Distributionally-Robust Optimization for Sustainable Exploitation of the Infinite-dimensional Superposition of Affine Processes with an Application to Fish Migration

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Abstract. We consider a novel modeling and computational framework of the distributionally-robust optimization problem of jump-driven general affine process focusing on its application to inland fisheries as a case study. In particular, we consider an exploitation problem of migrating fish population as a major fishery issue. Our target process is the superposition of Ornstein–Uhlenbeck processes (supOU process) serving as a fundamental model of mean-reverting phenomena with (sub-)exponential memory. It is an affine process and admits a closed-form characteristic function being useful in applications. A theoretical novelty here is the assumption that the supOU process is allowed to have uncertain model parameter values as often encountered in engineering applications. Another novelty is the formulation of a long-term exploitation problem of the supOU process where the uncertainty is penalized through a generalized divergence between benchmark and distorted models. We present a strictly convex discretization of the optimization problem based on the model identified using the existing data of migrating fish population of a river in Japan. Further, the statistical analysis results in this paper are new by themselves. The computational results suggest the optimal harvesting policy of the fish population.

Keywords: Computational Optimization; supOU Processes; Generalized Divergence

1 Introduction

Computational optimization plays a vital role in the planning of resource and environmental management because the problems of interest are not always analytically solvable [1-2]. Furthermore, in such problems, target dynamics (e.g., resource, environmental, and biological dynamics) are often stochastic [3-4] and hence the objective function to be minimized is given as some expectation. The expectation can then be evaluated

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DOI: 10.1007/978-3-031-36030-5_45

numerically [5] although it may be computationally prohibitive if the underlying dynamics are complicated and high-dimensional.

Another potential issue encountered in the modelling and optimization related to resource and environmental management is that the target dynamics need to be identified using only a limited amount of data that is not necessarily accurate. The identified model therefore usually contains modelling errors emerging as incorrect parameter values and/or functional forms of coefficients [6-7]. The model uncertainty should then be taken into account when considering an optimization problem using the identified model in a computationally feasible way.

In addition to the above-mentioned issues, processes of interest in resource and environmental dynamics have sub-exponential memory [8], postulating the use of a mathematical model that can capture this property. Volterra and mixed moving average processes are such candidates from both theoretical and engineering viewpoints [9-10], while their computational optimization under model uncertainty has not been studied well to the best of the authors' knowledge.

Motivated by the issues reviewed above, the objectives of this paper are the formulation and application of a computational optimization approach for resource and environmental management under model uncertainty. We focus on the problem of exploiting a part of migrating fish population at a fixed point in a river reach of an anthropogenically-disturbed river system and then distribute them to other parts of the same river system to sustain the regional fish population [11]. This is a major problem in inland fisheries in Japan whose bottleneck is the optimization of the harvesting rate of the population. We show that the dynamics can be identified as the superposition of Ornstein–Uhlenbeck processes (supOU process) [12] whose characteristic function is available in a closed form. This useful property allows us to compute the probability density function (PDF) by a discrete Fourier transform.

Our optimization problem is to harvest the migrating fish population so that it is not excessively harvested because completely exploiting the population triggers a local extinction of the fish. The risk of the overexploitation is evaluated by a Conditional-Value-at-Risk (CVaR) measure [13], while the model uncertainty is penalized by a generalized (Tsallis) divergence [14]. The latter is a generalization of the Kullback–Leibler divergence serving as a more flexible mathematical tool.

The optimization problem is a feasible convex one once the PDF is given, which is father made strictly convex by regularizing the non-smoothness of the CVaR measure [15]; the strict convexity guarantees the unique solvability. We present computational examples using the real data [16] and analyze the impacts of model uncertainty on the distortion of the PDF and the optimal harvesting rate. Limitation and extendibility of the proposed computational approach are finally discussed. Our contribution thus covers both theory and application of a new computational optimization approach under uncertainty.

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DOI: 10.1007/978-3-031-36030-5_45

2 SupOU process

2.1 Formulation

We formulate and review the supOU process following the literature [10, 12]. The supOU process is a superposition (i.e., integration) of infinitely many continuous-time OU processes using the jump-type random fields called Lévy bases [12]. These are tractable models of infinite-dimensional white noises having independent increments. It is known that Lévy bases associate equivalent representations based on Poisson random measures that are physically easier to understand. We exploit this fact and set the supOU process X_t at time t as

$$X_t = \int_0^{+\infty} \int_0^{+\infty} \int_{-\infty}^{t} \exp\left(-\rho\left(t-s\right)\right) z N\left(\mathrm{d}z, \mathrm{d}\rho, \mathrm{d}s\right),\ t \in \mathbb{R}. \tag{1}$$

Here, N is a Poisson random measure on $\left(0,+\infty\right)\times\left(0,+\infty\right)\times\mathbb{R}$ with the compensator $v\left(\mathrm{d}z\right)\pi\left(\mathrm{d}\rho\right)\mathrm{d}s$, where π is a probability measure of a positive random variable, and v is a Lévy measure of pure-jump Lévy processes having bounded-variations and positive jump sizes. The representation (1) is intuitive as the right-hand side can be formally understood as a limit of the superposition:

$$X_t \approx \sum_{i=1} \int_0^{+\infty} \int_{-\infty}^{t} \exp\left(-\rho_i\left(t-s\right)\right) z_i N_i\left(\mathrm{d}z_i, \mathrm{d}s\right) \tag{2}$$

with a suitable positive and strictly increasing sequence $\left\{\rho_i\right\}_{i=1,2,3,...}$ and a family of mutually-independent Poisson random measures $\left\{N_i\right\}_{i=1,2,3,...}$ on $\left(0,+\infty\right)\times\mathbb{R}$. Indeed, this approximation is theoretically justified in the sense of distribution [12]. This kind of discretization method is called Markovian lifts.

The supOU process (1) is assumed to be stationary that is why we do not explicitly specify an initial condition. The parameter $\rho > 0$ represents the reversion speed distributed according to the probability measure π. To guarantee the existence of the stochastic integral of (1), we need to assume that the reversion speed is not too much accumulated around the origin $\rho = 0$:

$$R \equiv \int_0^{+\infty} \rho^{-1} \pi\left(\mathrm{d}\rho\right) < +\infty. \tag{3}$$

Physically, R represents the dominant time-scale of the supOU process, and hence it is a key quantity for better understanding its dynamics. The condition (3) guarantees the existence of the right-hand side of (1) as well as the stationarity and almost sure positivity of X_t [12, 17]. We will show that this condition is harmless in practice.

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DOI: 10.1007/978-3-031-36030-5_45

2.2 Statistical Properties

An advantage of the supOU process is that its characteristic function is obtained analytically. The characteristic function $c(\xi)$ is given by [10]

$$c(\xi)=\mathbb{E}\left[e^{\mathrm{i}\xi X_t}\right]=\exp\left(R\int_0^{+\infty}\int_0^{+\infty}\left(\exp\left(\mathrm{i}\xi z e^{-t}\right)-1\right)v(\mathrm{d}z)\mathrm{d}t\right),\ \xi\in\mathbb{R} \tag{4}$$

with the imaginary unit i ($\mathrm{i}^2=-1$) and a suitable expectation $\mathbb{E}$. The moment generating function, if necessary, is formally obtained as $m(\xi)=c(-\mathrm{i}\xi)$. Hereafter, we assume the Lévy measure of the form $v(\mathrm{d}z)=Az^{-(\alpha_v+1)}\exp(-\beta_v z)\mathrm{d}z$ ($A>0$, $\alpha_v<1$, $\beta_v>0$) that covers the cases considered in our application. More general v can also be utilized when preferred.

We close this section by presenting two remarks. Firstly, the closed-form availability of the characteristic function (4) allows for computing the corresponding PDF via a discrete Fourier transform [18]. We can therefore avoid computing expectations of the supOU process without resorting to a time-consuming statistical method like a Monte-Carlo method. Indeed, the infinite-dimensionality implied in (1)-(2) suggests that we need to design a Monte-Carlo method to generate a sufficiently large number of paths of a high-dimensional system for evaluating an expectation, which would be computationally expensive and maybe infeasible. At the same time, key statistics, such as mean, variance, and skewness can also be obtained from (4). Secondly, the autocorrelation function that can handle both exponential and sub-exponentially decaying cases is obtained in a closed-form, which becomes fully-explicit if we assume Gamma-type π complying with the condition (3). We effectively use these properties in the application.

3 Optimization Problem

3.1 Generalized Divergence

We define a generalized divergence before going to the formulation of the optimization problem. Given two equivalent positive PDFs p, r of a positive random variable Z, the generalized (Tsallis) divergence $D_q(r|p)$ from p to r is set as [14]:

$$D(r|p)\equiv\int_0^{+\infty}\frac{1}{1-q}\left(-\left(\omega(Z)\right)^q+q\omega(Z)+1-q\right)p(Z)\mathrm{d}Z\ , \tag{5}$$

where $q\in(0,1]$ with the Radon–Nikodým derivative $\omega=r/p$. If $q=1$, then (5) is understood as the classical Kullback–Leibler divergence

$$D(r|p)\equiv\int_0^{+\infty}\left(\omega(Z)\ln\omega(Z)-\omega(Z)+1\right)p(Z)\mathrm{d}Z\ . \tag{6}$$

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In both cases, we have $D(r|p) \geq 0$ and $D(r|p) = 0$ if and only if $p = r$, and that $D(p|r) \neq D(r|p)$ in general due to the asymmetry implied in (6). The divergence is therefore an index that can measure the probabilistic difference (i.e., model uncertainty) between two equivalent PDFs but is not a metric. The uncertainty is then evaluated by considering p as the benchmark model and r as a distorted model. A remark between the two cases (5)-(6) is that the integrand as a function of the Radon–Nikodým derivative $\omega(Z)$ is more strongly convex as well as becomes larger (i.e., has a sharper profile) for larger q. It implies that the model uncertainty in our context is evaluated to be larger for larger q. Hence, the Kullback–Leibler divergence is an extreme case of the generalized divergence. There is no model uncertainty if $\omega = 1$ almost everywhere.

For later use, define the q-exponential and q-logarithm functions:

$$\exp_q(x) = (1 + (1-q)x)^{\frac{1}{1-q}} \text{ and } \ln_q(x) = \frac{x^{1-q} - x}{1-q}, \ x \geq 0. \tag{7}$$

We conventionally set $\exp_1(x) = \exp(x)$ and $\ln_1(x) = \ln x$.

3.2 Problem Formulation

The optimization problem here is to determine the harvesting rate $h = h(X_t)$ ($0 \leq h \leq 1$) as a bounded measurable function of the observed process X_t now considered as a unit-time population of migrating fish species. The set of such functions is denoted as $\mathfrak{H}$. The fish should be harvested as large as possible by the decision-maker, a fishery cooperative, while fully exploiting them should be avoided to prevent a local extinction. The optimization problem without model uncertainty is then set as

$$\text{Find } \inf_{h \in \mathfrak{H}} J(h) \ \text{ with } \ J(h) = \mathbb{E}[g(1-h)] - \lambda \mathrm{CVaR}_\alpha((1-h)X). \tag{8}$$

Here, the expectation is based on the PDF p of the stationary supOU process. The first term of J with a non-negative, uniformly bounded, and strictly convex function g represents the averaged harvesting rate. The second term represents the risk of harvesting the migrating fish population when it becomes small. Here, we use the upper CVaR measure with the weight $\lambda > 0$ and the quantile $\alpha \in (0,1)$ for generic non-negative random variable Z

$$\mathrm{CVaR}_\alpha(Z) = \frac{\mathbb{E}[Z\mathbb{I}(Z \leq Z_\alpha)]}{\mathbb{E}[\mathbb{I}(Z \leq Z_\alpha)]} = -\inf_{u \geq 0}\left\{-u + \frac{1}{\alpha}\mathbb{E}[\max\{u - Z, 0\}]\right\} \tag{9}$$

with the indicator function $\mathbb{I}(Z \leq Z_\alpha)$ of the set $\{Z \leq Z_\alpha\}$ ($\mathbb{I}(Z \leq Z_\alpha) = 1$ if $Z \leq Z_\alpha$ and is 0 otherwise) and the quantile value Z_α such that $\mathbb{E}[\mathbb{I}(Z \leq Z_\alpha)] = \alpha$. The right-

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DOI: 10.1007/978-3-031-36030-5_45

most side of (9) is the optimization-oriented dual formula [13]. The risk-aversion of the decision-maker becomes stronger as the penalization (λ) increases or the quantile level (α) decreases.

The problem (8) is a convex optimization problem with the objective being lower-semicontinuous in a Hilbert space of bounded functions with the norm $\|\cdot\|$ given for generic $f:(0,+\infty)\to\mathbb{R}$ by

$$\|f\|^2=\int_0^{+\infty} f^2(X)\,p(X)\,\mathrm{d}X\ . \tag{10}$$

Owing to the convexity, the problem is feasible.

We extend the problem (8) so that the model uncertainty can be evaluated in the context of distributionally-robust optimization [e.g., 19]. The distributionally-robust problem assumes that the expectation $\mathbb{E}$ is distorted and evaluated by r equivalent to p but not necessarily by p. Given a PDF p, the admissible set of positive measurable functions ω such that $\int_0^{+\infty}\omega(Z)p(Z)\mathrm{d}Z=1$ is denoted as $\mathfrak{W}$.

We formulate the distributionally-robust optimization problem as follows:

$$\text{Find } W=\inf_{h\in\mathfrak{H}}\sup_{\omega\in\mathfrak{W}} J(h,\omega)\,, \tag{11}$$

$$J(h,\omega)=\mathbb{E}_q\left[g(1-h)\right]-\lambda\mathrm{CVaR}_\alpha\left((1-h)X\right)-\mu D\left(\omega p|\,p\right), \tag{12}$$

where the expectation $\mathbb{E}_q$ is such that $\mathbb{E}_q[\cdot]=\mathbb{E}\left[\omega^q\cdot\right]$ to consistently reformulate the problem under uncertainty in the sense of Tsallis [20]. The last term of (12) penalizes the model ambiguity with another weight $\mu>0$ in a way that larger model uncertainty and hence larger divergence $D\left(\omega p|\,p\right)$ is allowed for smaller μ. The case $\mu\to+\infty$ thus corresponds to the benchmark case (8).

The problem (11) is a min-max problem but can be rewritten as a minimization problem as shown below. Considering the dual representation formula (9), given $h\in\mathfrak{H}$, the inner maximization is achieved by

$$\omega^*(X)=\exp_q\left(\frac{F(X,u,h)}{\mu}\right)\left(\int\exp_q\left(\frac{F(X,u,h)}{\mu}\right)p(X)\,\mathrm{d}X\right)^{-1}\in\mathfrak{W} \tag{13}$$

with F given by

$$(0\le)F(X,u,h)=g(1-h)+\frac{\lambda}{\alpha}\max\left\{u-\left(1-h(X)\right)X,0\right\}(<+\infty)\,. \tag{14}$$

Plugging (14) into (11) yields

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DOI: 10.1007/978-3-031-36030-5_45

$$W=\inf_{u\geq 0,h\in\mathfrak{H}}\left\{-\lambda u+\mu\ln_q\left(\int\exp_q\left(\frac{F\left(X,u,h\left(X\right)\right)}{\mu}\right)p\left(X\right)\mathrm{d}X\right)\right\}. \tag{15}$$

This is a convex optimization problem because F of (14) is convex with respect to all the arguments and the second term of the right-hand side of (15) defines a convex certainty equivalent of the expectation $\int F\left(X,u,h\right)p\left(X\right)\mathrm{d}X$ [Theorem 5.1 of 21]. Further, we can replace the range $\left[0,+\infty\right)$ of the auxiliary decision variable u by a sufficiently large compact set $\left[0,U\right]$ with some constant $U>0$. This is proven by a simple contradiction argument [Proposition 4 of 19]. Therefore, this distributionally-robust optimization problem is feasible.

3.3 Regularization of CVaR

The problem (15) is convex but not necessarily strictly convex, which is due to the non-smoothness of the "max" function in the CVaR measure. We therefore regularize "max" function as the following strictly convex one with a small $\tau>0$ [15]:

$$\max_\tau\left(x\right)=\frac{x+\sqrt{x^2+4\tau^2}}{2}\quad\text{for } x\geq 0, \tag{16}$$

satisfying

$$\max\left\{0,x\right\}\leq\max_\tau\left(x\right)\leq\max\left\{0,x\right\}+\tau\quad\text{for } x\geq 0. \tag{17}$$

The difference between max and $\max_\tau$ functions is at most τ, and can be made arbitrary small by choosing a suitably small τ.

Now, the regularized distributionally-robust optimization problem reads

$$\text{Find } W_\tau=\inf_{u\geq 0,h\in\mathfrak{H}}\left\{-\lambda u+\mu\ln_q\left(\int\exp_q\left(\frac{F_\tau\left(X,u,h\left(X\right)\right)}{\mu}\right)p\left(X\right)\mathrm{d}X\right)\right\} \tag{18}$$

with F_τ given by

$$F_\tau\left(X,u,h\right)=g\left(1-h\right)+\frac{\lambda}{\alpha}\max_\tau\left\{u-\left(1-h\left(X\right)\right)X,0\right\}. \tag{19}$$

It is straightforward to show that the problem (18) is strictly convex, and that the range of the auxiliary decision variable u can be made compact without changing the value of W. Consequently, the problem (18) admits a unique minimizing pair $u\geq 0,h\in\mathfrak{H}$. The corresponding worst-case Radon–Nikodým derivative ω^* is then found as (13) with F replaced by F_τ. The regularized problem is therefore feasible and uniquely solvable. We numerically compute it with a small $\tau>0$ as explained below.

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3.4 Numerical Discretization

The regularized problem (18) is discretized through a replacement of the PDF p by its empirical version p_N ($N \in \mathbb{N}$) on a uniform grid, which is formally given by

$$p_N(X)\mathrm{d}X \approx \sum_{i=1}^{N} p(i)\delta(X - X_i), \quad \sum_{i=1}^{N} p(i) = 1 \tag{20}$$

with the Dirac's delta $\delta(\cdot)$, a positive sequence $\{p(i)\}_{1 \le i \le N}$ serving as discrete probabilistic weights, and a non-negative and strictly increasing sequence $\{X_i\}_{1 \le i \le N}$ as representative points at which the process is evaluated. The harvesting rate h and the Radon–Nikodým derivative ω are accordingly discretized on the same grid. The discretization of the regularized problem is then set as

$$\text{Find } W_{\tau,N} = \inf_{u \ge 0, h \in \mathfrak{H}} \left\{ -\lambda u + \mu \ln_q \left(\int \exp_q \left(\frac{F_\tau(X,u,h(X))}{\mu} \right) p_N(X)\mathrm{d}X \right) \right\}, \tag{21}$$

with the worst-case Radon–Nikodým derivative ω^* given by (13) with F replaced by F_τ and p by p_N.

The computational procedure of the problem (21) in practice is as follows. Firstly, we identify the supOU process (1), namely the measures π and v, from some time series data. Then, the characteristic function (4) is Fourier inverted on a sufficiently fine uniform grid to compute p_N following Hainaut [18]. The size and spacing of the grid are chosen sufficiently large and fine so that the computed PDF is entirely positive (i.e., no or at most small Gibbs oscillations). The minimizing pair u,h is computed where the former is a real constant and the latter is distributed at each grid point. This process is carried out by using a gradient descent method with an inertia [22], but other optimization methods can be utilized if preferred. Finally, the worst-case Radon–Nikodým derivative is accordingly obtained by the optimized pair u,h.

4 Application

4.1 Study Site

The study site of this paper is the weir called Meiji-yosui irrigation head works located at the midstream of Yahagi River pouring to Pacific Ocean, Aichi Prefecture, Japan (35° 02′ 52″ E 137° 10′ 43″). This weir is an observation point of the spring upstream migration of the fish ayu *Plecoglossus altivelis altivelis* from Pacific Ocean to the river [16]. *P. altivelis* is one of the most important inland fishery resources in Japan from ecological, fisheries, and cultural standpoints [23]. Recent river development critically degraded habitat and food availability for the fish, thereby their population as well as

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DOI: 10.1007/978-3-031-36030-5_45

catches have been decreasing. In some case, a part of the migrating fish population is harvested at a point in a river reach and distributed to the other reaches to prevent them from extinctions [11]. Such a project is usually planned and executed by a local inland fishery cooperative. When and how much of the fish should be harvested for this purpose has been a crucial issue for inland fishery cooperatives in Japan.

The supOU process is fitted against the data of each year based on the verified least-squares approach [10, 17] that firstly fits the probability measure π by comparing theoretical and empirical autocorrelation function and then the Lévy measure v by using mean, standard deviation, and skewness. We assume the Gamma-type $\pi(\mathrm{d}\rho) \propto \rho^{\alpha_\rho - 1} \exp(-\rho / \beta_\rho)\mathrm{d}\rho$ ($\alpha_\rho > 1$, $\beta_\rho > 0$). Then, the autocorrelation of the supOU process with lag $s \geq 0$ is sub-exponential given by $(1 + \beta_\rho s)^{1-\alpha_\rho}$, which approaches to the exponential case as $\alpha_\rho \to +\infty$ with fixed $\beta_\rho(\alpha_\rho - 1)$. We need $\alpha_\rho > 1$ by (3). We set $\alpha_v = -2$ as it has preliminary been found to effectively work.

The collected data of daily migrating populations of the *P. altivelis* from 2010 to 2020 is available in the recent report [16]. This kind of fine and large amount of data is rarely available, which is why we have chosen the present study site. **Fig.1** plots the reported daily migrating population in each year. **Table 1** shows the total, mean, standard deviation, skewness, and kurtosis of the time series data in each year. Hereafter, we normalize X_t in each year by the total migrated population without significant loss of generality. **Table 2** shows the fitted parameter values for the data and the time-scale R of each year for self-centeredness of the paper. Indeed, this kind of statistical results against the time series data, namely **Tables 1-2**, are unique contributions by themselves.

Tables 1-2 show that the time series data is probabilistically positively skewed and has sharper PDFs than Gaussian ones. In addition, the yearly difference of the memory structure is found to be significantly different; α ranges from $O(10^0)$ (polynomial) to $O(10^4)$ (almost exponential). The relative errors of the statistics are on average 0.014 (mean), 0.034 (standard deviation), 0.011 (skewness), and 0.0084 (kurtosis). Hence, the supOU process can capture the wide range of statistical behavior of the migrating fish population dynamics. The time scale R ranges from $O(10^{-1})$ (day) to $O(10^1)$ (day), suggesting that the decaying speed of peaks are broadly different among different years. Interestingly, the total migrated population is significantly different among different years. Biological understanding of this large difference is beyond the scope of this paper, but will be an interesting research topic to be resolved in the future.

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DOI: 10.1007/978-3-031-36030-5_45

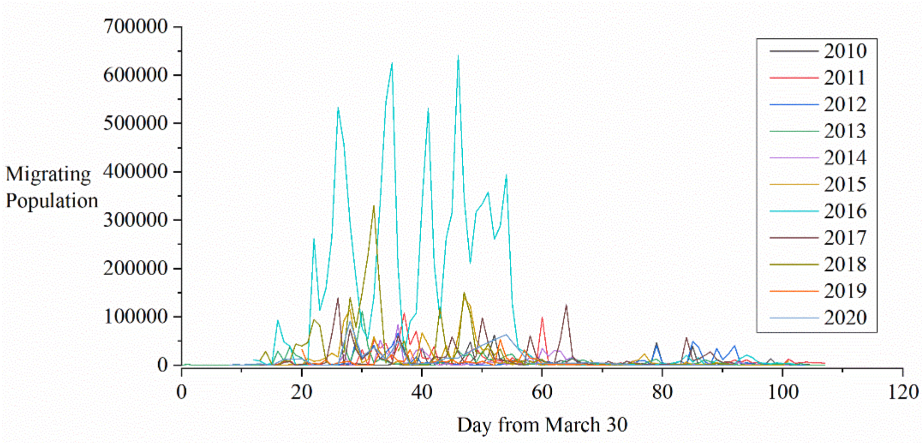

Fig. 1. Reported time series data of the daily migrating population of *P. altivelis* in each year.

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DOI: 10.1007/978-3-031-36030-5_45

Table 1. Total, mean, standard deviation, skewness, and kurtosis of the migrating population. The statistics are computed for the supOU process *X* normalized (divided) by the total.

Year	Total	Mean	Standard deviation	Skewness	Kurtosis
2010	487,951	0.0118	0.0205	3.39	12.5
2011	985,637	0.0106	0.0184	3.32	12.8
2012	761,990	0.0126	0.0141	2.30	4.94
2013	839,587	0.00741	0.0187	4.62	30.4
2014	601,147	0.0114	0.0215	3.47	15.8
2015	1,276,048	0.0124	0.0185	2.91	9.12
2016	10,030,840	0.0133	0.00890	1.57	1.69
2017	1,440,609	0.0133	0.0176	2.55	7.37
2018	2,307,520	0.0134	0.0223	3.09	11.7
2019	447,134	0.0185	0.0230	2.35	6.67
2020	1,103,486	0.0138	0.0128	1.85	3.44

Table 2. Fitted parameter values of the supOU process. The time unit is day.

Year	A	β_v	α_ρ	β_ρ	R
2010	1553.5	42.037	2.3720	2.5778	0.28273
2011	344.85	47.009	3.4618	0.25484	1.5940
2012	2375.1	95.032	3,754.2	0.00011700	2.2774
2013	80.293	31.808	25,577	0.000026310	1.4861
2014	162.09	36.756	12.838	0.048555	1.7397
2015	545.64	54.224	3.5410	0.217560	1.8089
2016	13,781	251.33	2,270.0	0.000057746	7.6321
2017	2,424.2	64.165	5,334.1	0.00025953	0.72249
2018	265.73	40.360	2.8859	0.32082	1.6528
2019	5,288.0	52.455	3.2520	1.7553	0.25298
2020	4,005.6	125.28	63,896	0.0000046289	3.3810

4.2 Computational Results and Discussion

We present demonstrative computational examples of the regularized distributionally-robust optimization problem for years 2018 and 2019. The year 2018 has a relatively longer memory with α_ρ closer to 2 than the other years and the PDF in 2018 is more skewed as well as sharper than 2019 (See, **Tables 1-2**). The PDF was computed using the discrete Fourier transform with the space increment of 1/2,500 and the degree-of-freedom $N = 2{,}500$. We needed a post-processing using an exponential spectral filter [23] to obtain oscillation-free PDF profiles. This procedure can be considered as a part of model uncertainty although its influences would be small in our case. The

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DOI: 10.1007/978-3-031-36030-5_45

regularization parameter is $\tau = 0.00000125$ that has been found to be sufficiently small. We report that if $\tau = 0$ then the gradient descent fails to converge due to discontinuous variational derivatives. We set the nominal parameter values $q = 0.5$, $\mu = 10$, and $\alpha = 0.30$ unless otherwise specified. All the decision variables below are obtained from (21). All the solutions have been obtained without numerical instabilities.

Firstly, we analyze the harvesting rate. **Fig. 2** compares the computed harvesting rates in 2018 and 2019 as a function of the (normalized) migrating population X and the weight λ. The optimal harvesting rate is 1 at $X = 0$, while it does not play a role in practice because no fish can be harvested in such as case. For $X > 0$, the harvesting rate is increasing in the migrating population. No fish should be harvested for relatively small positive X, which appears due to the use of the CVaR term effectively penalizing the local extinction. This non-harvesting area in the figure contracts as the weight λ and hence the extinction risk increases. This observation suggests that the decision-maker should not be too much afraid of the risk of local extinction for the global minimization of the objective (12). The impacts of the risk-aversion are concentrated more on the small population for the sharper PDF in 2018. The analysis below focuses on the year 2019 as the impacts of the uncertainty are more visible for a wider range of the population.

Secondly, we analyze the uncertainty and its influences on the distortion of the PDF. **Fig. 3** compares the computed worst-case uncertainties, the Radon–Nikodým derivatives, for the nominal $\mu = 10$ and smaller $\mu = 1$ potentially allowing for larger uncertainty. **Fig. 4** then compares the corresponding computed PDFs. Larger model uncertainty leads to the worst-case Radon–Nikodým derivatives and PDFs be more concentrated on small X, thereby underestimating the mean population.

Finally, we analyze impacts of the parameter q in the generalized divergence. **Fig. 5** shows the computed harvesting rates for $q = 0.25$ and $q = 1$. It is shown that the use of the sharper divergence (larger q) results in less conservative harvesting rate having smaller range of no-harvesting. This is due to that the Radon–Nikodým derivative is restricted to be smaller for the sharper divergence.

Consequently, the decision-maker can design his/her harvesting strategy flexibly as demonstrated in this paper.

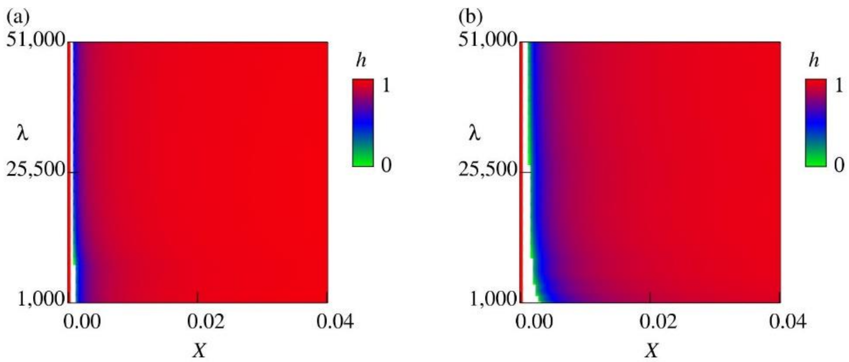

Fig. 2. Harvesting rate in (a) 2018 and (b) 2019. $h = 0$ is optimal in the white area.

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DOI: 10.1007/978-3-031-36030-5_45

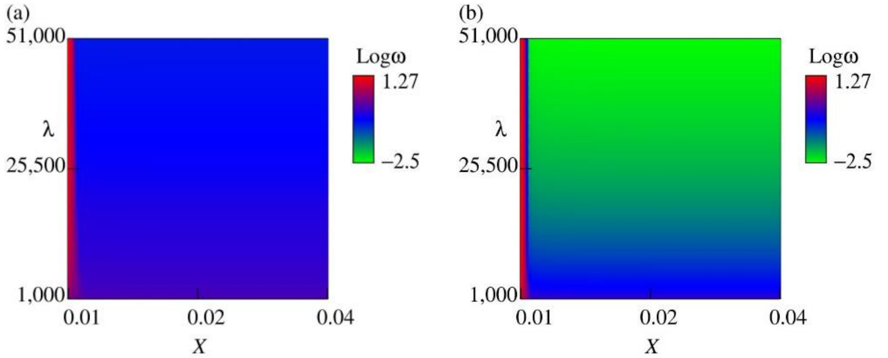

Fig. 3. Worst-case ambiguity in 2019 with (a) nominal and (b) larger uncertainty.

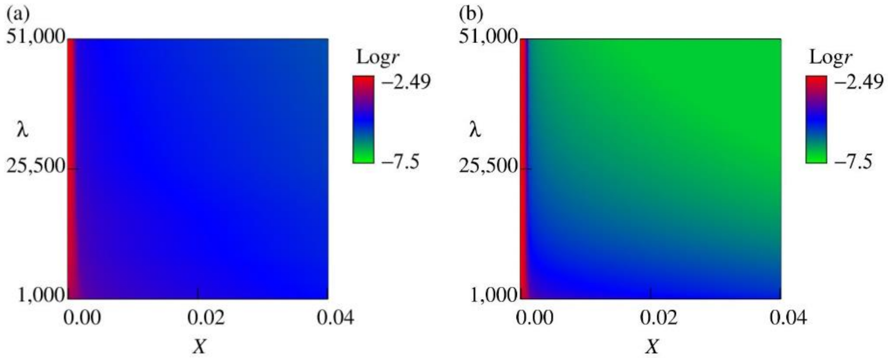

Fig. 4. Worst-case PDF in 2019 with (a) nominal and (b) larger uncertainty.

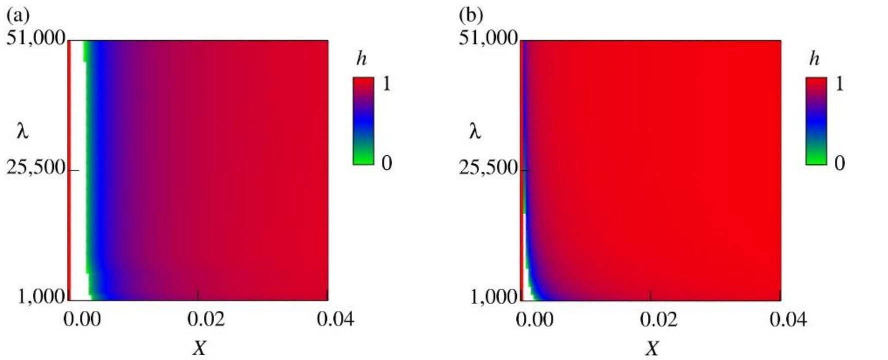

Fig. 5. Harvesting rate in 2019 with (a) $q = 0.25$ and (b) $q = 1$. Compare with **Fig.2**.

5 Conclusion

A distributionally-robust optimization problem of the supOU process was formulated focusing on a sustainable exploitation problem in fisheries. Key points in our

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formulation were the use of the generalized divergence covering the classical Kullback–Leibler divergence and the closed-form availability of the characteristic function of the supOU process with which the expectations can be evaluated without resorting to time-consuming statistical methods. The computational results based on the real data suggested that our framework can be utilized to support the decision-making for resolving the fisheries problem.

Our formulation can be extended to more complex cases provided that the characteristic function of the target process is accessible in a closed form. Affine stochastic Volterra processes [25] and self-exciting affine processes [10] would be such examples. In this view, a variety of modern engineering issues related to sustainability such as the water abstraction for hydropower generation can be analyzed by a suitable modification of our framework. We focused on a jump-driven process, while considering jump-diffusion processes would not encounter significant technical difficulties.

A problem that was not addressed in this paper is multi-stage problems as fish migration would be more reasonably considered as seasonal population dynamics. The (conditional) characteristic function would be still available in a closed-form even in such cases, while our preliminary investigations suggested that computing transient PDFs is highly subject to Gibbs oscillations. The oscillations will be mitigating by a more careful application of a spectral filter [24]. From a biological standpoint, better understanding of the fish migration would advance the modeling strategy and its application to fisheries optimization.

Acknowledgements

This work was supported by Japan Society for the Promotion of Science (22K14441, 22H02456) and Grant from MLIT Japan (B4R202002).

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DOI: 10.1007/978-3-031-36030-5_45

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DOI: 10.1007/978-3-031-36030-5_45