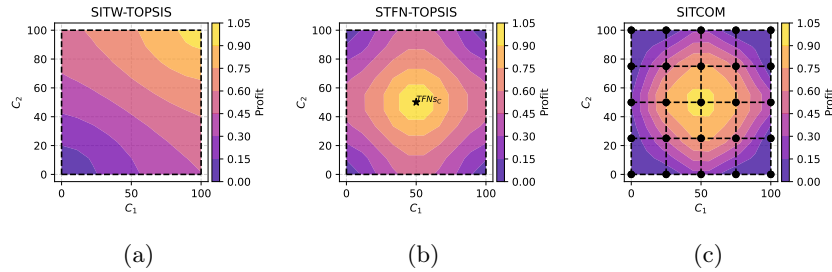


Fig. 7: Comparison of reconstructed preference surfaces for different reidentification methods applied to the single-extremum function.

Figure 8 clearly shows that SITW-TOPSIS exhibits the highest deviations, confirming that weight-based optimization struggles with this type of preference structure. STFN-TOPSIS produces minimal differences, making it the most reliable method for this scenario. SITCOM displays moderate performance, with

some areas of substantial error where the reference objects fail to capture the smooth transition of preference values.

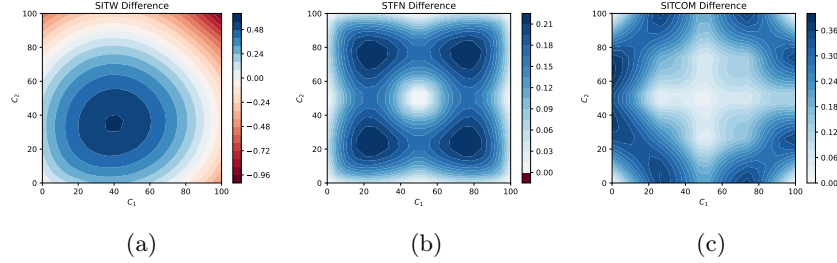


Fig. 8: Visualization of differences between the original single-extremum function and reconstructed models.

The results in Table 2 further support the conclusions drawn from the figures. SITW-TOPSIS has the highest error and variation, confirming that it is not suitable for single-extremum preferences. STFV-TOPSIS has the lowest deviation, demonstrating strong stability and accuracy. SITCOM provides intermediate results, but its performance varies depending on the distribution of reference objects.

Table 2: Statistical evaluation of reidentification accuracy for the single-extremum function.

| | SITW | STFV | SITCOM |
|------|-----------|-----------|----------|
| Min | -1.000000 | -0.000619 | 0.000000 |
| Mean | 0.133333 | 0.133841 | 0.215693 |
| Max | 0.566203 | 0.223607 | 0.389591 |
| Std | 0.307615 | 0.063766 | 0.096850 |

3.3 Nonmonotonic (multiple extrema)

The nonmonotonic function with multiple extrema represents a decision-making scenario where preference values exhibit multiple local optima rather than a single peak or a strictly increasing/decreasing pattern. Unlike the previous cases, this function simulates situations where decision-makers may have multiple preferred options, rather than a single optimal solution. This type of preference structure is common in financial portfolio optimization, supply chain management, and strategic decision-making, where multiple distinct alternatives may yield similarly favorable outcomes.

Figure 9 illustrates the shape of the function, highlighting the existence of multiple regions of high preference values, which presents a greater challenge for reidentification methods.

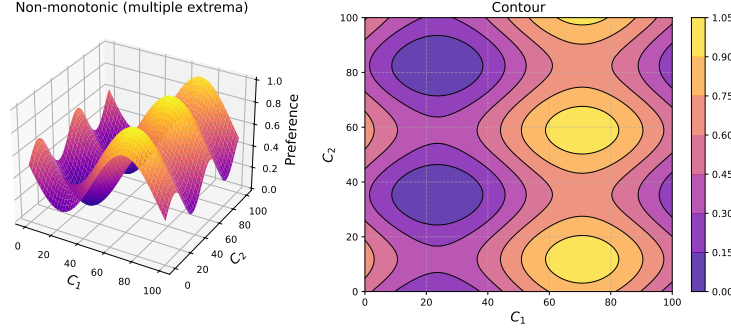


Fig. 9: Graphical representation of the nonmonotonic function with multiple extrema.

The same reidentification methods as in the previous cases were applied to reconstruct decision-maker preferences for this scenario, namely SITW-TOPSIS, STFNTOPSIS, and SITCOM. The key objective was to assess how well these methods handle a complex preference structure with multiple local optima, which is inherently more difficult to approximate compared to monotonic or single-extremum functions. Figure 10 presents the reconstructed preference surfaces for each model.

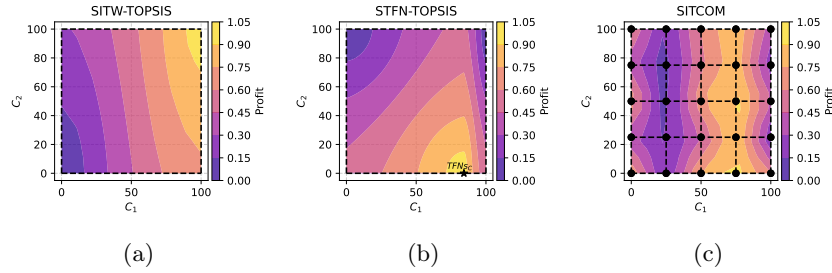


Fig. 10: Comparison of reconstructed preference surfaces for different reidentification methods applied to the multiple-extrema function.

Figure 11 further confirms that SITW-TOPSIS has the highest deviations, showing substantial errors across the entire domain. STFNTOPSIS shows lower but still notable deviations, as it fails to fully replicate all preference peaks.

SITCOM demonstrates the smallest differences, confirming that it is the most effective method for this type of preference structure.

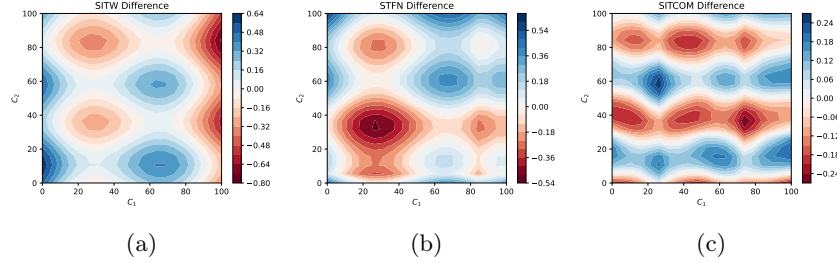


Fig. 11: Visualization of differences between the original multiple-extrema function and reconstructed models.

The results in Table 3 further support the conclusions from Figures 10 and 11. SITW-TOPSIS has the highest error margins and standard deviation, confirming its poor fit for multiple-extrema preferences. STFN-TOPSIS achieves lower deviations, but its performance is inconsistent across different regions. SITCOM has the lowest error values and standard deviation, demonstrating its suitability for reidentification in cases where multiple preference peaks exist.

Table 3: Statistical evaluation of reidentification accuracy for the multiple-extrema function.

| | SITW | STFN | SITCOM |
|------|-----------|-----------|-----------|
| Min | -0.736977 | -0.486978 | -0.267628 |
| Mean | -0.001813 | 0.019604 | -0.008462 |
| Max | 0.627975 | 0.615861 | 0.268190 |
| Std | 0.238251 | 0.212547 | 0.115615 |

4 Conclusions

This study examined the effectiveness of SITW-TOPSIS, STFN-TOPSIS, and SITCOM in reconstructing decision-maker preferences across different decision-making scenarios modeled by benchmark functions. The results indicate that SITW-TOPSIS is most suitable for monotonic preferences, providing accurate and stable approximations when preferences follow a strictly increasing or decreasing trend. STFN-TOPSIS demonstrated superior performance in cases with a single preference peak, effectively capturing the structured nonmonotonic preference model. SITCOM emerged as the most effective method for complex

decision-making problems characterized by multiple local optima, where reference-object-based modeling provided the highest accuracy. These findings highlight that the choice of reidentification method must be aligned with the structure of the decision-maker's preferences to ensure accurate modeling.

Future research should focus on extending the benchmark set to include asymmetrical and discontinuous preference structures, as well as validating these methods on real-world decision problems. Further investigations into hybrid approaches that integrate multiple reidentification techniques may enhance accuracy by leveraging the strengths of different methods. Additionally, incorporating uncertainty-based modeling, such as probabilistic or fuzzy approaches, could improve robustness in dynamic decision-making environments. The findings presented in this study contribute to the advancement of Multi-Criteria Decision Analysis by providing a structured evaluation of preference reidentification techniques, offering practical guidance for decision-makers in selecting appropriate methods based on the characteristics of their decision problems.

Acknowledgments. A bold run-in heading in small font size at the end of the paper is used for general acknowledgments, for example: This study was funded by X (grant number Y).

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