Model of perspective distortions for a vision measuring system of large-diameter bent pipes

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Abstract. The measuring system described in the article was designed for measuring large and heavy bent pipes with diameters up to 1.2 m. Currently, measurements of large-diameter bent pipes are taken using either simple and inaccurate protractors or measuring systems requiring a 3D model of the pipe created in graphics software, an optical scanner and markers. This paper presents methods of modeling distortions for measuring system based on one camera that is easy to install in an industrial plant. Those models allow to perform measurement quickly at any position of the pipe on a large industrial measurement table. The paper describes the mathematical models of perspective projections of single- and double- bent pipes, as well as the method of bending angle determination and detection of straight sections of the bent pipe. As part of the research, measurement accuracy of the designed system model were described and confirmed.

Keywords: Modelling · Vision system · Pipes measurement · Perspective distortions

1 Introduction

Today's vision systems are widely used in manufacturing processes. Growing quality requirements entail the use of continuous control and correction of the production process parameters. Cameras are used as sensors in many applications [4][10]. With the cameras, it is possible to recognise, classify, separate objects and determine their direction and location in space [9]. The use of a vision system in production lines allows for automatic and continuous control of the dimensional and shape quality of the manufactured parts [1]. The measurement accuracy is obviously lower than the accuracy of measurements performed by metrological machines and devices [7]. However, features such as: ability to track moving objects, no need to change the orientation of objects in space, quick and non-contact measurement, high degree of automation, durability or failure-free operation are the advantages that distinguish the vision systems. In

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this paper, the models of perspective distortions are provided because those are important in measurement vision systems to obtain correct metric measurement values.

New, elements worth emphasizing in the paper are: (1) determination of a mathematical model for single- and double-bent pipes, (2) determination of the pipe bend radius and straight sections using mathematical methods.

In one of the first works [2] on the multi-camera measuring system of bent pipes, the accuracy of bending radius measurement was not lower than 0.5 mm. Moreover, several works related to optical control and measurement systems of bent pipes and cables have been published. Most of them determine the surfaces of the 3D model of pipes using passive multi-view systems. Therefore, the proposed system requires adding a pipe model created using graphic software to the measurement, and then, based on this model, measurement errors are estimated. The presented work does not provide for using a 3D model, and the mathematical model created is built on the basis of a perspective projection. Moreover, there is a tendency to increase the measurement accuracy by connecting an increasing number of cameras. Like in works [12],[8], where a system consisting of 16 fixed cameras was used.

Work [11] presents a pipe measurement system using stereo-visions and a perspective model of a pipe to increase the accuracy of the spatial reconstruction of the bent pipe central axis. In work [6], an analysis of reconstruction errors for a 3D model of bent pipes is performed using a multi-camera system that is divided into pairs of stereo-vision cameras. Additionally, an illuminated work table was used in that case. Work [3] presents a system used for reconstructing the shape of a pipe with a constant diameter on the basis of photos taken with a three-view vision system. The work takes into account perspective distortions for each of the cameras, and the pipe's surface is described with the use of circles of the same diameter and centres on the centre line of the pipe. The mean square measurement error of the bending radius of 0.127 mm was obtained.

2 Description of the proposed measuring system

Due to the production costs, the pipe processing process requires shape and dimensional control in subsequent stages of production, especially after the following processes: bending, heat treatment, possible corrective works and in the preparation of the as-built report. Even small errors in the bending angle prevent correct installation of the pipe in the pipeline.

We assume that the measured object is a combination of cylindrical and toroidal surfaces with a constant cross-sectional radius. As a result of this assumption, it is possible to measure the parameters of an object based on the image obtained in the camera setting. It should be mentioned here that this is an approximation – in fact, the bend of the bent pipe consists of many smaller bends with different radii and, what is more, the pipe becomes oval (changing the shape from round to elliptical.

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Parameters describing the double-bent pipe are given in Figure 1. The developed system determines the following parameters for pipes bent in one plane:
The work also considers the measurement of pipes bent in planes inclined to each other at an angle ζ.

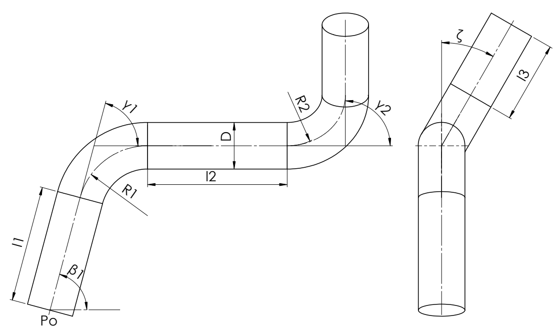

Fig. 1: Basic parameters of the bent pipe: bending angles γ_1, γ_2; pipe bending radii R_1, R_2; straight section lengths l_1, l_2, l_3; pipe diameter $D = 2r$; bending plane angles of deviation ζ

In the next section, we will show how to find those parameters for single and double bending.

3 Mathematical models of bent pipes

3.1 Single-bent pipe model

Let us begin with describing the parametric equations of the pipe surface centre line. We assume that the line consists of a straight line section $\mathbf{P}_0\mathbf{P}_1$, an arc $\mathbf{P}_1\mathbf{P}_2$ and another straight line section $\mathbf{P}_2\mathbf{P}_3$. The position of the measured object is determined by:

- co-ordinates x, y, z end of the centreline $\mathbf{P}_0 = [x_0 \; y_0 \; r]^T$,
- the slope of the centre line at point $\mathbf{P}_0$ to axis X: β_1.

One should note that the slope of the segment $\mathbf{P}_2\mathbf{P}_3$ will be equal to:

$$\beta_2 = \beta_1 + \gamma, \; \gamma > 0. \tag{1}$$

Suppose the parameter t is the length of the centre line from the point $\mathbf{P}_0$ to the point selected on the line $\mathbf{P}$. We will denote:

$$t_1 = l_1, \quad t_2 = l_1 + R\gamma, \quad t_3 = t_2 + l_2. \tag{2}$$

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So the parameters t_1, t_2, t_3 are the lengths of the line sections $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$. The parametric equation of the section $\mathbf{P}_0\mathbf{P}_1$ can take the following form:

$$\mathbf{M}(t) = \begin{bmatrix} x_0 + t\cos\beta_1 \\ y_0 + t\sin\beta_1 \\ r \end{bmatrix}, \quad t \in [0, t_1]. \tag{3}$$

As the centre line is smooth, the coordinates of the centre point of the circle of which the arc $\mathbf{P}_1\mathbf{P}_2$ is a part equal:

$$\mathbf{O}_c = \mathbf{P}_1 + \begin{bmatrix} -R\sin\beta_1 \\ R\cos\beta_1 \\ r \end{bmatrix}, \quad \mathbf{P}_1 = \mathbf{M}(l_1) \tag{4}$$

and the parametric equation for the arc of a circle:

$$\mathbf{M}(t) = \mathbf{O}_c + \begin{bmatrix} R\sin\beta \\ -R\cos\beta \\ r \end{bmatrix}, \quad \beta = \beta_1 + \frac{t - t_1}{R}, \quad t \in [t_1, t_2]. \tag{5}$$

Similarly to (3), the parametric equation of the section $\mathbf{P}_2\mathbf{P}_3$ has the following form:

$$\mathbf{M}(t) = \mathbf{P}_2 + \begin{bmatrix} (t - t_2)\cos\beta_2 \\ (t - t_2)\sin\beta_2 \\ r \end{bmatrix}, \quad \mathbf{P}_2 = \mathbf{M}(t_2), \quad t \in [t_2, t_3]. \tag{6}$$

Figure 2b shows the projection of the centre line onto a plane for example parameter values: $x_0 = 100$, $y_0 = -30$, $\beta_1 = 150°$, $\gamma = 100°$, $R = 40$, $l_1 = 130$, $l_2 = 85$, $r = 15$.

Based on equations (3), (5) and (6) we can write a parametric equation for the entire surface. Suppose $\mathbf{M}(t)$ is the selected centre line point, and $\beta(t)$ is the angle of inclination of the tangent of this line to axis XY.

$$\beta(t) = \begin{cases} \beta_1 & \text{dla } t \in [0, t_1] \\ \beta_1 + \frac{t - t_1}{R_1} & \text{dla } t \in [t_1, t_2] \\ \beta_2 & \text{dla } t \in [t_2, t_3] \end{cases} \tag{7}$$

The cross-section of an object perpendicular to the centre line is described by the following equation:

$$\mathbf{T}(t) = \begin{bmatrix} \cos\beta(t) & -sin\beta(t) & 0 \\ \sin\beta(t) & cos\beta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ r\cos\alpha \\ r\sin\alpha \end{bmatrix} + \mathbf{M}(t), \tag{8}$$

where $\alpha \in [0, 2\pi]$, $\beta \in [0, t_3]$. The resulting equation is a parametric equation for the surface of a bent pipe. Figure 2a shows a pipe's surface generated using the previously selected example parameters. The measurement of the object parameters consists in analysing the edge of the object in the image captured by the camera. Based on the developed model, we will determine the parametric

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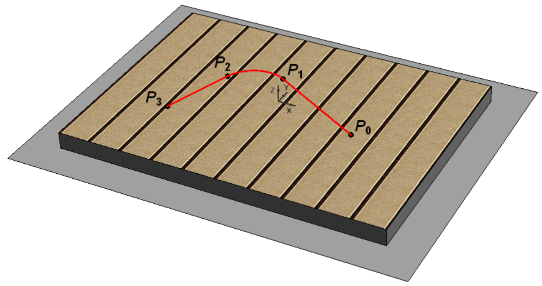
(a)



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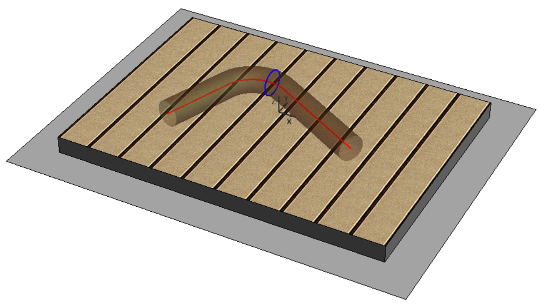

Fig. 2: (a) The centre line of a bent pipe (red) with points marking a straight line section $\mathbf{P}_0\mathbf{P}_1$, arc of a circle $\mathbf{P}_1\mathbf{P}_2$ and another straight line section $\mathbf{P}_2\mathbf{P}_3$ (b) Visualization of the pipe surface with its centreline (red) and the selected cross-section (blue). The reference coordinate system is also marked in the drawing

equations of these edges. To begin with, we will designate points $\mathbf{T}$ on the object surface for which the straight line drawn through the camera point $\mathbf{C} = [0\ 0\ z_c]^T$ and point $\mathbf{T}$ will be tangent to the surface of the pipe. We will determine these points based on the condition:

$$\overrightarrow{\mathbf{CT}} \perp \overrightarrow{\mathbf{TM}}, \tag{9}$$

where $\mathbf{M}$ is the point of the centre line of the cross section passing through the point $\mathbf{T}$ from the condition:

$$(\mathbf{C} - \mathbf{T})^T(\mathbf{T} - \mathbf{M}) = 0 \tag{10}$$

we obtain (for better clarity parameter t was omitted):

$$(r - z_c)\sin\alpha + (m_y \cos\beta - m_x \sin\beta)\cos\alpha + r = 0, \tag{11}$$

where m_x, m_y are the corresponding elements of vector $\mathbf{M}$. The above equation has two solutions α_1, α_2 dependent on parameter t. It is understandable, since

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from the point of camera one can draw two tangents to the pipe surface in a fixed cross-section. Intersection points **CT** of the straight line with plane XY are equal

$$\mathbf{E}_{1,2} = \begin{bmatrix} \frac{r z_c \cos\alpha_{1,2} \sin\beta - m_x z_c}{r \sin\alpha_{1,2}} + r - z_c \\ \frac{r z_c \cos\alpha_{1,2} \cos\beta - m_y z_c}{r \sin\alpha_{1,2}} + r - z_c \\ 0 \end{bmatrix}. \tag{12}$$

It must be noted that in the above equations variables β, M_y, M_y are functions t, so $\mathbf{E_1}$ and $\mathbf{E_2}$ are parametric equations of the edge of projection of the pipe surface onto the table plane. Equation (12) with example pipe parameters adopted from Figure 2b visualisation and for the adopted camera placement height $z_c = 130$ was crossed out and presented in Figure 3.

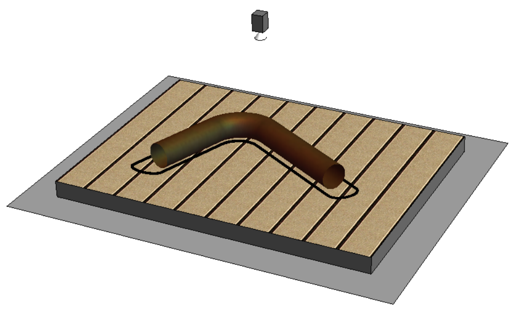

Fig. 3: Visualization of the projection of the pipe surface onto the table plane

It should be emphasized that the edges of the cylindrical surface section projection form two parallel lines. It results from the fact that the set of straight tangents to the cylinder and passing through the selected point creates a plane. Whereas the edges of the torus fragment projection are not, as one might assume, sections of circles, but a curve described by quite complex equations.

3.2 Twice-bent pipe model in one plane

The surface of an ideal pipe was recorded using parametric equations describing spatial figures: a cylinder and a torus. First, we will consider the case where the centre line is in a plane parallel to the plane of the table. Let us begin by describing the parametric equations of the pipe surface centre line. We assume that the line consists of a straight line section $\mathbf{P}_0\mathbf{P}_1$, an arc $\mathbf{P}_1\mathbf{P}_2$, another straight section $\mathbf{P}_2\mathbf{P}_3$, another arc $\mathbf{P}_3\mathbf{P}_4$ and another straight section $\mathbf{P}_4\mathbf{P}_5$. The position of the measured object is determined by:

- coordinates of the end of the centre line $\mathbf{P}_0 = [x_0\ y_0\ r]^T$,
- the slope of the centre line at a point $\mathbf{P}_0$ to the axis $X : \beta_1$.

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Let us presume the parameter t is the length of the centre line from the point $\mathbf{P}_0$ to the selected point on the centre line $\mathbf{M}$. We will denote:

$$\begin{aligned} t_1 &= l_1, \\ t_2 &= l_1 + R_1\gamma_1, \\ t_3 &= t_2 + l_2, \\ t_4 &= t_3 + R_2\gamma_2, \\ t_5 &= t_4 + l_3. \end{aligned} \tag{13}$$

So the parameters t_1, t_2, t_3,t_4, t_5 are the lengths of the line segments $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$, $\mathbf{P}_3\mathbf{P}_3$, $\mathbf{P}_4\mathbf{P}_5$. Parametric equation of the centreline for the first pipe section represented by segment $\mathbf{P}_0\mathbf{P}_1$ can be presented as follows:

$$\mathbf{M}(t) = \mathbf{P}_0 + t\begin{bmatrix}\cos\beta_1\\ \sin\beta_1\\ 0\end{bmatrix},\ t \in [0, t_1]. \tag{14}$$

As the centre line is smooth, the coordinates of the centre point of the circle of which the arc $\mathbf{P}_1\mathbf{P}_2$ equal:

$$\mathbf{O}_1 = \mathbf{P}_1 + R_1\begin{bmatrix}-\sin\beta_1\\ \cos\beta_1\\ 0\end{bmatrix},\ \mathbf{P}_1 = \mathbf{M}(t_1) \tag{15}$$

and the parametric equation of this arc of a circle:

$$\begin{aligned} &\mathbf{M}(t) = \mathbf{O}_1 + R_1\begin{bmatrix}\sin\beta\\ -\cos\beta\\ 0\end{bmatrix}, \\ &\beta = \beta_1 + (t - t_1)/R_1,\ t \in [t_1, t_2]. \end{aligned} \tag{16}$$

One should note that the slope of the segment $\mathbf{P}_2\mathbf{P}_3$ will be equal to:

$$\beta_2 = \beta_1 + \gamma_1, \tag{17}$$

so similarly to (14), the parametric equation of the section $\mathbf{P}_2\mathbf{P}_3$ has the following form:

$$\begin{aligned} &\mathbf{M}(t) = \mathbf{P}_2 + (t - t_2)\begin{bmatrix}\cos\beta_2\\ \sin\beta_2\\ 0\end{bmatrix}, \\ &\mathbf{P}_2 = \mathbf{M}(t_2),\ t \in [0, t_1]. \end{aligned} \tag{18}$$

Similarly to (15) and (16), the centre point and parametric equation of arc $\mathbf{P}_3\mathbf{P}_4$ take the following form:

$$\begin{aligned} &\mathbf{O}_2 = \mathbf{P}_3 + R_2\begin{bmatrix}-\sin\beta_2\\ \cos\beta_2\\ 0\end{bmatrix}, \\ &\mathbf{P}_3 = \mathbf{M}(t_3), \end{aligned} \tag{19}$$

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$$\mathbf{M}(t) = \mathbf{O}_2 + R_2 \begin{bmatrix} \sin\beta \\ -\cos\beta \\ 0 \end{bmatrix}, \qquad \beta = \beta_2 + (t - t_3)/R_2,\ t \in [t_3, t_4]. \tag{20}$$

Segment slope $\mathbf{P}_4\mathbf{P}_5$ to the axis X is:

$$\beta_3 = \beta_2 + \gamma_2 \tag{21}$$

and its parametric equation has the following form:

$$\mathbf{M}(t) = \mathbf{P}_4 + (t - t_4) \begin{bmatrix} \cos\beta_3 \\ \sin\beta_3 \\ 0 \end{bmatrix}, \qquad \mathbf{P}_4 = \mathbf{M}(t_4),\ t \in [t_4, t_5]. \tag{22}$$

Figure 4 shows the visualisation of the centre line of the pipe placed on the measurement table for example parameter values: $x_0 = 120$, $y_0 = 10$, $\beta_1 = 150°$, $\gamma_1 = 100°$, $\gamma_2 = 75°$, $R_1 = 40$, $R_2 = 40$, $l_1 = 130$, $l_2 = 50$, $l_3 = 50$, $r = 15$.

The next section describes image processing algorithms, which are necessary for automatically detecting edges of the pipes on the measuring table.

4 Determining the angle of bending and straight sections of pipes

There is assumed that the vision system correctly calculated the edge points of the measuring pipe. The obtained edge points of a selected pipe shown in Figure 5 that are expressed in the measurement table coordinate system, will be used to determine the parameters of the bent pipe. This task was divided into three stages: determination of two pairs of parallel lines (projections of straight pipe sections), determination of curves being a projection of a torus section and matching pipe parameters using the presented analytical model.

An effective method to reduce errors is to match a straight line or curve with the obtained data. For this purpose, simple equations were determined using an algorithm based on the Hough Transform [5]. In the case of a straight line, we will define its position in polar coordinates using: the inclination angle θ and the distance from the origin of the coordinate system ρ. Through each given point on the plane (x, y) we can route infinite number of lines which parameters θ and ρ are related by the equation

$$x\cos\theta + y\sin\theta = \rho. \tag{23}$$

The graph of this equation for a single point is a sine curve. We assume angle $\theta \in [0, \pi]$ and distance $\rho \in [0, \rho_{max}]$, where value ρ_{max} is the distance from the origin of the coordinate system to the most distant point (for an image it is the

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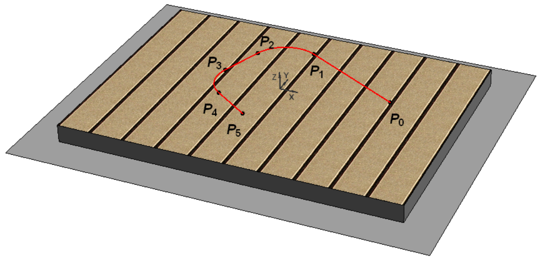
(a)



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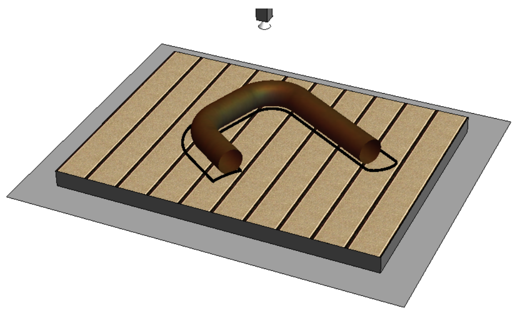

Fig. 4: (a) The centre line of a bent pipe (red) with points marking a straight line section $\mathbf{P}_0\mathbf{P}_1$, arc of a circle $\mathbf{P}_1\mathbf{P}_2$, straight line section $\mathbf{P}_2\mathbf{P}_3$, arc of a circle $\mathbf{P}_3\mathbf{P}_4$ and another straight line section $\mathbf{P}_4\mathbf{P}_5$, (b) Visualization of the projection of twice-bent pipe surface onto the table plane

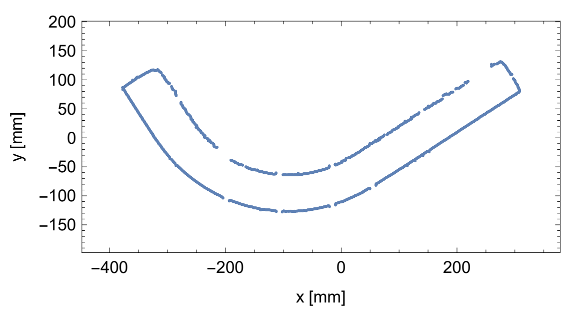

Fig. 5: The edge points of the recognized pipe converted to the measurement table coordinate system

diagonal length). Figure 6 shows the designated Hough plane $\Pi_h(\theta, \rho)$ with the step of discretisation $\Delta\theta = 0.1°$ and $\Delta\rho = 0.1$ mm. The parameters of the lines can be found by searching for local maxima, the value of which corresponds to the number of points lying on a given line. Note that the projection of the edge of

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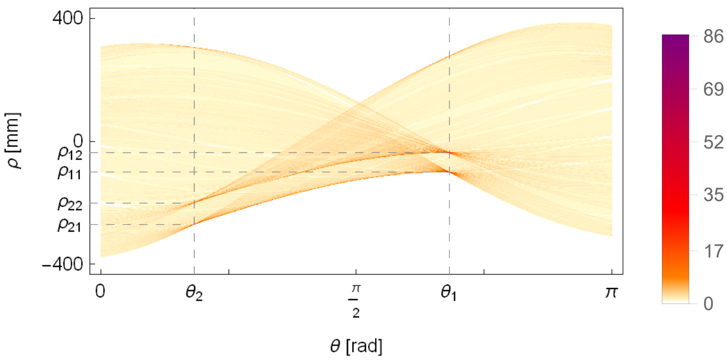

Fig. 6: The Hough plane represents the number of points on a straight line with an angle of inclination θ and distances from the origin of the coordinate system ρ

a straight pipe section lying on the table is a pair of parallel lines, and therefore of the same inclination angle θ_i but different distances ρ_{i1} and ρ_{i2}. Such a pair of simple lines can be found, for example, by solving the mathematical problem

$$\max_{\theta_i, \rho_{i_1}, \rho_{i_2}} (\Pi_h(\theta_i, \rho_{i_1}) + \Pi_h(\theta_i, \rho_{i_2})) \tag{24}$$

assuming the distance between the lines $|\rho_{i1} - \rho_{i2}|$ is sufficient (e.g. $|\rho_{i1} - \rho_{i2}| > \delta\rho$, where $\delta\rho$ is not greater than the nominal diameter D of the tested pipes).

In this work, when searching for pairs of lines, points lying in the vicinity of the straight line in the distance η were additionally taken into account and match index was defined as the following function:

$$g(\theta_i) = \max_{\rho_{i_1}, \rho_{i_2}} (\sum_{j=-\eta}^{\eta} \Pi_h(\theta_i, \rho_{i_1} + j) + \sum_{j=-\eta}^{\eta} \Pi_h(\theta_i, \rho_{i_2} + j)) \tag{25}$$

Solving the dependency (25) for a sample pipe and using parameters $\eta = 0.1$ mm and $\delta\rho = 10.0$ mm, the course of the quality index shown in Figure 7 was obtained. It has two maxima for angles θ_1 and θ_2. Additionally, when creating the algorithm it was assumed that the difference between angles $|\theta_1$ - $\theta_2|$ should be greater than the minimum bend angle $\delta\theta$, which is 0.5°.

The result of the described algorithm are the parameters of two pairs of parallel lines $(\theta_1, \rho_{1_1}, \rho_{1_2})$ and $(\theta_2, \rho_{2_1}, \rho_{2_2})$ that are visible on the Hough plane and are marked in Figure 6. The determined two pairs of parallel lines were marked on the edge points of the pipe in Figure 8.

The next task is to determine the mean line for the surface of the straight sections of the pipe.

In order to determine the parameters of the pipe, an equation for middle lines of two cylindrical parts of the pipe. Let us consider a cylindrical section of a pipe with an inclination angle θ_1. We will three planes: the plane of the table π_1 and

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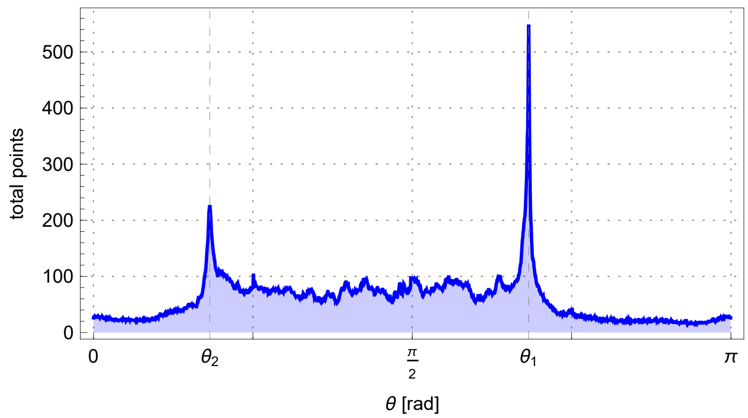

Fig. 7: Matching index for a pair of parallel lines inclined depending on angle θ

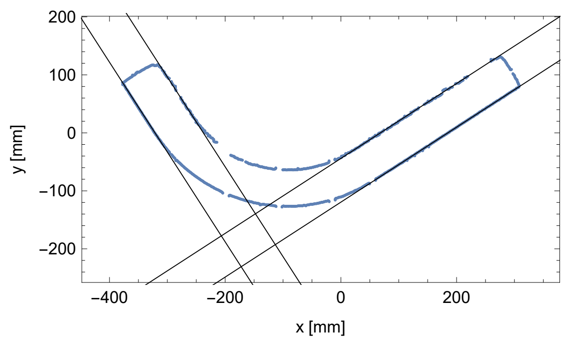

Fig. 8: Matching of the pairs of lines parallel to the edge points of the cylindrical sections of the bent pipe

two planes tangent to the cylinder passing through the camera point π_2, π_3, and then a section perpendicular to the three described planes passing through the camera point $\mathbf{C}$ (Figure 9).

Our goal is to determine the radius of the circle inscribed in the triangle ABC and the coordinate ρ_1 of the circle centre projection on the table plane, bearing in mind the coordinates of points $\mathbf{A}$ and $\mathbf{B}$ equal ρ_{11} and ρ_{12} respectively. We have:

$$r_1 = \frac{|\mathbf{AB}| z_c}{|\mathbf{AB}| + |\mathbf{BC}| + |\mathbf{CA}|}, \tag{26}$$

where $|\mathbf{AB}| = |\rho_{i_1} - \rho_{i_2}|$, $|\mathbf{BC}| = \sqrt{z_c^2 + \rho_{1_2}^2}$, $|\mathbf{CA}| = \sqrt{z_c^2 + \rho_{1_1}^2}$. Similarly we will find the value of radius r_2. Of course, the radius of the pipe is actually the same at both ends. We can therefore assume:

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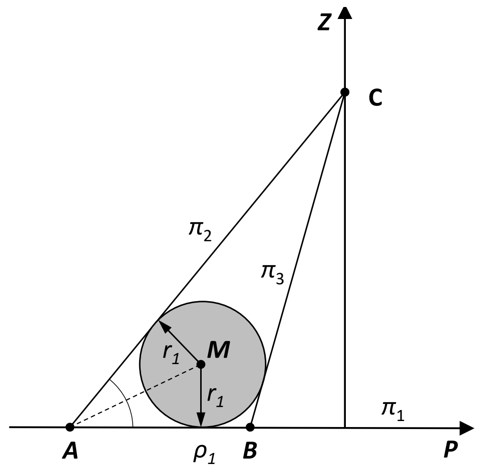

Fig. 9: Cross-section of the pipe with visible edge points **A** and **B** and a point on the centre line **M**

$$r = \frac{r_1 + r_2}{2}. \tag{27}$$

Figure 9 also shows the method of determining coordinate ρ_1. We have (I assume $r_1 = r$)

$$\rho_1 = \rho_{1_1} + r\cot(\frac{1}{2}\sphericalangle\mathbf{BAC}). \tag{28}$$

And in the same way, we will determine the parameter of the mean line of the second cylindrical segment.

As a result of the calculations, we have obtained parameters of the straight middle parts of both cylindrical parts of pipes (θ_1, ρ_1) and (θ_2, ρ_1).

On the basis of the obtained values, we can determine angle γ:

$$\gamma = \begin{cases} |\beta_2 - \beta_1| & \text{for the bend turning to the left side} \\ \pi - |\beta_2 - \beta_1| & \text{for the bend turning to the right side} \end{cases} \tag{29}$$

To determine the bend radius of pipe R, let us consider the projection of the edge of its bent fragment, which is described by the complex equations of analytical model (12). Note that the centre of torus fragment $\mathbf{O}_c$ is located on the bisector of an angle adjacent to γ between straight lines lying on the centre line of two cylindrical parts of the pipe and, at the same time, is offset from them to the distance of R. We will determine the point location $\mathbf{O}_c$ based on the real projection of the bent pipe edge.

Let us presume the distance between point $\mathbf{O}_c$ and the mean line, that is, value R falls within a certain range $[R_{min}, R_{max}]$. By making the interval discretisation by step δR we obtain values $\hat{R}_i$. Now, we define two accumulators A_{1i}

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and A_{2i} in the form of vectors with initial values equal to zero, indices of which are related to discrete values $\hat{R}_i$. For each edge point, we determine the distance from the two edge lines determined in the analytical model. If this distance is less than δP, we increase the value of accumulators A_{1i} or A_{2i} accordingly.

Finally, we designate index i and the corresponding value $R = \hat{R}_i$ for which sum:

$$\max_i = A_{1i} + A_{2i} \tag{30}$$

reaches its maximum.

The other parameters sought are the lengths of straight pipe sections l_1, l_2. By analysing the distance of the pipe edge points from the previously determined straight lines with parameters $(\theta_1, \rho_{11}, \rho_{12})$ and $(\theta_2, \rho_{21}, \rho_{22})$ we will define four points at both ends of the pipe profile. Based on these points, we will designate points on the centre line of the pipe $\mathbf{P}_0$, $\mathbf{P}_3$ which represent the position of the extreme cross sections of the pipe. Thus, the lengths of the straight pipe sections are, respectively:

$$l_1 = |\mathbf{P}_0\mathbf{P}_1|, \quad l_2 = |\mathbf{P}_2\mathbf{P}_3|. \tag{31}$$

The determined parameters are finally verified using the analytical model (12), by comparing the calculated outline of the pipe with the edges detected in the camera image (Figure 10). A similar method is used to determine the bend angles for double-bent pipes.

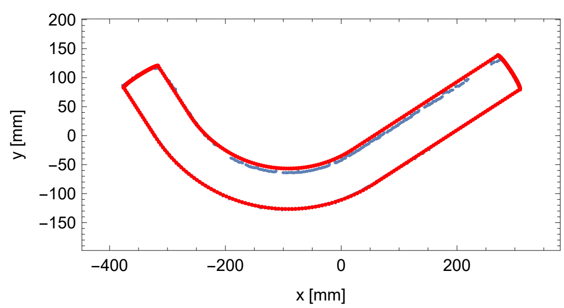

Fig. 10: Comparison of the pipe edge points (blue colour) to the perspective projection of the pipe calculated using the analytical model (red colour)

5 Experimental research

The measurement accuracy of the developed models was checked by comparing model results with results from a certified CMM Zeiss Prismo Navigator machine, designed for very accurate contact measurements. The measurement was taken along the outer and inner surface of the pipe, so that the outline of the pipe corresponded to its basic projection. The results of the accuracy are shown in

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Table 1. The CMM results are given in the first column and are considered a reference standard for the check. Then, a series of 14 measurements of the bent pipe were taken and the average value of these measurements is shown in the second column of Table 1. The last column presents the standard deviation of the measurements. From the industrial measurements point of view, the two most important parameters, i.e. the bend radius error and the bend angle error of the pipe, are respectively 0.85 mm, 0.34° and are similar to the results obtained in other scientific studies mentioned in the introduction of this paper. Moreover, they are accepted by the industry in pipelines components prefabrication.

Table 1: The results of the measuring system accuracy series of measurements of a reference pipe compared to the reference values measured by a Zeiss – Prismo Navigator CMM

	Reference measurement	**Series of measurements**	
		Average value	**Standard deviation**
Pipe diameter D [mm]	60.8	61.1	0.34
Bending angle [°]	89.9	89.8	0.34
Bending radius [mm]	238.6	238.4	0.85
Straight section length 1 [mm]	296.4	295.5	1.99
Straight section length 2 [mm]	100.9	101.7	1.58
Length of the centre line [mm]	771.5	770.8	1.68

6 Conclusions

The paper presents some aspects of optical measuring system intended for measuring large-diameter bent pipes. Measurement stages such as, mathematical models, the method of determining the bending angle of pipes and other parameters important for the pipeline industry are described. In particular, the focus was on mathematical models and analytical models of perspective projections for single- and double-bent pipes were developed. For pipes bent twice in one plane (torsional angle $\zeta = 0°$) taking into account the perspective deformations of the pipe contour in the developed algorithms positively influenced the increase in the accuracy and repeatability of the measurement results. Optical measurements with the use of a developed mathematical model for a single-bent pipe were compared with measurements results obtained on a certified CMM. The auxiliary measurements performed clearly confirm that the developed measurement system meets the assumptions as to the accuracy of the measurement and its repeatability. During the tests a measurement accuracy of 0.34° for the bending angle and 0.85 mm for the bend radius was achieved.

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As part of further research, it is necessary to investigate the effect of the pipe flattening that occurs during the bending process on the final results.

The use of the developed vision measuring system can be used as a quality control tool for controlling products in subsequent stages of pipeline production, as well as a tool for preparation of as-built reports. The measurements data collected can also be used to correct the settings of process machines and contribute to the reduction of production costs.sts.

References

1. Borzykowski, J., Domańska, A.: Współczesna metrologia. WNT, Warszawa (2004)
2. Bösemann, W.: The optical tube measurement system olm photogrammetric methods used for industrial automation and process control. International Archives of Photogrammetry and Remote Sensing **31**, 55–58 (1996)
3. Cheng, X., Sun, J., Zhou, F., Xie, Y.: Shape from apparent contours for bent pipes with constant diameter under perspective projection. Measurement **182**, 109787 (2021)
4. Giancola, S., Valenti, M., Sala, R.: A survey on 3D cameras: Metrological comparison of time-of-flight, structured-light and active stereoscopy technologies. Springer (2018)
5. Hart, P.E., Duda, R.: Use of the hough transformation to detect lines and curves in pictures. Communications of the ACM **15**(1), 11–15 (1972)
6. Huang, H., Liu, J., Liu, S., Jin, P., Wu, T., Zhang, T.: Error analysis of a stereo-vision-based tube measurement system. Measurement **157**, 107659 (2020)
7. Isa, M.A., Sims-Waterhouse, D., Piano, S., Leach, R.: Volumetric error modelling of a stereo vision system for error correction in photogrammetric three-dimensional coordinate metrology. Precision Engineering **64**, 188–199 (2020)
8. Jin, P., Liu, J., Liu, S., Wang, X.: A new multi-vision-based reconstruction algorithm for tube inspection. The International Journal of Advanced Manufacturing Technology **93**(5), 2021–2035 (2017)
9. Ma, Y., Soatto, S., Košecká, J., Sastry, S.: An invitation to 3-d vision: from images to geometric models, vol. 26. Springer Science & Business Media (2012)
10. Schwenke, H., Neuschaefer-Rube, U., Pfeifer, T., Kunzmann, H.: Optical methods for dimensional metrology in production engineering. CIrP Annals **51**(2), 685–699 (2002). https://doi.org/https://doi.org/10.1016/S0007-8506(07)61707-7, https://www.sciencedirect.com/science/article/pii/S0007850607617077
11. Sun, J., Zhang, Y., Cheng, X.: A high precision 3d reconstruction method for bend tube axis based on binocular stereo vision. Optics express **27**(3), 2292–2304 (2019)
12. Wang, X., Liu, J., Liu, S., Jin, P., Wu, T., Wang, Z.: Accurate radius measurement of multi-bend tubes based on stereo vision. Measurement **117**, 326–338 (2018)

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