# Incorporating Performance Ordering in MCDA: A Study of the Frobenius SPOTIS Method

Andrii Shekhovtsov[1][0000-0002-0834-2019], Jean Dezert[2][0000-0003-3474-9186], and Wojciech Sałabun[1,3][0000-0001-7076-2519]

[1] National Institute of Telecommunications Szachowa 1, 04-894 Warsaw, Poland
{a.shekhovtsov, w.salabun}@il-pib.pl
[2] The French Aerospace Lab - ONERA 91120 Palaiseau, France
jean.dezert@onera.fr
[3] West Pomeranian University of Technology Żołnierska 49, 71-210 Szczecin, Poland

**Abstract.** Most Multi-Criteria Decision Analysis (MCDA) methods encode a decision-maker's preferences through criterion weights, yet even a well-weighted model can yield ties or virtually indistinguishable scores. We propose Frobenius SPOTIS (Fro-SPOTIS), which is a generalization of the classical SPOTIS method that incorporates ordering information to resolve such ambiguities. Each alternative's attribute-based ranking of criteria is compared with a reference ranking derived from the criteria weights. This comparison is performed by converting both rankings into pairwise preference-score matrices and computing the Frobenius distance between them. This distance, modulated by a tolerance parameter $\tau \in [0, 1]$, is used to modify to the native SPOTIS score: $\tau = 0$ recovers the original SPOTIS results, while higher values increasingly favor alternatives whose performance ordering aligns with the reference. A three-alternative example shows how Fro-SPOTIS untangles an otherwise unresolved tie, and two sensitivity analysis studies trace how rankings shift with (i) changes in the underlying data and (ii) variations in $\tau$. The results confirm that Fro-SPOTIS retains the simplicity of SPOTIS while offering a more flexible and expressive approach to tie-breaking in MCDA.

**Keywords:** SPOTIS · Frobenis Distance · MCDA.

## 1 Introduction

Multi-Criteria Decision Analysis (MCDA) is a subfield of operational research focused on providing methodologies and algorithms to support decision-makers in complex decision scenarios involving multiple, often conflicting criteria. MCDA methods assist in structuring decision problems, incorporating both qualitative and quantitative data, and deriving well-informed, rational choices. Many MCDA methods rely on subjective expert knowledge to determine the relative importance of criteria, whereas others employ mathematical formulations to objectively analyze alternatives and establish rankings or recommendations [15]. Consequently, these techniques offer a comprehensive framework, balancing

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

subjective judgment with rigorous quantitative analysis, thereby enhancing the reliability and transparency of decision-making processes.

Most MCDA methods incorporate importance weights to capture the knowledge and preferences of the decision maker or expert [19]. These weights reflect the relative significance of each criterion and play a key role in shaping the final ranking of alternatives. However, in practice, weighting alone may be insufficient—particularly when it leads to ties or nearly indistinguishable rankings [17]. To overcome this limitation, some approaches incorporate additional information that assesses how well alternatives align with the desired order of performance across criteria [3].

While traditional MCDA techniques effectively encode expert input via criterion weights, they often fall short when it comes to distinguishing between alternatives with similar aggregated scores. Moreover, they typically lack the means to evaluate the degree to which alternatives adhere to the intended prioritization of criteria. This limitation highlights the need for more advanced methods that not only consider the relative importance of criteria but also quantify the alignment of alternatives with these priorities in a meaningful and discriminative way.

In this paper, we propose the Frobenius Stable Preference Ordering Toward Ideal Solution (Fro-SPOTIS) method, a generalization of the classical SPOTIS method, which utilizes the Frobenius distance [3] to measure discrepancies between the desirable ordering of criteria derived from their assigned weights and their actual performance ordering across alternatives. By introducing this ordering-based distance measure, criteria that deviate significantly from the preferred ordering are naturally penalized, resulting in their lower positioning within the final ranking of alternatives. Consequently, Fro-SPOTIS effectively addresses and resolves ranking ties and closely positioned alternatives, providing clearer discrimination among them.

Furthermore, we introduce a tolerance parameter $\tau \in [0, 1]$, enabling analysts to regulate the strictness of incorporating ordering discrepancies into the final assessment. When $\tau = 0$, the method reduces to the original SPOTIS formulation, ignoring ordering information, whereas increasing $\tau$ progressively amplifies the influence of ordering consistency. To illustrate and validate the capabilities of Fro-SPOTIS, we present an illustrative three-alternative example and perform two sensitivity analyses that explore the impact of varying the tolerance parameter and examine the robustness of rankings under data perturbations. Our results confirm that Fro-SPOTIS maintains the intuitive simplicity of the SPOTIS approach while enhancing its flexibility, interpretability, and practical utility in handling tie-breaking scenarios within multi-criteria decision analysis.

The remainder of the paper is structured as follows. In Section 2, we provide context for our study, iterating on recent work in the domain. In Section 3 we describe the algorithms and methods used in the study, the Frobenius distance algorithm and the SPOTIS method. In Section 3.3, we describe the proposed Frobenius SPOTIS method. In Section 4, we provide experiments showing the features and limitations of the proposed method and discuss its possible exten-

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

sions. Finally, in Section 5, we conclude our work and provide some ideas for future research directions.

## 2 Related Works

To compare different alternatives in the decision-making process, many MCDA methods rely on distance metrics to assess the proximity of each alternative to an ideal solution. One of the most commonly used metrics for this purpose is the Euclidean distance. It plays a central role not only in classical MCDA approaches such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [1], but also in more advanced modern methods, including COmbinative Distance-based ASsessment (CODAS) [13] and Preference Ranking On the Basis of Ideal-average Distance (PROBID) [18]. The Euclidean distance is also employed in the Stable Preference Ordering Towards Ideal Solution (SPOTIS) method [5] to evaluate the performance of alternatives.

In certain cases, researchers and decision-makers utilize even more sophisticated methodologies based on generalized fuzzy sets, necessitating adaptations or variations of classical distance metrics tailored specifically to the chosen fuzzy generalization [14,16]. Such generalizations often involve intuitionistic, hesitant, neutrosophic, or type-2 fuzzy sets, each of which requires unique adaptations of standard metrics to appropriately handle increased uncertainty and imprecision. Consequently, this underscores the significance of developing diverse distance measures, as well as thoroughly investigating their suitability, effectiveness, and robustness across different MCDA contexts. A comprehensive exploration and comparative analysis of these adapted metrics can significantly enhance the accuracy and interpretability of multi-criteria evaluations in complex decision environments.

The Frobenius distance is a recently introduced metric designed specifically to measure distances between rankings. It is constructed as a genuine distance metric, explicitly satisfying Kemeny's axiomatic principles for ranking comparisons [3,10]. Furthermore, similar metrics, notably the Kemeny distance itself, have found extensive applications beyond MCDA, such as in voting theory, preference aggregation, and social choice, highlighting their broader relevance and applicability across various decision-making domains.

Several recent studies have demonstrated the versatility of the Kemeny distance in various analytical contexts. For instance, in [6], the authors proposed robust fuzzy clustering methods combining Kemeny distance with medoid-based clustering algorithms. Their results indicated that the proposed approach effectively mitigates the impact of noise and outliers in datasets, improving the robustness of the clustering outcomes. Another noteworthy application of the Kemeny distance involves constructing median rankings from multiple individual rankings. This problem was addressed by Emond and Mason [7], who introduced a weighted version of the Kemeny-Snell distance for consensus ranking problems. Their comparative analysis highlighted the advantages of their method over Kendall's Tau measure, demonstrating superior performance in generating

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

consensus rankings. Additionally, Kemeny distance has been successfully applied within Multi-Criteria Decision Analysis, notably in the KEmeny Median Indicator Ranks Accordance (KEMIRA) method, to determine criteria weights when handling two distinct groups of criteria [12].

Although the Frobenius distance was originally introduced to demonstrate that metrics other than the Kemeny distance could also satisfy Kemeny's axiomatic principles, its applicability and properties have been further investigated in contexts involving consensus among multiple rankings. For instance, Dezert et al. [4] proposed utilizing both Kemeny and Frobenius distances to find optimal solutions in compromise ranking problems. However, their analysis revealed that not all scenarios yielded results aligning intuitively with 'common sense' expectations. Additionally, in [2], the authors explored the potential of the Frobenius distance by proposing a methodology specifically designed to quantify differences between partial orderings, further expanding its applicability in ranking-related problems.

This highlights the relevance of both Kemeny and Frobenius distances within the MCDA domain, underlining the need for continued exploration of their theoretical properties, comparative performance, and practical applicability across diverse decision-making contexts. Further research in this direction may lead to the development of more robust, interpretable, and context-sensitive MCDA tools that can better accommodate complex preference structures and ranking-based evaluations.

## 3 Methodology

In this section, we briefly introduce the methods used in this study: the Frobenius distance and the SPOTIS method. The former is used to measure deviations between performance matrices, while the latter supports ranking alternatives based on their distance to ideal solutions.

### 3.1 Frobenius Distance

Consider a set $X$ consisting of $n \geq 2$ objects, each ranked by two information sources. We denote the total preference orderings (TPOs) provided by these sources as $\text{Pref}_1$ and $\text{Pref}_2$. For example, consider a set of three objects $X = \{x_1 = A, x_2 = B, x_3 = C\}$. Source 1 might provide the preference $\text{Pref}_1$ and source 2 might provide $\text{Pref}_2$, with the following TPOs: $\text{Pref}_1 \triangleq A \succ B \succ C$ and $\text{Pref}_2 \triangleq B \succ C \succ A$.

The Frobenius distance between two TPOs (orderings) of $N$ objects is computed by first constructing an $N \times N$ pairwise Preference-Score Matrix (PSM) based on the ordering given by each source. By convention, the row index $i$ of the PSM corresponds to the index of elements $x_i$ on the left side of the preference ordering $x_i \succ x_j$, and the column index $j$ corresponds to the index of the element $x_j$ on the right side of the preference ordering $x_i \succ x_j$. Thus, we denote

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

a pairwise Preference-Score Matrix $\boldsymbol{M}(X) = [M(i,j)]$ where its components $M(i,j)$ for $i,j = 1,2,\ldots,N$ are defined as

$$\boldsymbol{M}(i,j) = \begin{cases} 1, & \text{if } x_i \succ x_j, \\ -1, & \text{if } x_i \prec x_j, \\ 0, & \text{if } x_i = x_j. \end{cases} \tag{1}$$

Note, that all diagonal elements $M(i,i)$ $(i = 1,2,\ldots,N)$ of the matrix $\boldsymbol{M}$ are always zero. Additionally, the PSM is inherently anti-symmetric because the preference $x_i \succ x_j$ implies $x_j \prec x_i$. Therefore, if $x_i \succ x_j$ holds, meaning $M(i,j) = 1$, then necessarily $x_j \succ x_i$ is false, implying $x_j \prec x_i$ is true, and thus $M(j,i) = -1$, and vice versa. As a result, $\boldsymbol{M}(X)^T = -\boldsymbol{M}(X)$, and $\mathrm{Tr}(\boldsymbol{M}(X)) = 0$.

The distance between two TPOs, $\mathrm{Pref}_1$ and $\mathrm{Pref}_2$, is defined using the Frobenius distance as follows [3]:

$$d_F(\boldsymbol{M}_1, \boldsymbol{M}_2) = ||\boldsymbol{M}_1 - \boldsymbol{M}_2||_F, \tag{2}$$

where $||\boldsymbol{M}||_F$ is the Frobenius norm of a square matrix $\boldsymbol{M} = [M(i,j), i,j = 1,\ldots,N]$, defined by [8,9]

$$||\boldsymbol{M}||_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} |M(i,j)|^2} = \sqrt{\mathrm{Tr}(\boldsymbol{M}^T\boldsymbol{M})}, \tag{3}$$

and where $\boldsymbol{M}^T$ is the transpose of the matrix $\boldsymbol{M}$, and Tr(.) is the trace operator for matrix.

Frobenius distance can be normalized to $[0,1]$ range by dividing the value $d_F(\boldsymbol{M}_1, \boldsymbol{M}_2)$ by the maximum distance value $d_F^{\max}$ computed by considering two TPOs in full contradiction (i.e. a preference and its opposite defined by reversing the preference order). For instance, if a preference ordering is $\mathrm{Pref} = A \succ B \succ C \succ D$, its opposite is $\neg\mathrm{Pref} = A \prec B \prec C \prec D = D \succ C \succ B \succ A$.

### 3.2 Stable Preference Ordering Towards Ideal Solution (SPOTIS)

The Stable Preference Ordering Toward Ideal Solution (SPOTIS) method, introduced by [5], is a MCDA technique that employs reference objects to assess decision alternatives. Unlike other methods that typically derive reference objects from the decision matrix data, SPOTIS requires the decision-maker to explicitly define these reference objects by defining criteria bounds.

To apply the SPOTIS method, the decision-maker must first establish the criteria bounds that will serve as reference objects for evaluating alternatives. For each criterion $C_j$ $(j \in \{1,2,\ldots,N\})$, the maximum $S_j^{max}$ and minimum $S_j^{min}$ bounds must be specified. Subsequently, the Ideal Solution Point (ISP) $\boldsymbol{S}^* = \{S_1^*, \ldots, S_j^*, \ldots, S_m^*\}$ is determined such that $S_j^* = S_j^{max}$ is for the profit criteria and $S_j^* = S_j^{min}$ for cost criteria. The decision matrix is represented as

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

$S = (S_{ij})_{M \times N}$, where $S_{ij}$ denotes the attribute value of the $i$-th alternative $A_i$ for the $j$-th criterion $C_j$.

The full algorithm of the SPOTIS method presented in [5] is as follows:

**Step 1.** Calculation of the normalized distances to ISP (4).

$$d_{ij}(A_i, S_j^*) = \frac{|S_{ij} - S_j^*|}{|S_j^{max} - S_j^{min}|} \tag{4}$$

**Step 2.** Calculation of the weighted normalized distances from ISP $d(A_i, \boldsymbol{S}^*) \in [0, 1]$, according to (5).

$$d(A_i, \boldsymbol{S}^*) = \sum_{j=1}^{N} w_j d_{ij}(A_i, S_j^*) \tag{5}$$

**Step 3.** Determine the final ranking by ordering the alternatives by the values $d(A_i, \boldsymbol{S}^*)$. The better alternatives have smaller values of $d(A_i, S^*)$.

The important features of this method include its simplicity, robustness against the rank reversal paradox, and the ability to utilize the Expected Solution Point (ESP). The ESP allows the decision-maker to define an expected outcome and construct the ranking based on this point rather than the Ideal Solution Point (ISP). To apply the SPOTIS method with a selected ESP $\boldsymbol{S}^+$, one should follow the standard SPOTIS procedure but replace the values of the Ideal Solution Point $S_j^*$ with the values of the ESP $S_j^+$. The decision-maker should select the values of $\boldsymbol{S}^+$ to align with the specific decision problem.

However, it is important to ensure that the selected ESP falls within the problem's scope, meaning $S_j^+$ should satisfy $S_j^{min} \leq S_j^+ \leq S_j^{max}$ for every value of $j$.

### 3.3 Proposed Fro-SPOTIS Method

We propose using Frobenius distance to incorporate information about performance order in an alternative to the final result. In order to do that, we need to extract information about the preferred performance order from the criteria weights. The intuition behind this is as follows: if criterion $C_i$ is most important to us according to the weights, then we want it to have the best performance in the top alternatives. To create this order, we need to create a ranking of $R(C)$ for the criteria in the problem.

For example, the importance weight vector $\mathbf{w} = [0.3, 0.5, 0.2]$ indicates that $C_2$ is the most important criterion, $C_1$ is the second most important criterion and $C_3$ is the least important criterion in the given problem. This means that the importance-based (decreasing) order of the criteria is $C_2 \succ C_1 \succ C_3$, which is characterized by the ranking vector $\mathbf{r}(C) = [r(C_i), i = 1, 2, 3]$ where $r(C_i)$ is the rank of the criterion $C_i$ in the importance-based decreasing ordering. In this example, we have $\mathbf{r}(C) = [2, 1, 3]$.

**Step 1.** Calculation of the normalized distances to the Ideal Solution Point as was shown in (4).

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

**Step 2.** Calculation of the normalized weighted distances $d_i = d(A_i, S^*) \in [0, 1]$, according to (5).

**Step 3.** The ranking matrix $\mathbf{R}$ as defined (6) is created by the ranked distances normalized from the ISP $(d_{ij}(A_i, S_j^*))$ for each alternative. The ranking should be done from the lowest to the highest values, as the values $d_{ij}(A_i, S_j^*)$ are lower if the alternative performs better (that is, it is closer to the ideal solution).

$$\mathbf{R} = \begin{bmatrix} r(A_1)_1 & r(A_1)_2 & \cdots & r(A_1)_m \\ r(A_2)_1 & r(A_2)_2 & \cdots & r(A_2)_m \\ \vdots & \vdots & \ddots & \vdots \\ r(A_n)_1 & r(A_n)_2 & \cdots & r(A_n)_m \end{bmatrix} \tag{6}$$

**Step 4.** Calculate the vector of normalized Frobenius distances between each row of the ranking matrix and the ideal order of the criteria $d_F(\mathbf{r}(C), \mathbf{r}(A_i))$ according to (7).

$$f_i = \{d_F(\mathbf{r}(C), \mathbf{r}(A_i))\}, \quad i \in \{1, 2, \ldots, n\} \tag{7}$$

**Step 5.** Define the parameter $\tau \in [0, 1]$, which defines the tolerance within which two alternatives are considered to be in a tie.

Next, for each two alternatives $A_i$ and $A_j$ apply the Equation (8), which modifies their preferences $d_i$ based on the $\tau$. Note that Equation (8) should be applied to both $A_i$ and $A_j$ for each pair. This means that if $d_i$ is modified and increased, $d_j$ should be decreased accordingly.

$$d_i' = \begin{cases} d_i - \frac{\tau}{2} & if \quad |d_i - d_j| \leq \tau \ and \ f_i < f_j \\ d_i + \frac{\tau}{2} & if \quad |d_i - d_j| \leq \tau \ and \ f_i > f_j \\ d_i & otherwise \end{cases} \tag{8}$$

Finally, when there is no pairs of alternatives for which a change in $d_i$ is required, the final ranking should be determined based on $d_i'$ values, where smaller values suggest that the alternative is better. Flowchart presented in Figure 1 summarize Frobenius-SPOTIS method algorithm by visually representing it.

For a better understanding of the proposed approach, see a simple example in the following section.

## 4 Experiments and Results

In this section, we present the results of our experiments using the proposed Fro-SPOTIS method. We begin with a simple example that demonstrates how the Fro-SPOTIS method can resolve tie situations in preferences. Following this, we conduct two sensitivity analysis experiments to further explore the features and limitations of the proposed method. Lastly, we discuss the application of the Fro-SPOTIS method with the Expected Solution Point (ESP).

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

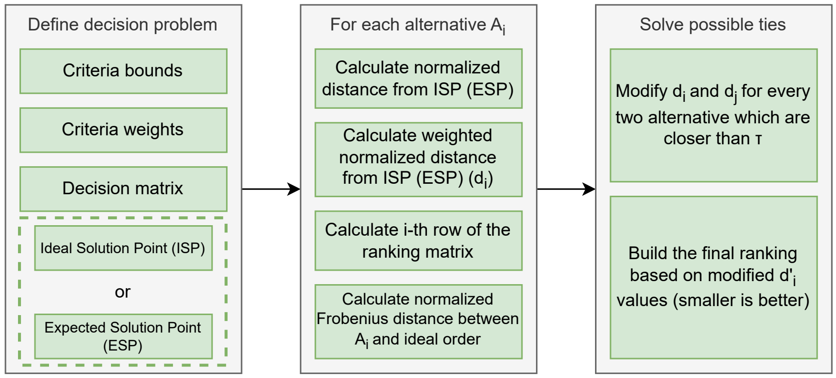


**Fig. 1.** Flowchart of the Fro-SPOTIS method.

### 4.1 Simple Example

Assume that the following decision problem is consistent with the three alternatives presented in Table 1, as well as with the criteria weights and the criteria bounds. To keep the example simple, we consider using three criteria with values in the range $[0, 10]$ and criteria weights $\mathbf{w} = [0.1, 0.3, 0.6]$. All criteria are considered profit; therefore, a value of 10 is the most desirable value in all criteria.

**Table 1.** Decision matrix, criteria weights and criteria bounds for the simple example.

| $A_i$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $A_1$ | 8 | 3 | 3 |
| $A_2$ | 5 | 8 | 2 |
| $A_3$ | 2 | 3 | 5 |
| $w_j$ | 0.1 | 0.3 | 0.6 |
| $S_j^{min}$ | 0 | 0 | 0 |
| $S_j^{max}$ | 10 | 10 | 10 |

If we proceed with this example with the original SPOTIS algorithm, the alternative $A_1$ will be evaluated as 0.65, while the other two alternatives will be in a tie, with the weighted normalized distance from the ISP equal to 0.59 (for each of them). This makes these two alternatives incomparable, making it impossible to prioritize one or another in the decision-making process. However, analyzing the performance of the alternatives, we can see that $A_3$ performs 2.5 times better in $C_3$ (which is most important to us) than in $A_2$. Of course, $A_2$ performs better in $C_1$ and $C_2$; however, the importance of these criteria is low for us, making them less desirable.

However, if we use the proposed Fro-SPOTIS algorithm that uses additional information on the performance order in the alternatives, we will get the calculation shown in Table 2. First, for each alternative, the normalized distance from

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

ISP $d_{ij}(A_i, S_j^*)$ is calculated; then it is multiplied by weights. Furthermore, based on $d_{ij}(A_i, S_j^*)$ performance rankings $\mathbf{r}(A_i)$ are determined for each alternative.

**Table 2.** Calculation process for Frobenius SPOTIS.

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $d_{1j}(A_1, S_j^*)$ | 0.2000 | 0.7000 | 0.7000 |
| $d_{2j}(A_1, S_j^*)$ | 0.5000 | 0.2000 | 0.8000 |
| $d_{3j}(A_2, S_j^*)$ | 0.8000 | 0.7000 | 0.5000 |
| $w_j d_{1j}(A_1, S_j^*)$ | 0.0200 | 0.2100 | 0.4200 |
| $w_j d_{2j}(A_1, S_j^*)$ | 0.0500 | 0.0600 | 0.4800 |
| $w_j d_{3j}(A_2, S_j^*)$ | 0.0800 | 0.2100 | 0.3000 |
| $\mathbf{r}(A_1)$ | 1 | 2 | 2 |
| $\mathbf{r}(A_2)$ | 2 | 1 | 3 |
| $\mathbf{r}(A_3)$ | 3 | 2 | 1 |

Next, if we apply Equations (7) - (8), we obtain the results shown in Table 3. The value $d_i$ is the original SPOTIS output, and $f_i$ is the Frobenius distance between $\mathbf{r}(A_i)$ and $\mathbf{r}(C)$. Finally, the values $P_i$ show the final evaluation, and $R_i$ determines the rank of the alternatives; these values were obtained with $\tau = 0.05$. The addition of the Frobenius distance makes all three alternatives comparable, and according to the results, $A_3$ is the best.

**Table 3.** Final calculations for the Fro-SPOTIS method.

| $A_i$ | $d_i$ | $f_i$ | $P_i$ | $R_i$ |
|---|---|---|---|---|
| $A_1$ | 0.6500 | 0.8660 | 0.6500 | 3 |
| $A_2$ | 0.5900 | 0.8165 | 0.6150 | 2 |
| $A_3$ | 0.5900 | 0.0000 | 0.5650 | 1 |

This small experiment shows how the proposed Fro-SPOTIS method solves ties in the rankings and preferences obtained with the original SPOTIS method. The tolerance parameter $\tau$ also introduces some level of flexibility in terms of adapting the method to the specific decision problem.

### 4.2 Sensitivity Analysis

In this section, we want to demonstrate the limitation of the proposed Frobenius SPOTIS method, which appears due to the nature of the Frobenius SPOTIS in some cases. In specific cases, the proposed method can violate the dominance principle [11]. The dominance principle states that if one option is better than another in at least one aspect and not worse in all other aspects, then the former option dominates the latter, meaning that dominant one should be chosen over the dominated one. Respectively, a situation in which a dominated alternative is

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

preferred violates the dominance principle. To demonstrate how Fro-SPOTIS can violate the dominance principle, we designed a sensitivity analysis experiment presented further in this Section.

Consider a four-criterion decision problem, with criteria bounds $[0, 10]$ for each of the four criteria. Criteria weights are selected as $\mathbf{w} = [0.4, 0.3, 0.2, 0.1]$, so we can show how the preference value changes with the change of the performance order in the alternative. Each criterion is considered profit, therefore, making 10 the most desirable value, and in all calculations, $\tau = 0.01$ was used.

Suppose that we have the alternative $A = [4, 3, 2, 1]$, which is quite far from the ISP. Figure 2 is divided into four parts, and each of them demonstrates the change in preference values for the Frobenius SPOTIS and the classic SPOTIS depending on the value of $C_j$ in the alternative. We can see that in three of the four cases, Frobenius SPOTIS provides the same results as the original SPOTIS because there are no ties or close values. However, in the case of criterion $C_4$, the gradual change of its value causes a violation of the dominance principle. The cause of this behavior is that with a small weight $w_4 = 0.1$, a change of the $C_4$ criterion value can trigger correction of the preference according to (8).

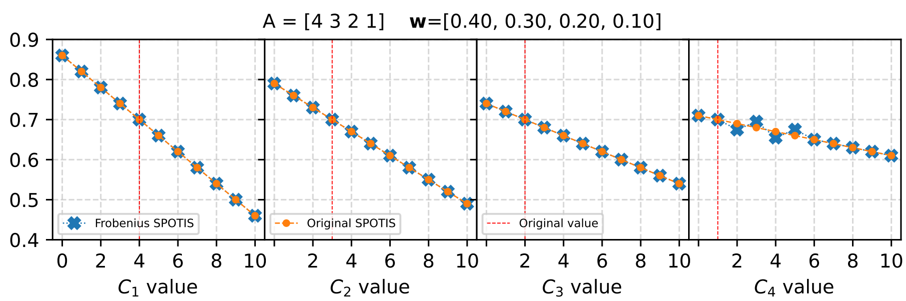


**Fig. 2.** Change in the preference value depending on $C_j$ value for alternative $A = [4, 3, 2, 1]$ for SPOTIS and Fro-SPOTIS methods.

This case, however, is synthetically created, and normally such situations should not occur. Additionally, if one wants to avoid such situations, Frobenius distance can be used with a small value of $\tau$ or only in the case of ties in the ranking.

### 4.3 Influence of $\tau$ value on results

In this section, we describe the experiment, which demonstrates how different values of $\tau$ influence the final results after modification of the original SPOTIS preferences. To address this, we design a simple simulation study on random decision matrices. Each decision matrix consists of four criteria and ten alternatives and is filled with integers drawn from the uniform probability distribution

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

with a range [0, 100]. The first two criteria were profit and the other two costs. The weights of the criteria were drawn from a uniform probability distribution [0, 1] and normalized to sum up to 1. Next, alternatives were evaluated with both original SPOTIS and proposed Fro-SPOTIS (with $\tau \in \{0.01, 0.05, 0.10, 0.15\}$), providing the same or different results, depending on the data. The correlation between two corresponding rankings was calculated using Weighted Spearman's correlation $r_w$. The described experiment was repeated 10,000 times, and results were presented as boxplots in Figure 3 grouped by different values of $\tau$.

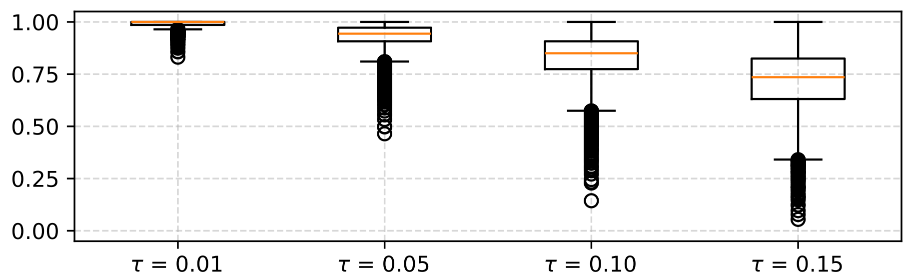


**Fig. 3.** Distibution of $r_w$ correlation values calculated between original and Frobenius SPOTIS rankings for different values of $\tau$.

In Figure 3, we can see how much Fro-SPOTIS results are correlated with original SPOTIS results. It can be seen that for $\tau = 0.01$, values of the $r_w$ correlation are pretty close to 1, which means that there were little changes in the rankings, but some outliers are present. For this value of $\tau$, we expect that only ties on very similar alternatives will be affected. For $\tau = 0.05$, the correlation falls, with worst outliers below $r_w = 0.5$, while the median is still quite close to 1.0. In the other two cases, the median falls further, but we do not have any outliers lower than zero, which means that even despite the low correlation, the results are still correlated.

However, even if it is possible to use high values of $\tau$, we advise against it. The reason is that with the larger value of $\tau$, it is possible for the Fro-SPOTIS to violate the dominance principle. Therefore, we suggest using only small values of $\tau$, mainly to help bring additional information to tie-solving or to strongly differentiate close alternatives.

### 4.4 Usage of the Expected Solution Point

The SPOTIS method stands out among most MCDA methods due to its flexibility in allowing the use of the Expected Solution Point (ESP) in place of the Ideal Solution Point (ISP) during calculations. This feature enables the decision-maker to guide the ranking process toward a solution that better reflects specific goals, expectations, or preferences relevant to the decision problem.

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

Similarly, in the Fro-SPOTIS variant, the ESP can be used instead of the ISP, maintaining compatibility with the core SPOTIS mechanism. To apply this, the ISP values are simply replaced with ESP values. This substitution is valid because the Frobenius distance values $f_i$, which capture deviation across the entire performance matrix, are calculated independently of the reference point (ISP or ESP). Meanwhile, the distance values $d_i$ retain their role by representing the weighted normalized distance from each alternative to the selected reference point, whether it be the ISP or ESP.

## 5 Conclusions

In this paper, we propose a novel MCDA method named Fro-SPOTIS, which explicitly incorporates information regarding preference ordering into the decision-making framework. The proposed approach leverages the Frobenius distance to quantify deviations between the desired ordering of alternatives, defined by the decision-maker's preferences, and the actual observed performance order of alternatives. Additionally, Fro-SPOTIS introduces a tolerance parameter $\tau$, enabling the decision-maker to control the conditions under which certain alternatives are classified as incomparable, thereby providing greater flexibility in handling decision uncertainty or preference ambiguity.

We illustrate the practical application of Fro-SPOTIS through a numerical example, complemented by sensitivity analyses examining the robustness of the method to variations in both the attribute values of alternatives and the tolerance parameter $\tau$. These analyses demonstrate how the ranking outcomes evolve in response to changes in input data and preference structures, providing insight into the method's adaptability to diverse decision-making scenarios. Furthermore, we present an experimental investigation highlighting potential limitations of Fro-SPOTIS, notably cases where the method may violate the dominance principle. Identifying these limitations offers valuable direction for future research, in which we aim to address these issues and subsequently apply Fro-SPOTIS to more complex, real-world decision problems to further validate and enhance its practical utility.

**Disclosure of Interests.** The authors have no competing interests to declare that are relevant to the content of this article.

## References

1. Çelikbilek, Y., Tüysüz, F.: An in-depth review of theory of the TOPSIS method: An experimental analysis. Journal of Management Analytics **7**(2), 281–300 (2020)
2. Dezert, J., Shekhovtsov, A., Salabun, W.: Distances between partial preference orderings. arXiv preprint arXiv:2407.19869 (2024)
3. Dezert, J., Shekhovtsov, A., Sałabun, W.: A new distance between rankings. Heliyon **10**(7) (2024)

ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21

4. Dezert, J., Shekhovtsov, A., Sałabun, W., Tchamova, A.: On optimal solution of the compromise ranking problem. In: 2024 27th International Conference on Information Fusion (FUSION). pp. 1–8. IEEE (2024)
5. Dezert, J., Tchamova, A., Han, D., Tacnet, J.M.: The SPOTIS rank reversal free method for multi-criteria decision-making support. In: 2020 IEEE 23rd International Conference on Information Fusion (FUSION). pp. 1–8. IEEE (2020)
6. D'Urso, P., Vitale, V.: A kemeny distance-based robust fuzzy clustering for preference data. Journal of Classification **39**(3), 600–647 (2022)
7. Emond, E.J., Mason, D.W.: A new rank correlation coefficient with application to the consensus ranking problem. Journal of Multi-Criteria Decision Analysis **11**(1), 17–28 (2002)
8. Golub, G.H., Van Loan, C.F.: Matrix computations. JHU press (2013)
9. Horn, R.A., Johnson, C.R.: Matrix analysis. Cambridge university press (2012)
10. Kemeny, J.G.: Mathematics without numbers. Daedalus **88**(4), 577–591 (1959)
11. Kourouxous, T., Bauer, T.: Violations of dominance in decision-making. Business Research **12**, 209–239 (2019)
12. Krylovas, A., Zavadskas, E.K., Kosareva, N., Dadelo, S.: New KEMIRA method for determining criteria priority and weights in solving MCDM problem. International journal of information technology & decision making **13**(06), 1119–1133 (2014)
13. Kumari, A., Acherjee, B.: Selection of non-conventional machining process using CRITIC-CODAS method. Materials Today: Proceedings **56**, 66–71 (2022)
14. Rani, P., Mishra, A.R.: Single-Valued Neutrosophic SWARA-TOPSIS-Based Group Decision-Making for Prioritizing Renewable Energy Systems. Computer and Decision Making: An International Journal **2**, 425–439 (2025)
15. Sałabun, W., Wątróbski, J., Shekhovtsov, A.: Are MCDA methods benchmarkable? a comparative study of TOPSIS, VIKOR, COPRAS, and PROMETHEE II methods. Symmetry **12**(9), 1549 (2020)
16. Suvitha, K., Saraswathy, R., Thilagasree, C.S., Kalaiselvan, S., Narayanamoorthy, S.: Designing and analyzing fermatean fuzzy decision models: A comprehensive decision making approach. Journal of Intelligent Decision Making and Information Science **2**, 272–288 (2025)
17. Tammi, I., Kalliola, R.: Spatial MCDA in marine planning: Experiences from the Mediterranean and Baltic Seas. Marine Policy **48**, 73–83 (2014)
18. Wang, Z., Rangaiah, G.P., Wang, X.: Preference ranking on the basis of ideal-average distance method for multi-criteria decision-making. Industrial & Engineering Chemistry Research **60**(30), 11216–11230 (2021)
19. Więckowski, J., Kizielewicz, B., Shekhovtsov, A., Sałabun, W.: RANCOM: A novel approach to identifying criteria relevance based on inaccuracy expert judgments. Engineering Applications of Artificial Intelligence **122**, 106114 (2023)


ICCS Camera Ready Version 2025
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-97632-2_21