Issues Importance Analysis for Reaching High-Quality Consensus in Preference-Based Conflict Scenarios

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Abstract. The paper introduces a novel, systematic method for identifying the most critical issues impacting consensus quality in preference-based conflict situations. By analyzing the effect of removing individual issues on consensus outcomes – both for the entire group of agents and within smaller coalitions – the proposed approach provides an understanding of prioritizing negotiation efforts and reducing conflict intensity. A case study demonstrates the practical effectiveness of the method in real-world decision-making contexts, showing how it can guide agents toward higher-quality agreements. While the approach offers a structured framework for issue importance analysis, its current application is limited to scenarios with clearly defined agent preferences.

Keywords: Preference-based conflict, Multi-Criteria Decision-Making, Consensus, Coalition formation, Decision support systems.

1 Introduction

Achieving consensus in Multi-Criteria Decision-Making (MCDM) scenarios involving multiple agents with conflicting preferences is a fundamental challenge in areas such as corporate strategy, resource allocation, and policy development. The ability to identify key issues that significantly influence consensus quality is crucial for improving negotiation efficiency and reducing conflict intensity. Addressing this challenge requires systematic approaches that account for both individual and collective preferences while balancing competing priorities across diverse stakeholders.

MCDM research focused on developing a range of techniques, such as multi-attribute utility theory, analytic hierarchy process, and ELECTRE, to tackle different types of decision problems, including selection, ranking, and sorting of alternatives [1, 6, 14]. The development of MCDM has been further enhanced

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by the incorporation of concepts from fuzzy set theory and three-way decision-making, which have significantly improved the capacity to handle uncertainty and vagueness inherent in many real-world decision scenarios [7, 12, 13]. This paper follows a more qualitative approach proposed by Pawlak [8, 9] and considers a formal framework to analyze preference-based conflict situations proposed in [5]. Conflict analysis plays a crucial role in decision-making and has been extensively explored from various perspectives in the literature [2, 4, 10]. Various approaches, such as preference aggregation techniques and conflict resolution models, have been proposed to facilitate decision-making in complex scenarios [3, 11]. However, these methods often fail to systematically assess the contribution of individual issues to overall consensus quality. While traditional MCDM techniques provide structured frameworks for ranking and selecting alternatives based on multiple criteria, they often require explicit weighting and aggregation of preferences, which may not fully capture the dynamic interaction of individual issues in consensus formation, especially in conflicting stakeholder interests. In contrast, the proposed method systematically evaluates the impact of removing individual issues on consensus quality, offering a more granular analysis of issue importance. Unlike sensitivity analysis that measures output variation under input perturbations, our method introduces structural conflict resolution by identifying which issue removals maximally improve consensus quality through inverse problem-solving – determining optimal negotiation pathways rather than assessing parameter sensitivity.

This paper introduces a novel method to assess the importance of individual issues in achieving high-quality consensus. By systematically analyzing the effect of removing specific issues on consensus quality, both across the entire set of agents and within smaller coalitions, the proposed approach provides actionable insights for prioritizing negotiation efforts. The results demonstrate the effectiveness of this method in reducing conflict and improving consensus quality, offering valuable guidance for decision-makers in multi-agent environments.

The remainder of this paper is structured as follows: Section 2 presents the methods, including key definitions and the proposed algorithm. Section 3 provides a case study to illustrate the practical application of the approach. Section 4 offers a comparative discussion of the findings, while Section 5 concludes the paper with insights and future research directions.

2 Methods

Multi-criteria decision-making methods involve determining preferences among a set of alternatives. In this study, we employ a conflict model consisting of a set of agents – that can be interpreted as different evaluation criteria or perspectives – and a set of issues, which can be recognized as a representation of the available alternatives. The paper uses the conflict model with preferences [5], defines consensus, and investigates the impact of issues on reaching consensus. In other words, we investigate how the omission of a particular issue affects the ability to reach a higher-quality consensus.

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Definition 1. *A preference-based conflict situation is represented as a triplet* $PS = (A, I, \{\succ_a | a \in A\})$*, where* A *is a set of agents,* I *is a set of issues and* $\succ_a$ *is a strict preference relation over issues for each agent.*

Preferences are expressed through pairwise comparisons of issues by agents. Specifically, for each agent a, the strict preference relation $\succ_a$ represents a set of ordered pairs of issues (i, j), where agent a prefers issue i over issue j. Formally, $\succ_a = \{(i, j) \in I \times I | a \text{ supports } i \text{ more than } j\}$. When an agent does not express a strict preference between two issues, they are considered indifferent. The indifference relation $\sim_a$ captures this lack of preference and is defined as: $\sim_a = \{(i, j) \in I \times I | \neg(i \succ_a j) \wedge \neg(j \succ_a i)\}$. To complete the set of possible relations, we introduce the reverse preference relation. This relation inverts the direction of the strict preference ordering. For an agent $a \in A$, it is defined as: $\prec_a = \{(j, i) \in I \times I | (i, j) \in \succ_a\}$.

For a pair of issues (i, j), the conflict degree between two agents a and b (denoted by $c^{ij}(a, b)$) is determined by their preferences. Agreement: In the event that both agents exhibit a preference for i over j or conversely j over i, or if both agents show indifference to i and j, the degree of conflict is indicated as $\lambda^{=}$. Partial Agreement: In circumstances where one agent demonstrates indifference while the other expresses a preference, the degree of conflict is represented as $\lambda^{\approx}$. Disagreement: When one agent expresses a preference for i over j, while the other exhibits a preference for j over i, the degree of conflict is identified as $\lambda^{\neq}$.

The overall conflict degree between two agents over a set of issues $J \subseteq I$ is the sum of the conflict degrees for all pairs of issues from J: $c^J(a, b) = \sum_{\{i,j\} \in \hat{P}^J} c^{ij}(a, b)$ where $c^{ij}(a, b)$ is the conflict degree between agents a and b for the pairs of issues $\{i, j\}$ coming from a set $\hat{P}^J$ of all unordered pairs in J.

To enable thorough comparison, the conflict degree is standardized to a continuum $[0, 1]$. This process requires modifying the degree of unrefined conflict according to its minimum and maximum potential values: $\bar{c^J}(a, b) = \frac{c^J(a,b) - \min(c^J)}{\max(c^J) - \min(c^J)}$, where $\min(c^J) = \sum_{\{i,j\} \in \hat{P}^J} \lambda^{=} = (\frac{|J|(|J|-1)}{2} + |J|)\lambda^{=}$ and $\max(c^J) = \sum_{\{i,j\} \in \hat{P}^J} \lambda^{\neq} = (\frac{|J|(|J|-1)}{2} + |J|)\lambda^{\neq}$.

The relationships among the agents can be characterized according to the established normalized degree of conflict. For example, when the conflict degree between two agents falls below a predetermined threshold, they are deemed to be in a collaborative alliance. A more precise explanation is provided in Definition 2.

Definition 2. *For a preference-based conflict situation* PS*, the alliance relation* R^A*, neutrality relation* R^N*, and conflict relation* R^C *on* A *with respect to a set of issues* $J \subseteq I$ *are defined as:* $R^A = \{(a, b) \in A \times A \mid \bar{c^J}(a, b) < \beta\}$, $R^N = \{(a, b) \in A \times A \mid \beta \leq \bar{c^J}(a, b) \leq \alpha\}$, $R^C = \{(a, b) \in A \times A \mid \bar{c^J}(a, b) > \alpha\}$, *where* $\alpha, \beta \in [0, 1]$ *are two given thresholds satisfying* $\beta \leq c_0 \leq \alpha$ *with* $c_0 \in (0, 1)$ *representing the neutral point of the normalized conflict degree.*

The above approach allows us to identify a set of alliance agents. The concept of consensus is defined to determine agreement among agents and establish

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Algorithm 1: Evaluation the issue's importance in obtaining consensus

Data: A preference-based conflict situation $PS = (A, I, \{\succ_a | a \in A\})$. Method of calculating γ

Result: X – a set of issues essential in obtaining consensus for a set of agents A. X_C – a set of issues essential in obtaining consensus for coalitions.

begin
  $X \leftarrow$ empty list; `// Key issues required to reach consensus in` A
  $X_C \leftarrow$ empty list; `// Issues for achieving consensus in coalitions`
  foreach $i \in I$ **do**
    Consider a preference-based conflict situation $PS^i = (A; I^i; \{\succ_a^{I^i} | a \in A\})$ where $I^i = I \setminus \{i\}$;
    foreach *pair* $a, b \in A$ **do**
      Calculate normalized degree of conflict $C^{I^i}(a, b)$
    Calculate γ using a specific method;
    Determine coalitions $C_1, \ldots, C_n$;
    Determine the preference-based consensus $PCons_A^{I^i}$ and the weak consensus $WCons_A^{I^i}$ for the set of agents A;
    foreach *coalition* C_j **do**
      Determine the preference-based consensus $PCons_{C_j}^{I^i}$ and the weak consensus $WCons_{C_j}^{I^i}$ for the coalition C_j;
    Calculate the quality of the consensus $PConQ_A^{I^i}$ and $WConQ_A^{I^i}$;
    Calculate the average quality of consensus for coalitions $PConQ_{C_1,\ldots,C_n}^{I^i} = \frac{1}{n}\sum_{j=1}^{n} PConQ_{C_j}^{I^i}$ and $WConQ_{C_1,\ldots,C_n}^{I^i} = \frac{1}{n}\sum_{j=1}^{n} WConQ_{C_j}^{I^i}$
  $X = \arg\max_{i \in I}\{PConQ_A^{I^i}, WConQ_A^{I^i}\}$; `//` $\arg\max_x f(x)$ `denotes the value of` x `that maximizes the function` $f(x)$
  $X_C = \arg\max_{i \in I}\{PConQ_{C_1,\ldots,C_n}^{I^i}, WConQ_{C_1,\ldots,C_n}^{I^i}\}$;
return X, X_C

a partially or fully compatible ordering of issues. We propose two types of consensus. The first type (Definition 3) is more restrictive and requires that all agents agree on the preference of issue i over issue j. The second type of consensus (Definition 4) is more gentle and recognizes a preference for issue i over issue j provided that at least one agent agrees with it and none of the other agents oppose such a preference. Thus, a possible indifference relation is considered in favor of the agent who prefers one issue over the other. To better understand the role of individual attributes in driving conflict, we systematically compare the degree of conflict before and after removal of each attribute. This comparison allows us to quantify how the presence or absence of specific attributes affects both global and coalition-based conflict levels.

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Definition 3. *For any subset of agents $B \subseteq A$ the preference-based consensus over a set of issues $J \subseteq I$ is defined as:* $PCons_B^J = \bigcap_{a \in B} \succ_a^J$

Definition 4. *For a subset of agents $B \subseteq A$ a weak consensus over a set of issues $J \subseteq I$ is defined as follows* $WCons_B^J = (\bigcup_{a \in B} \succ_a^J) \setminus (\bigcup_{a \in B} \prec_a^J)$

The consensus obtained does not have to be of weak order (satisfies irreflexivity, asymmetry, transitivity, and negative transitivity). It will be just a selected subset of issue preferences agreed upon by all agents. The more common preferences a consensus contains, the better its quality. To be able to compare the quality of consensus we introduce a general measure in Definition 5.

Definition 5. *For a subset of agents $B \subseteq A$ the quality of consensus over the set of issues $J \subseteq I$ is defined by:* $Q_B^J = \frac{|Cons_B^J|}{|P^J \setminus T^J|}$ *where $Cons_B^J$ is any set of issues defined in Definitions 3 and 4, and P^J is the set of all ordered pairs in a set of issues J. In the definition, we subtract the pairs of the same issue i.e. $T^J = \{(i,i) : i \in J\}$ as they are not causing any conflict and can exist only in the indifference relation.*

It should be further noted that consensus can be determined in the full set of agents or for a subset of compatible agents. Based on the alliance relation and the conflict degree, we define the coalition of agents for which the consensus will be determined.

Definition 6. *The subset of agents $C \subseteq A$ is forming a coalition with a threshold γ if all pairs of agents (a,b) from C are within an alliance relation i.e. $\bar{c^J}(a,b) < \gamma$ or the coalition consists of one agent $\{a\}$ if there is no agent $\{b\}$ such that $\bar{c^J}(a,b) < \gamma$.*

Our goal is to investigate how reaching an agreement on a specific issue affects the state of the conflict and the level of consensus. In addition, how does the elimination of an issue affect the quality of the consensus? To analyze this, we can assess the impact of each issue by systematically removing them one by one from the set I. This approach will enable us to assess the contribution of each issue to the overall conflict dynamics and consensus formation. Such an analysis will allow us to determine the issue that is most important in a given situation and provide important guidelines in the negotiation process to establish consensus.

The pseudocode of the algorithm to evaluate the importance of issues in obtaining consensus is presented in Algorithm 1. In line 4 of Algorithm 1, a loop iterates over each issue, defining a preference-based conflict situation while omitting the current issue. Coalitions $C \subseteq A$ are formed when pairwise conflict degrees $\bar{c^J}(a,b)$ fall below threshold γ. It can be calculated as $\gamma = \frac{1}{|A|^2} \sum_{a,b \in A} \bar{c^J}(a,b)$. This ensures coalition members have conflict levels below the group average. The consensus quality is then evaluated for both the entire set of agents A and coalitions. Once the loop completes, the issues are selected in such a way that removing them results in consensus with the highest

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quality. A selected set of key issues demands the most attention during negotiations. Achieving agreement on these issues ensures the highest quality of consensus. Of course, the algorithm can be easily modified in case one is interested in determining only one of the consensuses: preference-based consensus or weak consensus.

3 Case study – Critical issues in achieving consensus

Consider a multi-criteria decision-making scenario where six managers $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, must agree on the prioritization of five strategic projects $I = \{i_1, i_2, i_3, i_4, i_5\}$, for a company's annual development plan. Each project represents a different focus area, such as innovation (i_1), cost reduction (i_2), employee training (i_3), marketing expansion (i_4) and sustainability (i_5). Each manager evaluates these projects based on their individual priorities and goals, leading to preference relations as follows:
a_1 Prefers cost reduction, training, marketing, and sustainability over innovation, indicating a focus on operational efficiency.
a_2 Values innovation above all other projects, but also ranks cost reduction higher than training, marketing, and sustainability.
a_3 Prioritizes innovation and sustainability while placing a lower importance on cost reduction and marketing.
a_4 Sees innovation as critical but also emphasizes marketing and cost reduction over training and sustainability.
a_5 Consistently prioritizes innovation across all comparisons, suggesting a strong alignment with the company's long-term growth vision.
a_6 Has a mixed set of preferences, balancing innovation, cost reduction, and sustainability, while de-emphasizing training and marketing.
In this scenario, the managers must navigate their conflicting preferences to identify a consensus that aligns with the company's overall strategy. More formally, the preference relations are: $\succ^I_{a_1} = \{(i_2, i_1), (i_3, i_1), (i_4, i_1), (i_5, i_1)\}$, $\succ^I_{a_2} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4), (i_1, i_5), (i_2, i_3), (i_2, i_4), (i_2, i_5)\}$, $\succ^I_{a_3} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4), (i_1, i_5), (i_5, i_2), (i_5, i_3), (i_5, i_4)\}$, $\succ^I_{a_4} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5), (i_4, i_2), (i_4, i_3), (i_4, i_5)\}$, $\succ^I_{a_5} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4), (i_1, i_5)\}$, $\succ^I_{a_6} = \{(i_1, i_3), (i_2, i_1), (i_2, i_3), (i_2, i_4), (i_4, i_3), (i_5, i_1), (i_5, i_3), (i_5, i_4)\}$. The corresponding indifference relations are: $\sim^I_{a_1} = \{(i_1, i_1), (i_2, i_2), (i_2, i_3), (i_2, i_4), (i_2, i_5), (i_3, i_2), (i_3, i_3), (i_3, i_4), (i_3, i_5), (i_4, i_2), (i_4, i_3), (i_4, i_4), (i_4, i_5), (i_5, i_2), (i_5, i_3), (i_5, i_4), (i_5, i_5)\}$, $\sim^I_{a_2} = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_3, i_4), (i_3, i_5), (i_4, i_3), (i_4, i_4), (i_4, i_5), (i_5, i_3), (i_5, i_4), (i_5, i_5)\}$, $\sim^I_{a_3} = \{(i_1, i_1), (i_2, i_2), (i_2, i_3), (i_2, i_4), (i_3, i_2), (i_3, i_3), (i_3, i_4), (i_4, i_2), (i_4, i_3), (i_4, i_4), (i_5, i_5)\}$, $\sim^I_{a_4} = \{(i_1, i_1), (i_1, i_4), (i_2, i_2), (i_2, i_3), (i_2, i_5), (i_3, i_2), (i_3, i_3), (i_3, i_5), (i_4, i_1), (i_4, i_4), (i_5, i_2), (i_5, i_3), (i_5, i_5)\}$, $\sim^I_{a_5} = \{(i_1, i_1), (i_2, i_2), (i_2, i_3), (i_2, i_4), (i_2, i_5), (i_3, i_2), (i_3, i_3), (i_3, i_4), (i_3, i_5), (i_4, i_2), (i_4, i_3), (i_4, i_4), (i_4, i_5), (i_5, i_2), (i_5, i_3), (i_5, i_4), (i_5, i_5)\}$, $\sim^I_{a_6} = \{(i_1, i_1), (i_1, i_4), (i_2, i_2), (i_2, i_5), (i_3, i_3), (i_4, i_1), (i_4, i_4), (i_5, i_2), (i_5, i_5)\}$

For the considered example and the set of agents A the consensus is equal $PCons^I_A = \emptyset$ and the weak consensus $WCons^I_A = \{(i_2, i_3), (i_4, i_3), (i_5, i_3)\}$. Now,

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let us consider the consensus within the coalition. Table 1 lists the conflict degree expressed with $\lambda^{=}$, $\lambda^{\approx}$, and $\lambda^{\neq}$. Table 2 shows the normalized degree of conflict with $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, and $\lambda^{\neq} = 2$.

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	$15\lambda^{=}$	$8\lambda^{=} + 3\lambda^{\approx} + 4\lambda^{\neq}$	$8\lambda^{=} + 3\lambda^{\approx} + 4\lambda^{\neq}$	$8\lambda^{=} + 4\lambda^{\approx} + 3\lambda^{\neq}$	$11\lambda^{=} + 0\lambda^{\approx} + 4\lambda^{\neq}$	$8\lambda^{=} + 6\lambda^{\approx} + 1\lambda^{\neq}$
a_2		$15\lambda^{=}$	$10\lambda^{=} + 4\lambda^{\approx} + 1\lambda^{\neq}$	$9\lambda^{=} + 5\lambda^{\approx} + 1\lambda^{\neq}$	$12\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$8\lambda^{=} + 5\lambda^{\approx} + 2\lambda^{\neq}$
a_3			$15\lambda^{=}$	$9\lambda^{=} + 5\lambda^{\approx} + 1\lambda^{\neq}$	$12\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$8\lambda^{=} + 5\lambda^{\approx} + 2\lambda^{\neq}$
a_4				$15\lambda^{=}$	$11\lambda^{=} + 4\lambda^{\approx} + 0\lambda^{\neq}$	$9\lambda^{=} + 2\lambda^{\approx} + 4\lambda^{\neq}$
a_5					$15\lambda^{=}$	$7\lambda^{=} + 6\lambda^{\approx} + 2\lambda^{\neq}$
a_6						$15\lambda^{=}$

Table 1. Conflict degree c^I express with $\lambda^{=}$, $\lambda^{\approx}$, and $\lambda^{\neq}$

Let us assume that the threshold value γ for an allied relation is equal to the average value of the normalized degrees of conflict between different agents. Therefore, in our case it is equal to $\gamma = \frac{116}{30} \div 15 = 0.258$ Thus, each pair of agents for which the degree of conflict is less than 0.258 is in an alliance relation. The graphical representation of this relation, where the nodes are agents and the alliance relation is represented by edges, is shown in Figure 1. As can be seen in the presented situation, we can identify three coalitions $\{a_2, a_3, a_4, a_5\}, \{a_1\}, \{a_6\}$. Coalitions represent departments with aligned priorities (e.g. first coalition as R&D-focused units). Threshold γ is set as the average conflict degree to balance group cohesion and practical manageability.

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	$\frac{11}{30}$	$\frac{11}{30}$	$\frac{10}{30}$	$\frac{8}{30}$	$\frac{8}{30}$
a_2		0	$\frac{6}{30}$	$\frac{7}{30}$	$\frac{3}{30}$	$\frac{7}{30}$
a_3			0	$\frac{7}{30}$	$\frac{3}{30}$	$\frac{7}{30}$
a_4				0	$\frac{4}{30}$	$\frac{10}{30}$
a_5					0	$\frac{10}{30}$
a_6						0

Table 2. Normalized conflict degree $\bar{c^I}$ – $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, $\lambda^{\neq} = 2$

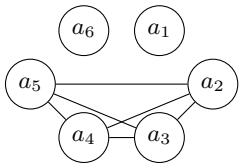

Fig. 1. Allied relation

Now, let us consider the consensus within each coalition separately. For coalition $C_1 = \{a_2, a_3, a_4, a_5\}$ we have: $PCons^I_{C_1} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5)\}$ and $WCons^I_{C_1} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4), (i_1, i_5), (i_2, i_3), (i_4, i_3), (i_5, i_3)\}$. For the remaining single-element coalitions $C_2 = \{a_1\}$, $C_3 = \{a_6\}$, consensus and weak consensus are the same as the preference relation of the agents belonging to the coalition. As we can see, coalitions provide a framework for analyzing the interactions of subsets of agents with similar or compatible preferences. This is especially relevant in scenarios where reaching full consensus is difficult due to conflicting views of agents. By examining coalitions, we can identify agent-specific patterns and influences on consensus formation. This is valuable for understanding the role of agents such as a_1 and a_6 that appear as outliers in their

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preferences. The coalition framework enables deeper analysis of how removing specific agents affects group agreements.

3.1 Importance of issue i_1

We start the analysis by removing the issue i_1. Let us consider a preference-based conflict situation $PS^{i_1} = (A; I^{i_1}; \{\succ_a^{I^{i_1}} | a \in A\})$, where $A = \{a_1, \ldots, a_6\}$, $I^{i_1} = \{i_2, i_3, i_4, i_5\}$, and the preference relations are: $\succ_{a_1}^{I^{i_1}} = \emptyset$, $\succ_{a_2}^{I^{i_1}} = \{(i_2, i_3), (i_2, i_4), (i_2, i_5)\}$, $\succ_{a_3}^{I^{i_1}} = \{(i_5, i_2), (i_5, i_3), (i_5, i_4)\}$, $\succ_{a_4}^{I^{i_1}} = \{(i_4, i_2), (i_4, i_3), (i_4, i_5)\}$, $\succ_{a_5}^{I^{i_1}} = \emptyset$, $\succ_{a_6}^{I^{i_1}} = \{(i_2, i_3), (i_2, i_4), (i_4, i_3), (i_5, i_3), (i_5, i_4)\}$

Table 3 lists the conflict degree expressed with $\lambda^{=}$, $\lambda^{\approx}$, and $\lambda^{\neq}$. Table 4 shows the normalized degree of conflict with $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, and $\lambda^{\neq} = 2$ - we compare it to the baseline (Table 2). For agents a_1 and a_5, the conflict degree decreases significantly, indicating that i_1 was a polarizing issue in their preferences. However, the conflict degree increases slightly between a_2 and a_6, suggesting that i_1 played a unifying role for this pair. This indicates that i_1 has a mixed effect, reducing conflict for some agents (e.g., a_2, a_3, a_5) while destabilizing others (e.g., a_1, a_6).

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	$10\lambda^{=}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$10\lambda^{=} + 0\lambda^{\approx} + 0\lambda^{\neq}$	$5\lambda^{=} + 5\lambda^{\approx} + 0\lambda^{\neq}$
a_2		$10\lambda^{=}$	$5\lambda^{=} + 4\lambda^{\approx} + 1\lambda^{\neq}$	$5\lambda^{=} + 4\lambda^{\approx} + 1\lambda^{\neq}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$6\lambda^{=} + 4\lambda^{\approx} + 0\lambda^{\neq}$
a_3			$10\lambda^{=}$	$5\lambda^{=} + 4\lambda^{\approx} + 1\lambda^{\neq}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$6\lambda^{=} + 4\lambda^{\approx} + 0\lambda^{\neq}$
a_4				$10\lambda^{=}$	$7\lambda^{=} + 3\lambda^{\approx} + 0\lambda^{\neq}$	$6\lambda^{=} + 2\lambda^{\approx} + 2\lambda^{\neq}$
a_5					$10\lambda^{=}$	$5\lambda^{=} + 5\lambda^{\approx} + 0\lambda^{\neq}$
a_6						$10\lambda^{=}$

Table 3. Conflict degree $c^{I^{i_1}}$ – $\lambda^{=}$, $\lambda^{\approx}$, and $\lambda^{\neq}$

Assume the threshold value for an allied relation is defined as the average of the normalized degrees of conflict between different agents. In this case, it is calculated as $\frac{60}{20} \div 15 = 0.2$. Figure 2 provides a graphical representation of this relation, where nodes represent agents and edges depict allied relations. In the current scenario, we can identify four coalitions: $\{a_1, a_2, a_5\}$, $\{a_1, a_3, a_5\}$, $\{a_1, a_4, a_5\}$, and $\{a_6\}$.

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{3}{20}$	0	$\frac{5}{20}$
a_2		0	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{4}{20}$
a_3			0	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{4}{20}$
a_4				0	$\frac{3}{20}$	$\frac{6}{20}$
a_5					0	$\frac{5}{20}$
a_6						0

Table 4. Normalized conflict degree $c^{I^{i_1}}$ – $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, $\lambda^{\neq} = 2$

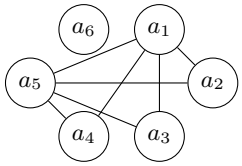

Fig. 2. Allied relation for preference-based conflict situation PS^{i_1}

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The consensus and weak consensus for the entire set of agents are $PCons_A^{I^{i_1}} = \emptyset$ and $WCons_A^{I^{i_1}} = \{(i_2, i_3), (i_4, i_3), (i_5, i_3)\}$. This reflects the fact that each agent prefers one of the issues over the i_3 issue. In fact, agent a_2 prefers issue i_2 over the others, agent a_3 prefers issue i_5 over the others, agent a_4 prefers issue i_4 over the others and agent a_6 prefers issues i_2 and i_5 over the others. All agents have in common that they least prefer the i_3 issue and this is illustrated in the weak consensus for them.

Now, let us consider the consensus and weak consensus obtained for each individual coalition. For the coalition $C_1 = \{a_1, a_2, a_5\}$ we have the consensus $PCons_{C_1}^{I^{i_1}} = \emptyset$ and the weak consensus $WCons_{C_1}^{I^{i_1}} = \{(i_2, i_3), (i_2, i_4), (i_2, i_5)\}$. For the coalition $C_2 = \{a_1, a_3, a_5\}$ we have the consensus $PCons_{C_2}^{I^{i_1}} = \emptyset$ and the weak consensus $WCons_{C_2}^{I^{i_1}} = \{(i_5, i_2), (i_5, i_3), (i_5, i_4)\}$. For the coalition $C_3 = \{a_1, a_4, a_5\}$ we have the consensus $PCons_{C_3}^{I^{i_1}} = \emptyset$ and the weak consensus $WCons_{C_3}^{I^{i_1}} = \{(i_4, i_2), (i_4, i_3), (i_4, i_5)\}$. For the coalition $C_4 = \{a_6\}$, the consensus and the weak consensus are both equal to the agent a_6 relations. In this situation the consensus for the coalitions is less interesting than the consensus for all agents A because the consensus for each coalition consists of preferences of single agents (for $C_1 - a_2$; for $C_2 - a_3$; for $C_3 - a_4$). This is a reflection of the fact that after the removal of issue i_1 agents a_1 and a_5 have no preferences.

Let us now evaluate the quality of consensus obtained for both the full set of agents and the individual coalitions after removing issue i_1. For the full set of agents, the quality of consensus is $PConQ_A^{I^{i_1}} = 0$ and the quality of weak consensus is $WConQ_A^{I^{i_1}} = \frac{3}{20}$. On the other hand, for coalitions, the average consensus quality is equal $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_1}} = (0+0+0+\frac{5}{20})/4 = \frac{5}{80}$ and the quality of weak consensus is $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_1}} = (\frac{3}{20}+\frac{3}{20}+\frac{3}{20}+\frac{5}{20})/4 = \frac{7}{40}$. It is evident that the highest quality of consensus after removing issue i_1 is achieved for weak consensus when considering coalitions.

3.2 Importance of issue i_2

We continue our analysis by removing the i_2 issue. Let us consider a preference-based conflict situation $PS^{i_2} = (A; I^{i_2}; \{\succ_a^{I^{i_2}} | a \in A\})$, where $A = \{a_1, \ldots, a_6\}$, $I^{i_2} = \{i_1, i_3, i_4, i_5\}$, and the preference relations are as follows: $\succ_{a_1}^{I^{i_2}} = \{(i_3, i_1), (i_4, i_1), (i_5, i_1)\}$, $\succ_{a_2}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5)\}$, $\succ_{a_3}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5), (i_5, i_3), (i_5, i_4)\}$, $\succ_{a_4}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_5), (i_4, i_3), (i_4, i_5)\}$, $\succ_{a_5}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5)\}$, $\succ_{a_6}^{I^{i_2}} = \{(i_1, i_3), (i_4, i_3), (i_5, i_1), (i_5, i_3), (i_5, i_4)\}$

Table 5 presents the normalized degree of conflict with $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, and $\lambda^{\neq} = 2$. Comparing this with the normalized degree of conflict after removing issue i_1 (Table 4), we observe that for agents a_2, a_3, a_4 and a_5, the values are lower when issue i_2 is removed compared to i_1. The opposite is true for agents a_1 and a_6. The average normalized degrees of conflict between the agents is now $\frac{69}{20} \div 15 = 0.23$. Figure 3 shows a graphical representation of allied relations for the threshold value 0.23 – we can identify four coalitions: a_2, a_4, a_5, a_2, a_3, a_5,

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a_3, a_6, and a_1. Comparing the normalized conflict degrees after removing i_2 to the baseline (Table 2), we note a reduction in conflict between a_2, a_3, and a_5, facilitating the formation of the coalition a_2, a_3, a_5. Conversely, conflict increases between a_1 and a_6, indicating that i_2 was pivotal in maintaining alignment between these agents. This suggests that i_2 is a significant driver of conflict in coalitions with high variability in preferences but acts as a stabilizing factor for outliers.

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	$\frac{6}{20}$	$\frac{8}{20}$	$\frac{7}{20}$	$\frac{6}{20}$	$\frac{6}{20}$
a_2		0	$\frac{2}{20}$	$\frac{3}{20}$	0	$\frac{0}{20}$
a_3			0	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{4}{20}$
a_4				0	$\frac{3}{20}$	$\frac{5}{20}$
a_5					0	$\frac{6}{20}$
a_6						0

Table 5. Normalized conflict degree $c^{I^{i_2}}$ – $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, $\lambda^{\neq} = 2$

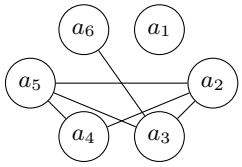

Fig. 3. Allied relation for preference-based conflict situation PS^{i_2}

The consensus and weak consensus for all agents are $PCons_A^{I^{i_2}} = \emptyset$ and $WCons_A^{I^{i_2}} = \{(i_4, i_3), (i_5, i_3)\}$. The weak consensus indicates that agents prefer issue i_4 and i_5 over issue i_3.

For coalition $C_1 = \{a_2, a_4, a_5\}$: $PCons_{C_1}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_5)\}$ and $WCons_{C_1}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5), (i_4, i_3), (i_4, i_5)\}$. Based on the weak consensus, a clear hierarchy of issues appears: the highest-rated issue is i_1 , followed by i_4, while the lowest-rated issues are i_3 and i_5. For coalition $C_2 = \{a_2, a_3, a_5\}$: $PCons_{C_2}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5)\}$ and $WCons_{C_2}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5), (i_5, i_3), (i_5, i_4)\}$. For both consensus and weak consensus the hierarchy of issues appears. Issue i_1 is the highest-rated, while the other three issues – i_3, i_4, and i_5 – share the second position. In contrast, under weak consensus, i_1 is followed by i_5, with i_3 and i_4 being the least preferred. In both consensus scenarios, some agents from coalition C_2 make concessions. In the consensus case, agent a_3 gives up its preference for issues $i_5 \succ i_3$ and $i_5 \succ i_4$. In contrast, under weak consensus, agents a_2 and a_5 agree on preferences regarding issues they were previously indifferent to. For coalition $C_3 = \{a_3, a_6\}$: $PCons_{C_3}^{I^{i_2}} = \{(i_1, i_3), (i_5, i_3), (i_5, i_4)\}$ and $WCons_{C_3}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_4, i_3), (i_5, i_3), (i_5, i_4)\}$. In the weak consensus case, the hierarchy of issues is as follows: i_1 and i_5 remain the highest-rated, i_4 is in second place, and i_3 is the least preferred. For the coalition $C_4 = \{a_1\}$, the consensus and the weak consensus are equal to the agent a_1 preferences.

We now analyze the quality of consensus achieved for both the complete set of agents and the individual coalitions after the removal of issue i_2. For the entire set of agents, the consensus quality is $PConQ_A^{I^{i_2}} = 0$ and the quality of weak consensus is $WConQ_A^{I^{i_2}} = \frac{2}{20}$. On the other hand, for coalitions, the average consensus quality is equal $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_2}} = (\frac{2}{20} + \frac{3}{20} + \frac{3}{20} + \frac{3}{20})/4 = \frac{11}{80}$ and the quality of weak consensus is $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_2}} = (\frac{5}{20} + \frac{5}{20} + \frac{5}{20} + \frac{3}{20})/4 = \frac{9}{40}$.

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These results show that weak consensus yields the highest quality when coalitions are considered. It is also evident that the quality of consensus achieved after removing issue i_2 is higher than that obtained after removing issue i_1. This indicates that agents' opinions on issue i_2 are more divergent and contribute to greater conflict.

3.3 Importance of issue i_3

We perform an analogous analysis for the issue i_3 removal. Let us consider a preference-based conflict situation $PS^{i_3} = (A; I^{i_3}; \{\succ_a^{I^{i_3}} | a \in A\})$, where $A = \{a_1, \ldots, a_6\}$, $I^{i_3} = \{i_1, i_2, i_4, i_5\}$, and the preference relations are as follows: $\succ_{a_1}^{I^{i_3}} = \{(i_2, i_1), (i_4, i_1), (i_5, i_1)\}$, $\succ_{a_2}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_4), (i_1, i_5), (i_2, i_4), (i_2, i_5)\}$, $\succ_{a_3}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_4), (i_1, i_5), (i_5, i_2), (i_5, i_4)\}$, $\succ_{a_4}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_5), (i_4, i_2), (i_4, i_5)\}$, $\succ_{a_5}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_4), (i_1, i_5)\}$, $\succ_{a_6}^{I^{i_3}} = \{(i_2, i_1), (i_2, i_4), (i_5, i_1), (i_5, i_4)\}$.

Table 6 shows the normalized degree of the conflict with $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, and $\lambda^{\neq} = 2$. The graphical representation of the alliance relation for the threshold value (average normalized conflict degree) 0.273 is shown in Figure 4. As can be seen in the current situation, we can recognize two coalitions $\{a_2, a_3, a_4, a_5\}, \{a_1, a_6\}$

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	$\frac{8}{20}$	$\frac{8}{20}$	$\frac{7}{20}$	$\frac{6}{20}$	$\frac{2}{20}$
a_2		0	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{1}{20}$
a_3			0	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{2}{20}$
a_4				0	$\frac{3}{20}$	$\frac{8}{20}$
a_5					0	$\frac{7}{20}$
a_6						0

Table 6. Normalized conflict degree $c^{I^{i_3}}$ – $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, $\lambda^{\neq} = 2$

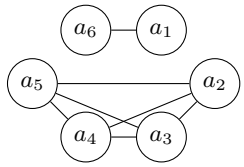

Fig. 4. Allied relation for preference-based conflict situation PS^{i_3}

The consensus and weak consensus for the set of agents are $PCons_A^{I^{i_3}} = \emptyset$ and $WCons_A^{I^{i_3}} = \emptyset$, therefore the quality is equal to $PConQ_A^{I^{i_3}} = WConQ_A^{I^{i_3}} = 0$. As can be seen with the removal of the i_3 issue, the agents have such different preferences that no common view can be found.

For coalition $C_1 = \{a_2, a_3, a_4, a_5\}$: $PCons_{C_1}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_5)\}$ and $WCons_{C_1}^{I^{i_3}} = \{(i_1, i_2), (i_1, i_4), (i_1, i_5)\}$. Thus, both consensus indicate that issue i_1 is preferred over issues i_2 and i_5. For the weak consensus issue i_4 is also preferred below i_1. For coalition $C_2 = \{a_1, a_6\}$: $PCons_{C_2}^{I^{i_3}} = \{(i_2, i_1), (i_5, i_1)\}$ and $WCons_{C_2}^{I^{i_3}} = \{(i_2, i_1), (i_2, i_4), (i_4, i_1), (i_5, i_1), (i_5, i_4)\}$. The hierarchy of issues is as follows: i_2 and i_5 are the highest-rated, followed by i_4 (weak consensus), with i_1 being the least preferred. The average consensus quality is $PConQ_{C_1,C_2}^{I^{i_3}} = (\frac{2}{20} + \frac{2}{20})/2 = \frac{1}{10}$ and in the case of weak consensus $WConQ_{C_1,C_2}^{I^{i_3}} = (\frac{3}{20} + \frac{5}{20})/2 = \frac{1}{5}$. This time also the weak consensus obtained the highest quality when coalitions are considered. Removing issue i_3 improves the quality of consensus compared to when

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issue i_1 was removed. However, it results in a lower quality of consensus compared to the removal of issue i_2. Thus, issue i_3 generates fewer conflicts compared to issue i_2. Among the three issues analyzed so far, achieving consensus on issue i_2 emerges as the most crucial, for obtaining good quality consensus.

3.4 Importance of issue i_4

Once we remove the i_4 issue we get the following preference-based conflict situation $PS^{i_4} = (A; I^{i_4}; \{\succ_a^{I^{i_4}} | a \in A\})$, where $A = \{a_1, \ldots, a_6\}$, $I^{i_4} = \{i_1, i_2, i_3, i_5\}$, and the preference relations are as follows: $\succ_{a_1}^{I^{i_4}} = \{(i_2, i_1), (i_3, i_1), (i_5, i_1)\}$, $\succ_{a_2}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5), (i_2, i_3), (i_2, i_5)\}$, $\succ_{a_3}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5), (i_5, i_2), (i_5, i_3)\}$, $\succ_{a_4}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5)\}$, $\succ_{a_5}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5)\}$, $\succ_{a_6}^{I^{i_4}} = \{(i_1, i_3), (i_2, i_1), (i_2, i_3), (i_5, i_1), (i_5, i_3)\}$

Table 7 shows the normalising degree of conflict with $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, and $\lambda^{\neq} = 2$. A graphical representation of allied relation for the threshold value (average of the normalized conflict degree) 0.227 is shown on Figure 5. We obtained the same situation as in Figure 4 when we removed the i_3 issue – two coalitions $\{a_2, a_3, a_4, a_5\}, \{a_1, a_6\}$ are identified.

A	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	$\frac{8}{20}$	$\frac{8}{20}$	$\frac{6}{20}$	$\frac{6}{20}$	$\frac{4}{20}$
a_2		0	$\frac{4}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{6}{20}$
a_3			0	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{6}{20}$
a_4				0	$\frac{0}{20}$	$\frac{6}{20}$
a_5					0	$\frac{6}{20}$
a_6						0

Table 7. Normalized conflict degree $c^{I^{i_4}}$ – $\lambda^{=} = 0$, $\lambda^{\approx} = 1$, $\lambda^{\neq} = 2$

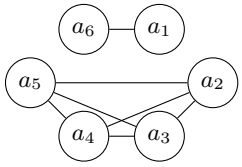

Fig. 5. Allied relation for preference-based conflict situation PS^{i_4}

The consensus and weak consensus for the set of agents are $PCons_A^{I^{i_4}} = \emptyset$ and $WCons_A^{I^{i_4}} = \{(i_2, i_3), (i_5, i_3)\}$ with quality $PConQ_A^{I^{i_4}} = 0$ and $WConQ_A^{I^{i_4}} = \frac{2}{20}$.

For coalition $C_1 = \{a_2, a_3, a_4, a_5\}$: $PCons_{C_1}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5)\}$ and $WCons_{C_1}^{I^{i_4}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_5), (i_2, i_3), (i_5, i_3)\}$, i_1 is the highest-rated, issues – i_2, i_3, and i_5 – share the second position for consensus or i_3 is the least preferred for weak consensus. For coalition $C_2 = \{a_1, a_6\}$: $PCons_{C_2}^{I^{i_4}} = \{(i_2, i_1), (i_5, i_1)\}$ and $WCons_{C_2}^{I^{i_4}} = \{(i_2, i_1), (i_2, i_3), (i_5, i_1), (i_5, i_3)\}$, i_2 and i_5 are the highest-rated, followed by i_1 for consensus and i_3 for weak consensus. The average consensus quality is equal $PConQ_{C_1,C_2}^{I^{i_4}} = (\frac{3}{20} + \frac{2}{20})/2 = \frac{5}{40}$ and the quality of weak consensus is $WConQ_{C_1,C_2}^{I^{i_4}} = (\frac{5}{20} + \frac{4}{20})/2 = \frac{9}{40}$. The weak consensus obtained the highest quality when coalitions are considered. The quality of consensus achieved after removing issue i_4 is identical to that obtained after removing issue i_2. Therefore, reaching a consensus on agents' views regarding either of these issues is equally important for achieving a high-quality consensus.

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3.5 Importance of issue i_5

After removing the i_5 issue we get the following preference-based conflict situation $PS^{i_5} = (A; I^{i_5}; \{\succ_a^{I^{i_5}} | a \in A\})$, where $A = \{a_1, \ldots, a_6\}$, $I^{i_5} = \{i_1, i_2, i_3, i_4\}$, and $\succ_{a_1}^{I^{i_5}} = \{(i_2, i_1),\ (i_3, i_1),\ (i_4, i_1)\}$, $\succ_{a_2}^{I^{i_5}} = \{(i_1, i_2),\ (i_1, i_3),\ (i_1, i_4),\ (i_2, i_3),\ (i_2, i_4)\}$, $\succ_{a_3}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4)\}$, $\succ_{a_4}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3), (i_4, i_2), (i_4, i_3)\}$, $\succ_{a_5}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4)\}$, $\succ_{a_6}^{I^{i_5}} = \{(i_1, i_3), (i_2, i_1), (i_2, i_3), (i_2, i_4), (i_4, i_3)\}$

Due to the page limit, we omit some calculations. For entire set of agents $PCons_A^{I^{i_5}} = \emptyset$ and $WCons_A^{I^{i_5}} = \{(i_2, i_3), (i_4, i_3)\}$ with quality $PConQ_A^{I^{i_5}} = 0$ and $WConQ_A^{I^{i_5}} = \frac{2}{20}$. For the coalition $C_1 = \{a_1\}$, the consensus and the weak consensus are equal to the agent's a_1 preferences. For coalition $C_2 = \{a_2, a_3, a_5\}$: $PCons_{C_2}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3), (i_1, i_4)\}$ and $WCons_{C_2}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3),\ (i_1, i_4),\ (i_2, i_3), (i_2, i_4)\}$. In the consensus, issue i_1 is the highest-rated, while the other three issues – i_2, i_3, and i_4 – share the second position. For the weak consensus the hierarchy of issues is as follows: i_1 remains the highest-rated, followed by i_2 in second place, with i_3 and i_4 being the least preferred. For coalition $C_3 = \{a_3, a_4, a_5\}$: $PCons_{C_3}^{I^{i_5}} = \{(i_1, i_2), (i_1, i_3)\}$ and $WCons_{C_3}^{I^{i_5}} = \{(i_1, i_2),\ (i_1, i_3),\ (i_1, i_4),\ (i_4, i_2),\ (i_4, i_3)\}$, i_1 remains the highest-rated, followed by i_4 in the second place (in weak consensus), with i_2 and i_3 being the least preferred. For coalition $C_4 = \{a_2, a_6\}$: $PCons_{C_4}^{I^{i_5}} = \{(i_1, i_3),\ (i_2, i_3),\ (i_2, i_4)\}$ and $WCons_{C_4}^{I^{i_5}} = \{(i_1, i_3), (i_1, i_4), (i_2, i_3), (i_2, i_4), (i_4, i_3)\}$, i_1 and i_2 are the highest-rated, followed by i_4, with i_3 being the least preferred (in weak consensus). The average consensus quality is equal $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_5}} = (\frac{3}{20} + \frac{3}{20} + \frac{2}{20} + \frac{3}{20})/4 = \frac{11}{80}$ and the quality of weak consensus is $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_5}} = (\frac{3}{20} + \frac{5}{20} + \frac{5}{20} + \frac{5}{20})/4 = \frac{9}{40}$. Like before the weak consensus obtained the highest quality when coalitions are considered. Moreover, reaching consensus on i_5 turns out to be just as important as reaching agreement on i_2 and i_4.

The analysis of the importance of individual issues in achieving consensus reveals significant differences in their impact on conflict resolution and coalition formation. Of the issues examined, i_2, i_4, and i_5 emerged as the most critical for achieving high-quality consensus, as their removal resulted in more structured and higher-quality weak consensus in coalitions. In contrast, i_1 and i_3 showed comparatively lower influence. The results highlight that weak consensus, rather than strict consensus, tends to yield higher overall quality, especially within smaller, well-defined coalitions. This emphasizes the importance of addressing specific high-impact issues to facilitate meaningful agreements in multi-agent conflict scenarios.

4 Comparisons and discussions

The example presented above illustrates how preference-based conflict situations can emerge in real-world multi-criteria decision-making processes and highlights the importance of structured negotiation methods to achieve consensus among diverse stakeholders. Table 8 provides a concise summary of the results derived

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from the real-world example discussed in the previous section. When considering consensus for a set of agents A, the most important issue is i_1 – innovation. Removing this issue (achieving agreement on this issue) leads to a consensus of the form $WCons_A^{I^{i_1}} = (i_2, i_3), (i_4, i_3), (i_5, i_3)$. This can be interpreted as any of the three strategic projects i_2, i_4, or i_5 that can be selected for the company's annual development plan. If consensus can be achieved in coalitions (for example, implementing an annual development plan across departments), then the most important issues for achieving good-quality consensus are i_2 – cost reduction, i_4 – marketing expansion, and i_5 – sustainability. For example, the best-quality consensus after removing issue i_2 is as follows. $WCons_{C_1}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5), (i_4, i_3), (i_4, i_5)\}$, $WCons_{C_2}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_1, i_5), (i_5, i_3), (i_5, i_4)\}$, $WCons_{C_3}^{I^{i_2}} = \{(i_1, i_3), (i_1, i_4), (i_4, i_3), (i_5, i_3), (i_5, i_4)\}$, $WCons_{C_4}^{I^{i_2}} = \{(i_3, i_1), (i_4, i_1), (i_5, i_1)\}$. Which means that in departments supervised by managers $\{a_2, a_4, a_5\}$ annual development plan i_1 should be chosen; in departments supervised by managers $\{a_2, a_3, a_5\}$ also development plan i_1 should be chosen; in departments supervised by managers $\{a_3, a_6\}$ development plan i_1 or i_5 can be chosen; in departments supervised by manager $\{a_1\}$ any other than i_1 plan can be selected. Analyzing conflict degrees before and after attribute removal re-

Removed issue	Quality of consensus
i_1	$PConQ_A^{I^{i_1}} = 0$, $WConQ_A^{I^{i_1}} = 0.15$, $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_1}} = 0.0625$, $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_1}} = 0.175$
i_2	$PConQ_A^{I^{i_2}} = 0$, $WConQ_A^{I^{i_2}} = 0.1$, $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_2}} = 0.1375$, $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_2}} = 0.225$
i_3	$PConQ_A^{I^{i_3}} = 0$, $WConQ_A^{I^{i_3}} = 0$, $PConQ_{C_1,C_2}^{I^{i_3}} = 0.1$, $WConQ_{C_1,C_2}^{I^{i_3}} = 0.2$
i_4	$PConQ_A^{I^{i_4}} = 0$, $WConQ_A^{I^{i_4}} = 0.1$, $PConQ_{C_1,C_2}^{I^{i_4}} = 0.125$, $WConQ_{C_1,C_2}^{I^{i_4}} = 0.225$
i_5	$PConQ_A^{I^{i_5}} = 0$, $WConQ_A^{I^{i_5}} = 0.1$, $PConQ_{C_1,C_2,C_3,C_4}^{I^{i_5}} = 0.1375$, $WConQ_{C_1,C_2,C_3,C_4}^{I^{i_5}} = 0.225$

Table 8. Comparison of consensus quality

veals key trends. Removing i_1 and i_3 generally reduces conflict, especially within smaller coalitions, but can increase fragmentation in outlier groups (e.g., a_1 and a_6). Removing i_2, i_4, or i_5 results in better alignment among coalitions, highlighting these as critical issues for achieving broader consensus. This comparison emphasizes the need to focus on attributes that significantly reduce overall conflict when removed, such as i_2, while carefully managing attributes that stabilize key outliers, such as i_1. The presented method for assessing the importance of individual issues in achieving consensus demonstrates a structured and systematic approach to navigate preference-based conflict situations. The results show that weak consensus, compared to strict consensus, often yields higher-quality outcomes, particularly within smaller, well-defined coalitions. This emphasizes the importance of focusing negotiation efforts on resolving critical issues first, because eliminating or resolving them significantly improves the overall quality of consensus and reduces the intensity of the conflict.

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5 Conclusion

This study introduces a structured method to identify critical issues in preference-based conflicts, enabling high-quality consensus in multi-criteria decision-making. By analyzing how removing individual issues impacts consensus quality across entire groups and coalitions, the approach pinpoints key issues driving consensus outcomes. A real-world case study demonstrates its effectiveness in guiding decision-makers to prioritize negotiations, reduce conflict, and improve agreement quality in complex multi-agent scenarios. Future work will extend the method to diverse negotiation contexts.

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