

Unstructured conservative level-set (UCLS) simulations of film boiling heat transfer *

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Abstract. A novel unstructured conservative level-set method for film boiling is introduced. The finite-volume discretization of transport equations is performed on collocated unstructured grids. Mass transfer is driven by thermal phase change and computed with the temperature gradient in the liquid and vapour phases at the interface. The fractional-step projection method is used for solving the pressure-velocity coupling. The convective term of transport equations is discretized by unstructured flux-limiter schemes to avoid numerical oscillations around the interface and minimize numerical diffusion. The central difference scheme discretizes diffusive terms. Verification and validation for film boiling on a flat surface are performed.

Keywords: Unstructured Conservative Level-Set Method · Unstructured Flux-Limiters · Finite-Volume Method · Unstructured Meshes · Vapor-Liquid Phase Change · Film Boiling

1 Introduction

Liquid-vapour phase change, e.g., boiling and condensation, is frequent in nature and industrial applications. Multiple engineering devices, from steam generators to cooling towers in nuclear and conventional thermal power plants, from unit operations to chemical reactors in the chemical processing industry, entail bubbles or droplets generated by liquid-vapour phase change, i.e., boiling, condensation and evaporation. Although empirical correlations have been proposed to perform predictions in boiling heat transfer, the interaction between fluid mechanics and transport phenomena in liquid-vapour phase change still needs to be better understood.

Research in Liquid-vapour phase change follows three paths: i) Experimental measurements with the appropriate visualization and instrumentation systems, ii) theoretical methods based on the analytical solutions of mathematical models with substantial

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simplifications of physics, and iii) numerical methods and their computational implementation. Complexities of liquid-vapour phase change constrain the analytical methods [1]. On the other hand, optical access can limit experimental measurements. Indeed, computational methods can often be the only mechanism to explore boiling heat transfer. Three computational methods are remarked: Euler-Euler method (E-E) or two-fluid model [47], Euler-Lagrange method (E-L) [47] and Direct Numerical Simulation (DNS) [47,68]. The E-E method formulates the continuous and dispersed phases as a fully interpenetrating continuum. The E-L method uses the Eulerian framework to solve the continuous phase, whereas the Lagrangian approach tracks the position and velocity of the fluid particles (bubbles or droplets). Finally, DNS solves all scales of fluid flow and interfaces without physics simplifications. Supercomputer advances have empowered the DNS as a practical approach for designing numerical experiments of liquid-vapour phase change.

Multiple methods have been reported for DNS of two-phase flows: front-tracking (FT) [70,65], level-set (LS) [45,61,26], Volume of Fluid (VoF) [32,50,68], coupled VoF-LS [60,59,11], and conservative level-set (CLS) [44,10,16], as examples. Further extensions of these methods include liquid-vapour phase change. For instance, [71,31,72,69,46,40] report VoF implementations of phase change. [55,56,57,58,51] propose extensions of LS methods to boiling heat transfer. Hybrid LS method and ghost-fluid approach [27] were reported by [28,42,63]. Coupled VoF-LS methods for phase change were reported by [43,64,53]. [34,23,24,67,33,48] inform a FT method for boiling. Although significant numerical and physical findings have been reported in previous efforts, most proposed methods employ structured and Cartesian meshes. Therefore, multiple flow conditions and engineering interest configurations must be explored. On the other hand, to the authors' knowledge, the robustness and accuracy of the unstructured conservative level-set (UCLS) method [16,14,10,8,18] to tackle film boiling heat transfer are still to be proven. Indeed, this work is a systematic step to develop numerical methods for complex interface physics in the framework of the UCLS method proposed by Balcazar et al. [16,14,10,8,18].

This research is organized as follows: Section 2 reviews the mathematical formulation. Section 2.4 presents the numerical methodology of the UCLS method on collocated unstructured meshes. Section 3 gives validations and numerical experiments of film boiling heat transfer. Finally, Section 4 report the conclusions.

2 Mathematical Formulation and Numerical Methods

2.1 Incompressible two-phase flow with phase change

The Navier-Stokes equations for the vapour phase (Ω_v) and liquid phase (Ω_l) are presented in the framework of the so-called one fluid formulation [66,47,68]:

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot \mu (\nabla \mathbf{v}) + \nabla \cdot \mu (\nabla \mathbf{v})^T + (\rho - \rho_0) \mathbf{g} + \mathbf{f}_\sigma, \quad (1)$$

Here p is the pressure, \mathbf{v} denotes the fluid velocity, μ refers to the dynamic viscosity, ρ refers to the fluid density, \mathbf{g} is the gravity, \mathbf{f}_σ denotes the surface tension force per

unit volume, acting on the interface (Γ). If periodic boundary conditions are applied along the vertical axis (parallel to \mathbf{g}), the acceleration of the flow field in the direction of \mathbf{g} should be avoided. Accordingly Eq.(1) incorporates the force $-\rho_0\mathbf{g}$ [16,6,7], where $\rho_0 = V_\Omega^{-1} \int_\Omega (\rho_d H_d + \rho_c H_c) dV$. Otherwise, $\rho_0 = 0$.

Physical properties are constant at each fluid phase. Nevertheless, a jump discontinuity arises at the interface. Consequently, $\rho = \rho_l H_l + \rho_v H_v$, and $\mu = \mu_l H_l + \mu_v H_v$. Subscripts l and v refer to the liquid and vapour phases. H_v denotes the Heaviside step function, one in Ω_v and zero elsewhere. Moreover, $H_l = 1 - H_v$.

The mass conservation equation for the vapour phase and liquid phase are written as follows [8]:

$$\frac{\partial}{\partial t} H_v + \nabla \cdot (H_v \mathbf{v}) = \frac{\dot{m}_{lv}}{\rho_v} \delta_\Gamma, \quad \frac{\partial}{\partial t} H_l + \nabla \cdot (H_l \mathbf{v}) = -\frac{\dot{m}_{lv}}{\rho_l} \delta_\Gamma, \quad (2)$$

where δ_Γ is the Dirac delta function concentrated at Γ , \dot{m}_{lv} is the mass transfer rate promoted by the liquid-vapour phase change. As a consequence:

$$\nabla \cdot \mathbf{v} = \left(\frac{1}{\rho_v} - \frac{1}{\rho_l} \right) \dot{m}_{lv} \delta_\Gamma. \quad (3)$$

Eq.(3) poses an incompressible constraint in Ω , excluding the interface region (Γ).

2.2 Unstructured conservative level-set (UCLS) method

Interface capturing on collocated unstructured meshes is performed by the Unstructured Conservative Level-Set (UCLS) method proposed by Balcázar et al. [16][10]. An implicit function, $\phi = \frac{1}{2} (\tanh(\frac{d}{2\varepsilon}) + 1)$, represents the interface. Here, the parameter ε sets the interface thickness, and d is a signed distance function [45]. Furthermore, $\varepsilon_P = 0.5(h_P)^\alpha$ at the local cell Ω_P , h_P is the local grid size [16], $\alpha = 0.9$ unless otherwise stated. The Heaviside step function (H_l) introduced in Eq.(2) can be regularized by the UCLS function (ϕ). Therefore, $H_l^s = 1 - H_v^s = \phi$. And, consequently, Eq.(2) leads to an interface advection equation with phase change,

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \phi \mathbf{v} = -\frac{1}{\rho_l} \dot{m}_{lv} \delta_\Gamma^s. \quad (4)$$

Here, $\delta_\Gamma^s = \|\nabla \phi\|$ is the regularized Dirac delta function [16,10,15,8]. The UCLS profile is kept constant and sharp by solving the re-initialisation equation [10],

$$\frac{\partial \phi}{\partial \tau} + \nabla \cdot \phi(1 - \phi) \mathbf{n}|_{\tau=0} = \nabla \cdot \varepsilon \nabla \phi, \quad (5)$$

where $\mathbf{n}|_{\tau=0}$ refers to the normal unit vector evaluated at $\tau = 0$. Eq.(5) is advanced in the pseudo-time (τ) upon arriving at the steady state. Interface curvature (κ) and normal unit vector (\mathbf{n}) are computed as follows: $\mathbf{n}(\phi) = \nabla \phi_i / \|\nabla \phi_i\|^{-1}$ and $\kappa(\phi) = -\nabla \cdot \mathbf{n}$. Furthermore, the Continuous Surface Force (CSF) model [20] is employed to compute the surface tension force in the framework of the UCLS method [16,14,9,18,10]. Consequently, $\mathbf{f}_\sigma = \sigma \kappa \mathbf{n} \delta_\Gamma^s = \sigma \kappa(\phi) \mathbf{n}(\phi) \|\nabla \phi\| = \sigma \kappa(\phi) \nabla \phi$, where σ is the surface tension coefficient. Finally, the smoothed Heaviside step function (H_l^s and H_v^s) regularize physical properties. Indeed, $\mu = \mu_l H_l^s + \mu_v H_v^s$ and $\rho = \rho_l H_l^s + \rho_v H_v^s$. Consistently with Eq.(4), $H_l^s = \phi$ and $H_v^s = 1 - H_l^s$.

2.3 Liquid-vapour phase change and energy equation

The following thermal energy equation computes the temperature on Ω_v :

$$\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{v}T) = \frac{1}{\rho c_p} \nabla \cdot (\lambda \nabla T), \quad (6)$$

where $c_p = c_{p,v}$ is the specific heat capacity at constant pressure, $\lambda = \lambda_v$ is the thermal conductivity, and $\rho = \rho_v$ is the density of the vapour phase. The vapour-liquid interface is at the saturation temperature (T_{sat}) [54], $T(\mathbf{x}, t) = T_{sat}$ on Ω_l . The mass transfer (\dot{m}_{lv}) driven by the liquid-vapour phase change is computed on the interface [8]: $\dot{m}_{lv} = h_{lv}^{-1} (\lambda_v (\nabla T \cdot \mathbf{n})_v - \lambda_l (\nabla T \cdot \mathbf{n})_l)$. Here, h_{lv} refers to the heat of vaporization. On the other hand, $T(\mathbf{x}, t) = T_{sat}$ in Ω_l implies that $(\nabla T \cdot \mathbf{n})_l = 0$. Finally, $(\nabla T \cdot \mathbf{n})_v = \Theta_{n,v}$ is extended on $\phi_{cut} < \phi < 1$, by solving a transport equation proposed in the framework of the UCLS method by [8]: $\frac{\partial}{\partial \tau} \Theta_{n,v} + \mathbf{n} \cdot \nabla \Theta_{n,v} = 0$. In this research $\phi_{cut} = 0.3$.

2.4 Finite-volume method: Unstructured flux-limiters

The finite-volume method discretizes transport equations for 3D collocated unstructured meshes [16]. The convective term of energy equation (Eq.(6)), level-set advection equation (Eq.(4)), and momentum transport equation (Eq.(1)), is explicitly computed through unstructured flux-limiter schemes proposed by Balcazar et al. [16,10]. Accordingly, the convective term in the local cell Ω_P is discretized as follows: $(\nabla \cdot \beta \psi \mathbf{v})_P = V_P^{-1} \sum_f \beta_f \psi_f (\mathbf{v}_f \cdot \mathbf{A}_f)$. Here, $\mathbf{A}_f = \|\mathbf{A}_f\| \mathbf{e}_f$ is the area vector. The unit vector \mathbf{e}_f , perpendicular to the face f , points outside the local cell Ω_P , V_P is the volume of Ω_P , and sub-index f refers to the cell faces. On the other hand,

$$\psi_f = \psi_{C_p} + \frac{1}{2} L(\theta_f) (\psi_{D_p} - \psi_{C_p}), \quad (7)$$

where $\theta_f = (\psi_{C_p} - \psi_{U_p}) / (\psi_{D_p} - \psi_{C_p})$ is a monitor variable, and $L(\theta_f)$ is the flux limiter function. Consistently with [16,10], subindex C_p refers to the upwind point, subindex U_p denotes the far-upwind point, and subindex D_p denotes the downwind point. Multiple flux limiters functions are implemented [62,25,38,39,41]:

$$L(\theta_f) \equiv \begin{cases} \max\{0, \min\{2\theta_f, 1\}, \min\{2, \theta_f\}\} & \text{SUPERBEE,} \\ (\theta_f + |\theta_f|) / (1 + |\theta_f|) & \text{VANLEER,} \\ \max\{0, \min\{4\theta_f, 0.75 + 0.25\theta_f, 2\}\} & \text{SMART,} \\ 1 & \text{CD,} \\ 0 & \text{UPWIND.} \end{cases} \quad (8)$$

A SUPERBEE flux limiter unless otherwise stated. The compressive term (Eq. (5)) is discretized as proposed by [16]: $(\nabla \cdot \phi_i (1 - \phi_i) \mathbf{n}_i^0)_P = \frac{1}{V_P} \sum_f (\phi_i (1 - \phi_i))_f \mathbf{n}_{i,f}^0 \cdot \mathbf{A}_f$. Here, $(\phi_i (1 - \phi_i))_f$ and $\mathbf{n}_{i,f}^0$ are linearly interpolated. The central difference scheme discretizes the diffusive term of transport equations [16]. Linear interpolation (arithmetic or distance weighted) [16] approximates the cell-face values. The weighted least-squares method [16,18,8,10] evaluates the gradients in Ω_P . The pressure-velocity coupling is solved by the fractional-step method [21,47,68,39]. First, a predictor velocity

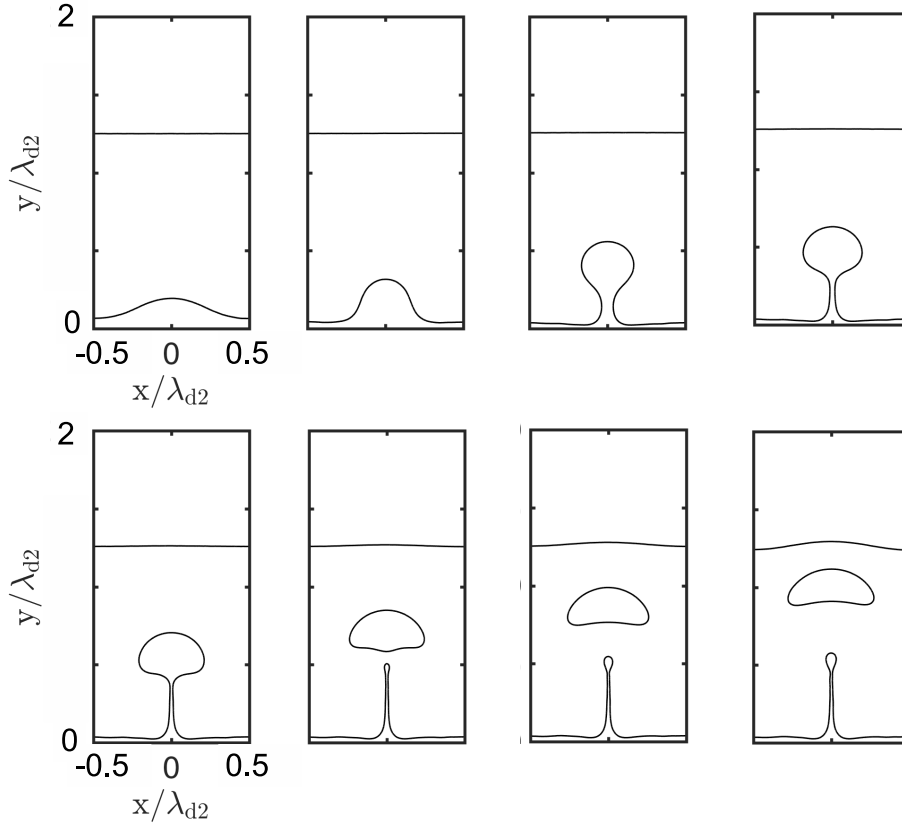


Fig. 1. Evolution of the liquid-vapour interface during film boiling. The dimensionless numbers are $Gr = 17.85$, $Pr = 4.2$ and $Ja = 0.064$. Physical properties ratios are $\mu_v/\mu_l = 0.386$, $\rho_v/\rho_l = 0.209$, $\lambda_v/\lambda_l = 0.281$, and $c_{p,v}/c_{p,l} = 1.830$. The grid size is $h = \lambda_{d2}300^{-1}$. $t^* = t/t_s = \{1.02, 3.06, 5.09, 6.11, 7.13, 8.15, 9.17, 10.19\}$.

(\mathbf{v}_P^*) is calculated:

$$\frac{\rho_P \mathbf{v}_P^* - \rho_P^0 \mathbf{v}_P^0}{\Delta t} = \mathbf{C}_{v,P}^0 + \mathbf{D}_{v,P}^0 + (\rho_P - \rho_0) \mathbf{g} + \sigma \kappa_P (\nabla \phi)_P, \quad (9)$$

where $\mathbf{C}_v = -\nabla \cdot (\rho \mathbf{v} \mathbf{v})$ and $\mathbf{D}_v = \nabla \cdot \mu \nabla \mathbf{v} + \nabla \cdot \mu (\nabla \mathbf{v})^T$. The superindex 0 refers to the previous time step. The incompressibility constraint with phase change (Eq.(3)) is applied to the corrector-step (Eq.(11)). Consequently, the following Poisson equation for the pressure arises:

$$\left(\nabla \cdot \frac{\Delta t}{\rho} \nabla p \right)_P = (\nabla \cdot \mathbf{v}^*)_P - \left(\frac{1}{\rho_v} - \frac{1}{\rho_l} \right) \dot{m}_{l,v,P} \delta_{\Gamma,P}^s, \quad \mathbf{e}_{\partial\Omega} \cdot \nabla p|_{\partial\Omega} = 0. \quad (10)$$

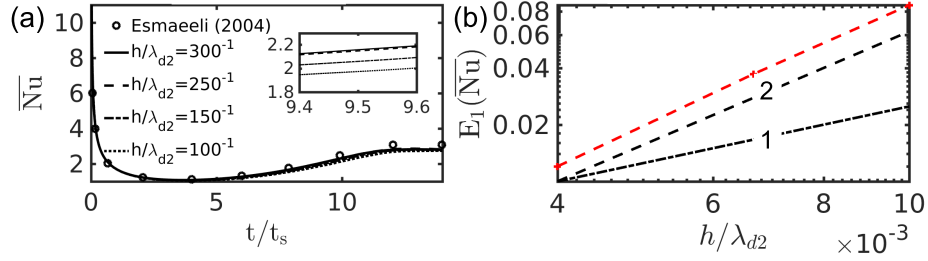


Fig. 2. Grid refinement test: (a) Time evolution of Nusselt number (\overline{Nu}). The dimensionless numbers are $Gr = 17.85$, $Pr = 4.2$ and $Ja = 0.064$. Physical properties ratios are $\mu_v/\mu_l = 0.386$, $\rho_v/\rho_l = 0.209$, $\lambda_v/\lambda_l = 0.281$, and $c_{p,v}/c_{p,l} = 1.830$. Benchmark results extracted from Esmaeeli and Tryggvason (2004) [23] (symbols). (b) Order of convergence: The red line denotes present simulations. Black lines for first-order (1) and second-order (2) convergence. $E_1(\overline{Nu}) = N^{-1} \sum_{i=1}^N \|\overline{Nu}_i - \overline{Nu}_{i,ref}\|$, $\overline{Nu}_{i,ref}$ refers to numerical results for the finest mesh, N is the number of sample points ($t_i t_s^{-1}$, \overline{Nu}_i).

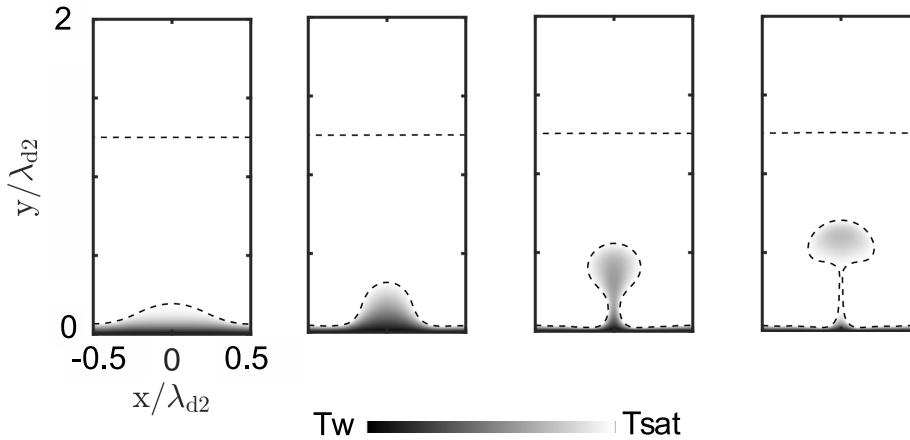


Fig. 3. Evolution of the liquid-vapour interface and temperature field during film boiling. Here, $Pr = 4.2$, $Gr = 17.85$, and $Ja = 0.064$. The ratio of thermophysical properties are $\rho_v/\rho_l = 0.209$, $\mu_v/\mu_l = 0.386$, $\lambda_v/\lambda_l = 0.281$, and $c_{p,v}/c_{p,l} = 1.830$. The grid size is $h = \lambda_{d2} 300^{-1}$.

Eq. (10) lead to a linear system, which is solved by the preconditioned conjugate gradient method [35]. In a further step, a corrected velocity (\mathbf{v}_P) is computed:

$$\frac{\rho_P \mathbf{v}_P - \rho_P \mathbf{v}_P^*}{\Delta t} = -(\nabla p)_P, \quad (11)$$

Furthermore, to avoid pressure-velocity decoupling on collocated meshes [49] and to fulfil the incompressibility constraint, a cell-face velocity \mathbf{v}_f is interpolated (see [14]).

The volume flux ($\mathbf{v}_f \cdot \mathbf{A}_f$), normal velocity ($\mathbf{v}_f \cdot \mathbf{e}_f$) or some equivalent variable are employed to solve the convective term of transport equations [14].

Technical details for the finite-volume discretization of the transport equations on 3D collocated unstructured meshes can be found in [16]. The new numerical methods for liquid-vapour phase change are developed in the framework of the UCLS solver proposed by Balcazar et al. [16]. The numerical code employs MPI (Message Passing Interface) for parallel communications and C++ for object-oriented design. The strong-speedup scalability is reported in [6,16].

3 Numerical Experiments

3.1 Validations and verifications of the UCLS method

The UCLS method was first introduced in [10]. A main contribution was the introduction of an accurate and original formulation of unstructured flux limiters for the finite-volume discretization of the level-set advection equation on collocated unstructured meshes. Systematic validations, verifications, and extensions of the UCLS method [10,16] include: thermocapillarity [14,4,18], gravity-driven rising bubbles [10,7,6,3,2], bubbly flows [12,6,16,17], binary droplet collision [12], collision of a droplet against a fluid-fluid interface [12], deformation of droplets [11], Taylor bubbles [29,30,2], falling droplets [9], atomization of gas-liquid jets [52], mass transfer in bubble swarms [16,17,5], and liquid-vapour phase change [8]. A comparison of the UCLS method [10] and coupled VoF-LS method [11] is reported in [9]. An extension of the UCLS method to tetrahedral adaptive mesh refinement has been reported in [3]. This research is a further step toward developing algorithms for complex interface physics in the framework of the UCLS method proposed by Balcazar et al. [16,18,13,7,12,10,15].

The following dimensionless numbers characterize the film boiling heat transfer:

$$\text{Gr} = \frac{\rho_v(\rho_l - \rho_v) \|\mathbf{g}\| X^3}{\mu_v^2}, \text{Pr} = \frac{\mu_v c_{p,v}}{\lambda_v}, \text{Ja} = \frac{c_{p,v}(T_w - T_{sat})}{h_{lv}}, \frac{\beta_v}{\beta_l}, \quad (12)$$

where Pr is the Prandtl number, Gr is the Grashof number, Ja denotes the Jakob number, $\beta = \{\rho, \mu, c_p, \lambda\}$. Here $X = l_s$ is a characteristic length scale. Furthermore, the computation of the Nusselt number (Nu) is performed as follows:

$$\begin{aligned} \text{Nu}(\mathbf{x}_w, t) &= \frac{X}{(T_w - T_{sat})} (\nabla T \cdot \mathbf{e}_w)(\mathbf{x}_w, t), \\ \overline{\text{Nu}}(t) &= \frac{1}{A_w} \int_{A_w} \text{Nu}(\mathbf{x}_w, t) dA, \\ \overline{\overline{\text{Nu}}} &= \frac{1}{T} \int_0^T \overline{\text{Nu}}(t) dt, \end{aligned} \quad (13)$$

where $\text{Nu}(\mathbf{x}_w, t)$ is the local Nusselt number. The unit vector \mathbf{e}_w , perpendicular to the bottom wall, points toward the fluids. $\overline{\text{Nu}}(t)$ refers to the space-averaged Nusselt number at the time t , A_w is the wall surface. $\overline{\overline{\text{Nu}}}$ denotes the time-averaged Nusselt number in the period T . On the other hand, $l_s = (\sigma \|\mathbf{g}\|^{-1} |\rho_l - \rho_v|^{-1})^{1/2}$ is the capillary length

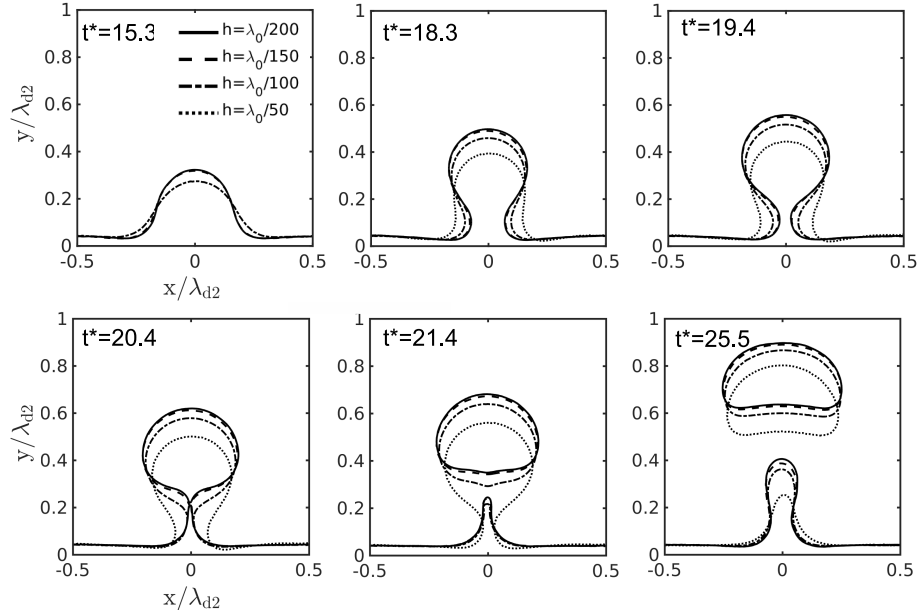


Fig. 4. Evolution of the liquid-vapour interface during film boiling. Here, $Pr = 1$, $Gr = 144.6$, and $Ja = 0.1$. The ratio of thermophysical properties are $\rho_v/\rho_l = 20^{-1}$, $\mu_v/\mu_l = 40^{-1}$, $\lambda_v/\lambda_l = 1$, and $c_{p,v}/c_{p,l} = 1$. The grid size is $h = \lambda_{d2}200^{-1}$.

scale, $v_s = (|\mathbf{g}|l_s)^{1/2}$ is the characteristic velocity, and $t_s = l_s/v_s$ is the characteristic time scale. The most dangerous wavelength [19] of Rayleigh-Taylor instability is defined as $\lambda_{d2} = 2\pi\sqrt{3}l_s$.

3.2 Film boiling on a horizontal plane

Ω is a rectangular domain $(L_x, L_y, L_z) = (\lambda_{d2}, 2\lambda_{d2}, h)$, discretized by triangular prisms with $h \approx \{\lambda_{d2}/150, \lambda_{d2}/200, \lambda_{d2}/250, \lambda_{d2}/300\}$. The initial film thickness is perturbed according to the interface shape: $y_I = y_0 + A \cos(2\pi x/\lambda_{d2})$ where $y_0 = 0.125\lambda_{d2}$, $A = 0.05\lambda_{d2}$ [23]. A vapour layer on top of the liquid layer is posed at $1.25\lambda_{d2}$. Neumann boundary conditions are set to all the boundaries except at the bottom wall. The no-slip boundary condition is applied for the bottom wall's velocity (\mathbf{v}_w). The temperature at the bottom wall is fixed to T_w . As initial conditions, the velocity field is zero, and the temperature field in Ω equals the saturation temperature T_{sat} . The computational setup is illustrated in Fig. 1.

A grid resolution test is depicted in Figure 2. The time evolution of the Nusselt number (\overline{Nu}) is shown in Figure 2a. The dimensionless numbers are $Gr = 17.85$, $Pr = 4.2$ and $Ja = 0.064$. Physical properties ratios are $\mu_v/\mu_l = 0.386$, $\rho_v/\rho_l = 0.209$, $\lambda_v/\lambda_l = 0.281$, and $c_{p,v}/c_{p,l} = 1.830$. Present results computed with the novel UCLS method for film boiling are in close agreement with front-tracking simulations reported

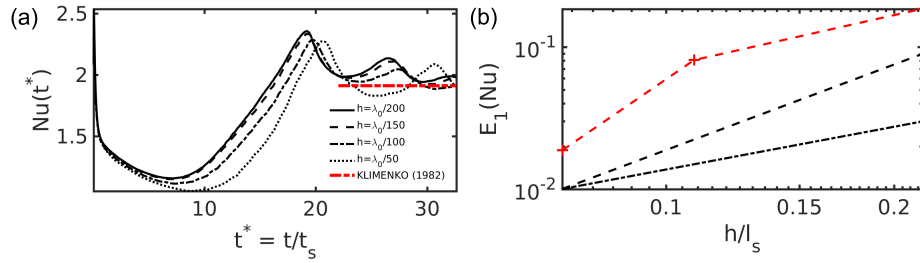


Fig. 5. Grid resolution test: (a) Time evolution of Nusselt number (\overline{Nu}). Here, $Pr = 1$, $Gr = 144.6$, and $Ja = 0.1$. The ratio of thermophysical properties are $\rho_v/\rho_l = 20^{-1}$, $\mu_v/\mu_l = 40^{-1}$, $\lambda_v/\lambda_l = 1$, and $c_{p,v}/c_{p,l} = 1$. Numerical results are compared against Klimenko's correlation [36,37]. (b) Order of convergence: The red line denotes present simulations. Black lines for first-order (1) and second-order (2) convergence. $E_1(\overline{Nu}) = N^{-1} \sum_{i=1}^N \|\overline{Nu}_i - \overline{Nu}_{i,ref}\|$, $\overline{Nu}_{i,ref}$ refers to numerical results for the finest mesh, N is the number of sample points ($t_i t_s^{-1}$, \overline{Nu}_i).

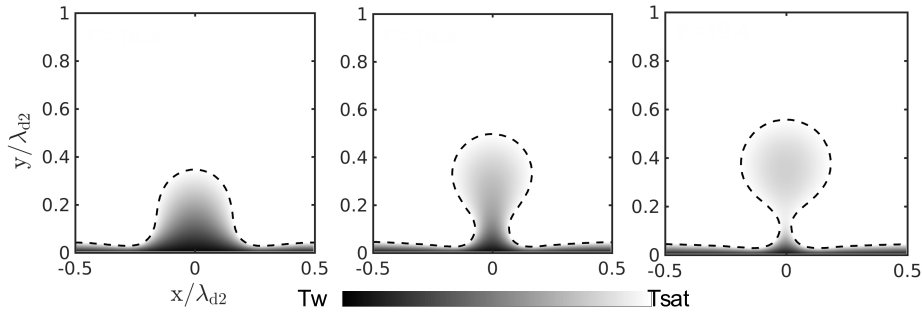


Fig. 6. Evolution of the liquid-vapour interface and temperature field during film boiling. Here, $Pr = 1$, $Gr = 144.6$, and $Ja = 0.1$. The ratio of thermophysical properties are $\rho_v/\rho_l = 20^{-1}$, $\mu_v/\mu_l = 40^{-1}$, $\lambda_v/\lambda_l = 1$, and $c_{p,v}/c_{p,l} = 1$. The grid size is $h = \lambda_{d2} 200^{-1}$.

by Esmaeeli and Tryggvason (2004) [23]. Figure 2b shows that the UCLS method presents second-order convergence. The red line denotes present simulations. Black lines for first-order (1) and second-order (2) convergence. The error is computed as follows: $E_1(\overline{Nu}) = N^{-1} \sum_{i=1}^N \|\overline{Nu}_i - \overline{Nu}_{i,ref}\|$, $\overline{Nu}_{i,ref}$ refers to numerical results for the finest mesh, N is the number of sample points ($t_i t_s^{-1}$, \overline{Nu}_i). The evolution of the liquid-vapour interface during film boiling is depicted in Figure 1, and the temperature field is shown in Figure 3.

A second case is performed on a rectangular domain $(L_x, L_y, L_z) = (\lambda_{d2}, \lambda_{d2}, h)$. Here, $Pr = 1$, $Gr = 144.6$, and $Ja = 0.1$. The ratio of thermophysical properties are $\rho_v/\rho_l = 20^{-1}$, $\mu_v/\mu_l = 40^{-1}$, $\lambda_v/\lambda_l = 1$, and $c_{p,v}/c_{p,l} = 1$. A test of grid convergence is illustrated in Figure 5, with $h \approx \{\lambda_{d2}/50, \lambda_{d2}/100, \lambda_{d2}/150, \lambda_{d2}/200\}$.

The evolution of the liquid-vapour interface during film boiling is depicted in Figure 4, and the temperature field is shown in Figure 6.

4 CONCLUSIONS

- The UCLS method introduced by Balcazar et al.[17,16,18,10,12,7,12] has been extended for film boiling heat transfer. The numerical model has been verified and validated against numerical results of [22] and Klimenko’s correlation [36,37].
- Unstructured flux-limiters schemes proposed by Balcazar et al. [10,16,11] for the discretization of the convective term, in the framework of the UCLS method, improves the numerical stability and minimize the numerical diffusion in simulations of film boiling heat transfer.
- Further verifications and validations, including three-dimensional cases, will be reported in a future work.

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