An Approach for Probabilistic Modeling and Reasoning of Voting Networks

Douglas O. Cardoso[2][0000−0002−1932−334X], Willian P. C. Lima[1][0000−0001−5281−9892], Guilherme G. V. L. Silva[1], and Laura S. Assis[1][0000−0003−3081−9722]

[1] Celso Suckow da Fonseca Federal Center of Technological Education, Rio de Janeiro, RJ, Brazil
{willian.lima, guilherme.lourenco}@aluno.cefet-rj.br, laura.assis@cefet-rj.br

[2] Smart Cities Research Center, Polytechnic Institute of Tomar, Tomar, Portugal
douglas.cardoso@ipt.pt

Abstract. This work proposes a methodology for a sounder assessment of centrality, some of the most important concepts of network science, in the context of voting networks, which can be established in various situations from politics to online surveys. In this regard, the network nodes can represent members of a parliament, and each edge weight aims to be a probabilistic proxy for the alignment between the edge endpoints. In order to achieve such a goal, different methods to quantify the agreement between peers based on their voting records were carefully considered and compared from a theoretical as well as an experimental point of view. The results confirm the usefulness of the ideas herein presented, and which are flexible enough to be employed in any scenario which can be characterized by the probabilistic agreement of its components.

Keywords: Node Centrality · Weighted Networks · Data Summarization · Social Computing · Voting Networks

1 Introduction

Information Systems can combine tools from various branches of knowledge, such as Data Science, enabling the discovery of interesting insights into complex systems from data respective to these [1, 13]. In this regard, a central aspect is the proper use of theoretical models which soundly fit available data leading to meaningful predictions. The trending importance of interpretability in such a context has been deservedly indicated in the recent literature [25]. Likewise, this has also happened in the more specific scenario of techniques related to complex networks: for example, spectral graph theory [34], random graph models [32], and social network analysis [3]. This work aims to contribute to the literature similarly while focusing on probabilistic networks modeling and analysis [16, 18, 19, 30, 31].

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

The importance of such modeling is evidenced by the wide range of concrete problems which can be approached from this point of view. This could be expected given the ubiquity of uncertainty and chance in all aspects of life. Such diversity also implies some challenges, considering the goal of developing tools whose use is not limited to a single probabilistic setting but ideally are flexible enough to help make better sense of most of these occasions. With that in mind, in this work a bottom-up strategy was carried out, from the reconsideration of basic probabilistic network features up to methods and results which rely on this fresher look.

Notwithstanding the just indicated ideas, since our work is oriented towards mining concrete systems modeled as complex networks, we decided to present the proposed methodology concomitantly with a use case regarding relationships between members of a political congress [14, 15, 26, 27] based on their voting records [8, 9, 11]. Therefore, in a rough description, the target of this paper is to show how to draw interesting conclusions from such political data, assessing the alignment between peers, which enables a multifaceted centrality analysis built on a solid yet straightforward mathematical foundation. And as a completing aspect, empirical results derived from applying this methodology on real data of the lower house of the Brazilian National Congress are reported and discussed.

The remainder of this paper is organized as follows. Section 2 surveys the theoretical background that substantiates this work. Section 3 addresses the related works and also highlights the contributions of this work in view of the existing literature. The proposed methodology is described in Section 4, while Section 5 presents its experimental evaluation and the respective discussion of the results. Some concluding remarks are stated in Section 6.

2 Fundamental Concepts

Many real-world circumstances can be represented using graph theory, popularly depicted as a diagram composed of a set of points and lines connecting pairs of this set. A simple undirected *graph* $G(V, E)$ consists of a set V of n *nodes*, and a set E of m *edges*. Each edge in a graph is specified by a pair of nodes $\{u, v\} \in E$, with $u \in V$, $v \in V$. A node u is *adjacent* to node v if there is an edge between them. The degree of a node is the number of edges that are incident upon the node. As an extension, in a *weighted graph* each edge $e_i \in E, i = 1, \cdots, m$ has a value $w(e_i) \in \mathbb{R}$ associated to it. This kind of graph can be denoted as $G = (V, E, w)$, where $w : \binom{V}{2} \to \mathbb{R}$.

A graph whose set of nodes equals the union of two disjoint sets (i.e., $V = T \cup U$, and $T \cap U = \varnothing$) such that any two nodes in the same set are not adjacent is called *bipartite graph, two-mode graph*, or *bigraph* [35]. A *projection* of a bipartite graph $G(T \cup U, E)$ is a graph whose nodes are a subset of T or U, and whose edges represent the existence of a common neighbor between its endpoints in G. Edges in a projection of a bipartite graph can be weighted in numerous ways, e.g. considering the absolute or relative number of shared neighbors of their endpoints in the original graph [6], or using a context-specific method [28].

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

Given a representation of a real-world system as a network (i.e., a concrete analog of a graph, which is an abstract structure), it is common to examine the characteristics and structural properties of the network. Many properties can be associated with graph nodes, such as distance and centrality. A certain awareness of the individual system elements' importance may be obtained by measures of how "central" the corresponding vertex is in the network. The search for "communities" and analogous types of indefinite "clusters" within a system may be addressed as a graph partitioning problem [21]. Some important definitions for this sort of network characterization are presented next.

Measures of centrality are designed to quantify notions of importance and thereby facilitate answering some questions related to the network. A common notion of a *central* node is a node that can easily reach many other nodes in the graph through its edges. There is an immense number of different centrality measures that have been proposed over the years [12]. This subsection discusses two classic measures on which the proposed methodology was developed: degree and strength.

The Degree Centrality [17] of a node is possibly the simplest centrality measure and refers to the number of incident edges upon this node, i.e., its own degree. From this point of view, the higher the degree, the more central the node is. This measure better represents influence from a local perspective on the graph. This is a straightforward yet effectual measure: in various contexts nodes with high degrees are ruled as notably central by other measures [5]. The Degree Centrality of the node u can be established as shown in Equation (1).

$$C_D(u) = |\{e \in E : u \in e\}| \ . \tag{1}$$

This measure was originally proposed for unweighted graphs. Afterwards, some generalizations were developed to contemplate valued networks. Barrat et al. [4] extended the notion of degree centrality of a node and called it the node *strength*, which was defined by the sum of the weights of all edges incident in it. Equation (2) presents the calculation of the strength of a given node u.

$$s_u = C_D^w(u) = \sum_{e \in E \,:\, u \in e} w(e) \ . \tag{2}$$

3 Literature Review

As stated in Section 1, probabilistic networks have a broad and rich history of applications. One of the oldest elements of such a collection is the work of Frank [16], which regards finding shortest paths in graphs whose edge weights are random variables instead of deterministic values. Sevon et al. [31] considered a similar problem but in graphs whose edge weights regard the probability of interaction between nodes. This setting was also explored by Potamias et al. [30], who proposed an approach to answering k-nearest neighbors queries in graphs as such. More recently, Fushimi et al. [18] used Monte Carlo simulations on the same kind of graph to establish *connected centrality* for suggesting the positioning of

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

evacuation facilities against natural disasters. And regarding social networks, the study of probabilistic ego networks can be mentioned [19].

In the political science context, network analysis has been recognized as an invaluable tool for the development of research at the highest level. [33]. Several studies have been conducted based on the relational fundamentals of politics, giving rise to different political networks models and methodologies to infer and validate contextual features [9]. As an example, Lee and Bearman [22] used that platform for arguing that political and ideological isolation and homogenization of U.S. population have never been so strong, according to data obtained through a survey.

Alternatively, studies concerning networks whose nodes are members of a political congress are very popular, even considering those which are not based on voting records: co-sponsorship was already used in this regard [14, 15, 27], while other dyadic actions as co-authoring and co-attendance were discussed [26]. These networks are commonly studied because it signals peer endorsement very explicitly and publicly. Networks induced by co-sponsorship of bills in legislative bodies have been used to examine, e.g., homophily [7] and characterize ideological evolution over time [20].

Lee et al. [24] presented a study of the community structure in time-dependent legislation co-sponsorship networks in the Peruvian Congress, which is compared with the networks of the US Senate. A multilayer representation of temporal networks is employed, and a multilayer modularity maximization is used to detect communities in these networks. How much the legislators tend to form ideological relationships with members of the opposite party is measured by Andris et al. [2]. The authors quantify the level of cooperation or lack thereof between Democrat and Republican party members in the U.S. House from 1949 to 2012.

A study about the Brazilian Congress network to investigate the relationships between the donations received by congressmen elected and their voting behaviors during next two years is presented in a paper by Bursztyn et al. [9]. Two networks are built and analyzed, the donation network, and the voting network. In both networks, the vertices are the congressmen. The homophily and cohesion of congressmen are investigated. The results indicate that regions exhibit stronger homophily than political parties in the donation network, while this trend is opposite for the voting network.

Using data from Twitter and other sources (e.g., roll-call vote data), Peng et al. [29] examined how and why the members of congress connect and communicate with one another on Twitter and also what effects such connection and communication have on their vote behavior. This study shows a high degree of partisan homogeneity and the homophily effect in social network research. A network with voting data from the Brazilian Deputies Chamber is determined in the work of Brito et al. [8]. A methodology for studying the evolution of political entities over time is presented. Although a multiparty political system characterizes the Brazilian Chamber of Deputies, the results obtained reveal that the expected plurality of ideas did not occur.

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

The network of relations between parliament members according to their voting behavior regarding political coalitions and government alliances was analyzed by Dal Maso et al. [11]. Existing tools for complex networks were used to exploit such a network and assist in developing new metrics of party polarization, coalition internal cohesion, and government strength. The methodology presented was applied to the Chamber of Deputies of the Italian Parliament. The results made it possible to characterize the heterogeneity of the governing coalition and the specific contributions of the parties to the government's stability over time.

4 Methodology

In this work, the starting point of the proposed methodology is establishing a bipartite network relating members of parliament (MPs) to ballots in which they participated. This network is unweighted, but each edge is marked with the option chosen for the single vote it represents, e.g.: *(i)* yes, *(ii)* no, *(iii)* obstruction, *(iv)* abstention. It is then possible to project an all-MPs network, whose links indicate at least one joint participation in the ballots considered, regardless of the options taken. For the sake of simplicity, the bipartite network is assumed to be connected, implying the same property for its projection.

4.1 Edge Weighting

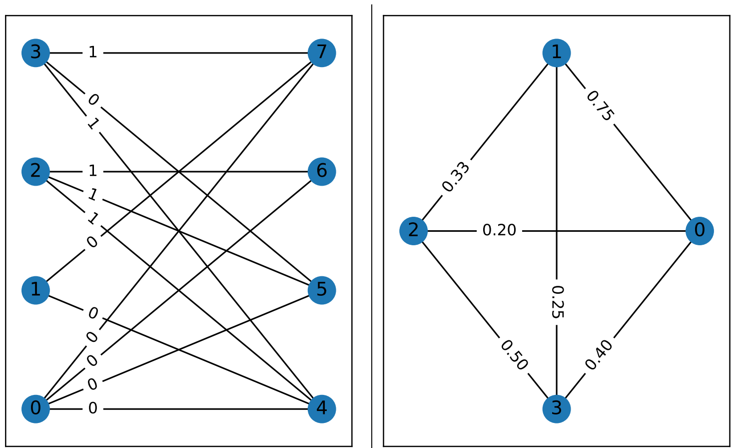
The edges of this projected network can be weighted for a deeper assessment of the relationships between MPs. An interesting approach in this regard is to model the empirical probability of agreement, i.e., voting likewise, or sharing a common interest in some political topics. With this in mind, let a matrix $\boldsymbol{V}_{m\times n}$ regard the participation of m MPs in n ballots, such that $v_{i,j}$ may represent the option chosen by the i-th MP in the j-th ballot, or it indicates that the MP did not take part in this ballot ($v_{i,j} = None$). Also let $H_i = \{j : v_{i,j} \neq None\}$ be the set of ballots in which the i-th MP indeed participated, and $O_i = \{(j, v_{i,j}) : j \in H_i\}$ be the set of pairs representing options chosen by an MP i in the respective ballots.

Then, the *agreement* between MPs x and y could be assessed according to ratio between of the number of votes they had in common and the number of ballots in which both participated, as shown in Equation (3), which is hereinafter referred to as *co-voting agreement*. Consider $\alpha > 0$ a parameter for additive smoothing of the Jaccard Index-like computation of $A(x, y)$ [10, 23]: by default, it was used in this work $\alpha = 1$.

$$A(x,y) = \frac{|O_x \cap O_y| + \alpha}{|O_x \cup O_y| + 2\alpha} \; , \tag{3}$$

A projected network of MPs whose edge weights are defined by this method is referred to as a Direct Agreement Network (DAN). The edge weights in the DAN have a positive interpretation as a proxy to voting alignment. Figure 1 provides an example of a voting network and the DAN which results from its projection as described.

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7



(a) Voting Network. (b) Direct Agreement Network.

Fig. 1: Hypothetical voting network (left) and its respective DAN (right), considering $\alpha = 1$. Nodes 0 to 3 represent voters, while nodes 4 to 7 represent polls. In (a), each edge implies the participation of the voter in one of its ends on the poll in its other end, by voting according to the edge label (in this example, 0 or 1). In (b), each edge implies that the voters it connects took part of the same poll at least once.

4.2 Probabilistic Centrality

Now consider evaluating node strength in this network: weights summation is inherent to such computation. However, this is questionable since there is no guarantee that the events the edges denote are mutually exclusive: the probabilistic understanding of such sums is unclear, as they could become even greater than 1.

The substitution of the probabilities by their logarithms may allow to counter these problems. This transformation also avoids the semantic loss previously imposed by weights summation. Adding log-probabilities is directly related to multiplying probabilities, which in turn can point to the joint probability of independent events occurrence. Although the independence of agreement events cannot be assured a priori, its assumption can be seen as a modeling simplification with no negative consequences observed so far.

The logarithm transformations can be conveniently employed to compute more insightful centrality indexes. The Strength can be computed for all DAN nodes so that it could be considered reasonable to affirm that the MP/node with the highest strength is very well-connected to its neighbors. However, comparing these values can be tricky: in the toy DAN shown in Figure 2, nodes A and C have the same strength (equal to 1.1), although the weight distribution over their edges differs.

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

Alternatively, it could be applied the logarithm transformation to the respective DAN, and then the strengths could be computed on the resulting network. Hence, in this case, cologDAN: $s_A = \log(0.9) + \log(0.2) = \log(0.9 \cdot 0.2) = \log(0.18)$, and $s_C = \log(0.30)$, so that $s_C > s_A$. This last approach has a significant advantage over the first. Each of these values is now meaningful, as the logarithm of a node's probability to simultaneously agree with all of its neighbors. Such measure was named Probabilistic Degree Centrality (PDC).

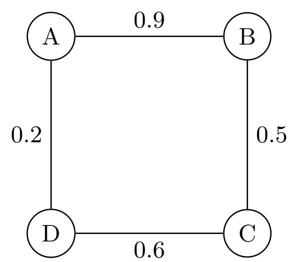


Fig. 2: A toy DAN. The edge weights regard the probability of agreement between the voters represented by the nodes.

4.3 Political Contextualization

The just introduced centrality indexes are flexible enough to be used for any network whose edge weights regard the chance of agreement, or some similar concept, between the nodes they connect. These are far from being limited to voting scenarios. However, in which context this work is focused allows specific interpretations and uses of some of these indexes as well as concepts used for their definition. The insights in this regard are detailed next.

First of all, in a broad sense, it is considered how the idea of node centrality provides a concrete and useful perspective of the probabilistic political networks in question. The most central node of a network is often regarded as the most important of all, for example, the one closest to its peers, the strongest influencer, or the one whose removal would result in the greatest loss of data traffic efficiency. Although it is tempting to consider the most central MP as the greatest leader of the house, there is no support to such hypothesis since this could only be reasonably stated if the agreement between peers was assessed in a directed fashion, which is not the case.

Despite this negative, a sensible characterization of the MPs according to their centralities is still possible. One perspective which enables reaching this goal is that of summarization, where the MP with whom there is a greater chance of an agreement with all others can serve as an archetype of the house members. Thus, it is processing a voting profile that resembles a medoid of

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

collecting all items of this kind. Moreover, the same principles can be employed in the identification of the member of a party that better represents it.

Going a little further in the just indicated direction, if known in advance the orientation of a central MP in a ballot, this information can be used as a predictor of its result. How confident one can be about such prediction can be conveniently evaluated since the outputs of the centrality measures proposed are probabilistically sound. These values can also objectively assess how cohesive the entire group of MPs or even a subset is. Thereby enabling compare different parties in a given period as well as the house in different periods, for example.

5 Experimental Evaluation

This section presents and discusses practical results obtained from the application of the proposed concepts to synthetic and naturally-produced data. This aimed at analyzing the proposed methodology under controlled circumstances, enabling a clearer perception of its features, as well as observing its behavior when subject to the idiosyncrasies which are inherent to real applications.

5.1 Synthetic Data

This first collection of experiments aimed at validating the idea which inspired all subsequent developments which were realized: taking into account the probabilistic structure of a voting network instead of ignoring it enables a more insightful analysis of its properties. In order to confirm this statement it was then hypothesized that the regular assessment of strength (Equation (2)) in a DAN would lead to a node ranking which could be notably dissimilar to that produced by its logDAN counterpart. This was tested as follows.

First, it was established a simple model to generate random voting networks. Its parameters were the number of polls to consider, the number of parties, the number of members of each party, the probability of attendance of the voters to polls, and the probability of loyalty of the voters to their parties: that is, in every poll, each of the voters can only be absent, or be loyal by voting in its own party, or be independent by voting in any party randomly. This model was used to produce numerous artificial voting networks, each of which was transformed to a DAN in order to compute the regular and probabilistic node centrality rankings, whose similarity was at last evaluated using the Kendall's τ non-parametric correlation coefficient [23].

To put the intended results into perspective, other methods for the projection of bipartite networks were also employed, despite the fact that they rely solely on the topological configuration of its input graph and but ignore attributes of its edges, as votes. Targeting to mimic the consideration of such attributes, instead of applying these methods on the original voting network, whose edges regarded every type of vote, they were applied on alternative versions of these in which only the edges of votes to party #1 were preserved. This way the coincidence between node neighborhoods, a concept which all these methods share, could be

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

better used for assessing voting alignment. The rivals methods considered were: weighted projection, Jaccard-based overlap projection, and maximum overlap ratio projection [6].

Figure 3 illustrates the results obtained considering 2 parties, each with 100 members, 100 polls, and varied scenarios with respect to the loyalty and attendance probabilities. In each scenario, a total of 100 random voting networks were generated and processed, targeting to ensure the statistical stability of the reported averages. It can be noticed that the proposed methodology (top-left subfigure) reflected more clearly than its counterparts the variation of both model parameters in question, exhibiting a smooth gradient. Moreover, it is possible to reason that as loyalty is diminished, what makes parties more irrelevant as voting becomes purely random, the distinction between regular and probabilistic degree centrality also vanishes.

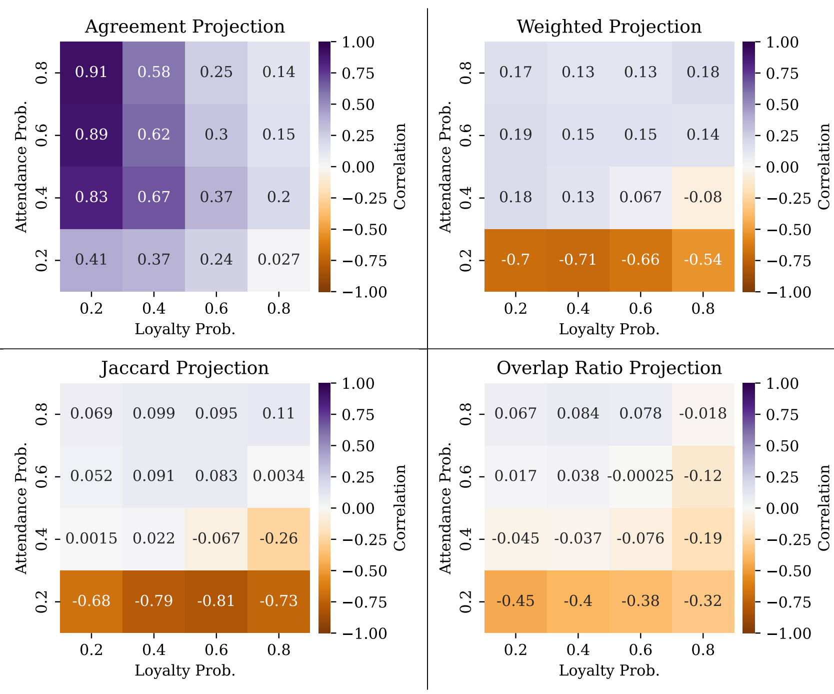


Fig. 3: Average Correlation between regular and probabilistic degree centrality rankings for synthetic data: 2 parties, 100 members per party, 100 polls. Compared to its alternatives, the proposed Agreement Projection better reflects variations in parameters of randomly generated voting networks.

As a final remark regarding artificial data, Figure 4 presents the results of the proposed methodology in a setting which is similar to the one just described but now with 3 parties. The aforementioned rival methods are unable to handle

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

this case since opting to keep only the edges regarding the votes of a single party means discarding the votes of the other two parties. Our methodology not only can handle this case, as evidenced by the same sensitiveness and smoothness in the results, but any number of different types of votes.

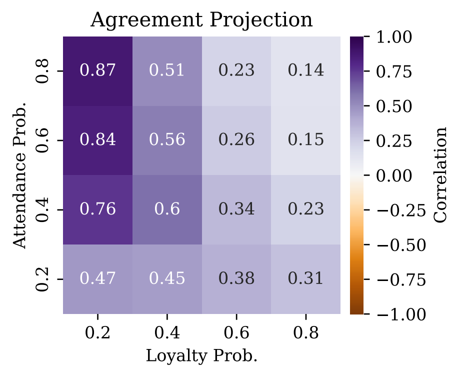


Fig. 4: Average Correlation between regular and probabilistic degree centrality rankings for synthetic data: 3 parties, 100 members per party, 100 polls. Agreement Projection can once again capture data peculiarities, even in this scenario which its counterparts are unable to properly represent by definition.

5.2 Real Data

These tests rely on publicly available records of Brazil's Chamber of Deputies, the lower house of the National Congress of the country: results from open votes that happened during 2021. First, a descriptive analysis of the dataset is provided, displaying some general information for the portrait of the context at hand. Then the network-based modeling and its developments are carefully reported.

As a start, it is presented next a broad description of the dataset focusing on its size. Figure 5 depicts the total number of votes respective to each of the 25 parties featured in the Congress ("S.PART." regards independent, unaffiliated deputies), which were declared in a total of 956 opportunities. Figure 6 also presents the total number of votes but according to its type: there is a total of 5 types, although two of those clearly dominate the distribution.

At last, Figure 7 displays the overall correlation between regular and probabilistic degree centrality node rankings of DANs produced from voting networks resulting from all votes grouped on a monthly basis. It also displays the correlation considering scenarios limited to some chosen pairs of parties: PSL and PT are the more numerous parties of the Congress, and represent the right-wing and left-wing spectrum, respectively; NOVO and PSOL are also from the right-wing

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

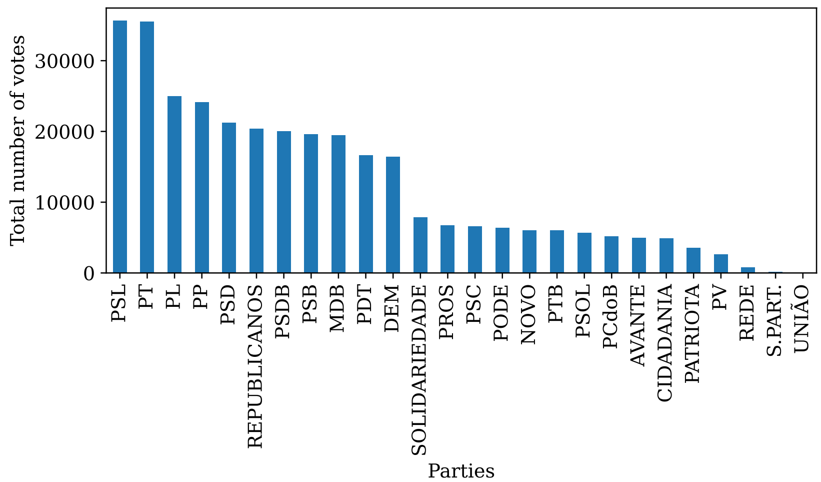


Fig. 5: Number of votes by each party.

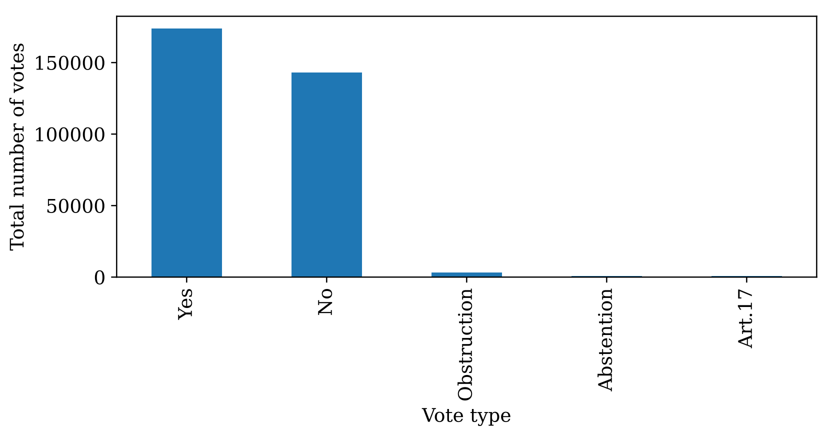


Fig. 6: Number of votes of each type. “Art. 17” is a vote type reserved to the president of the house, who can vote only in exceptional occasions.

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

and left-wing spectrum respectively but are smaller, more cohesive, and more ideologically-oriented than the first two mentioned.

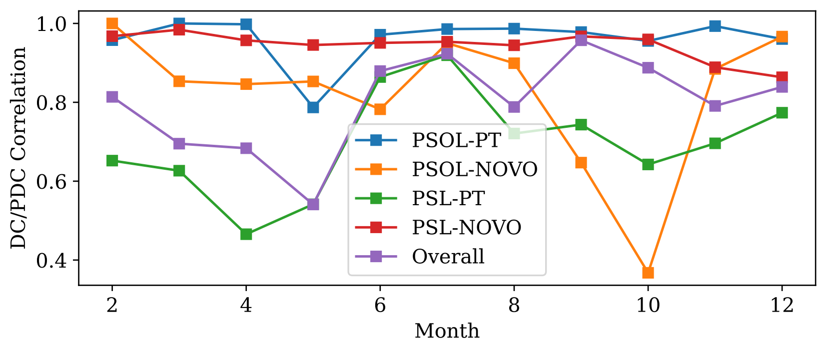


Fig. 7: Correlations between node rankings defined by their respective regular and probabilistic degree centralities, considering members of all parties or selected pairs of them, as well as monthly intervals. The behavior of the entire congress resembles that of the PSL-PT pair, which are the poles to the right and to the left of the political spectrum, respectively. The PSL-NOVO and PSOL-PT pairs are highly correlated, which is coherent with their ideological alignment.

As it can be observed, the results of the PSL-PT pair resemble those of the entire congress ("Overall"), which is consistent with the fact that these parties encompass the greatest number of MPs. Moreover, the correlation of this pair is in general smaller than that of all parties together, evidencing an antagonism in a higher level than that which is employed in general in the house. It is also interesting to confirm the ideological alignment of the PSOL-PT and PSL-NOVO pairs: the high correlation during the entire year indicates that they work with minimal discordance, in a similar fashion to the scenarios of random voting discussed in the previous subsection.

6 Conclusion

The inference and analysis of relationships in a political context are undeniably valuable, not only for politicians themselves but for the population in general, contributing for a better understanding of the social landscape in which we are all inserted. The employment of methods from network science in this regard is not new. However, generic approaches sometimes fail to take into account the specificities of the scenario in which they are used.

We believe the probabilistic centrality indexes proposed in this work directly tackle this issue, providing the means to a deeper and more reliable understanding of voting networks. Moreover, the proposed methodology is flexible enough

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

to be used on other systems with similar basic attributes, such as having the alignment of its components as random events. As a future work, a comparison of parliaments of different countries based on the proposed methodology could lead to interesting results and discussions. Another idea that could be explored is to use such the proposed ideas in the context of recommender systems, considering the agreement between customers given their behavioral records.

Acknowledgements Douglas O. Cardoso acknowledges the financial support by the Foundation for Science and Technology (Fundação para a Ciência e a Tecnologia, FCT) through grant UIDB/05567/2020, and by the European Social Fund and programs Centro 2020 and Portugal 2020 through project CENTRO-04-3559-FSE-000158. The authors acknowledge the motivational and intellectual inspiration by Artur Ziviani (*in memoriam*) which preceded this work.

References

1. Agarwal, R., Dhar, V.: Editorial—Big Data, Data Science, and Analytics: The Opportunity and Challenge for IS Research. Information Systems Research **25**(3), 443–448 (Sep 2014). https://doi.org/10.1287/isre.2014.0546, https://pubsonline.informs.org/doi/10.1287/isre.2014.0546
2. Andris, C., Lee, D., Hamilton, M.J., Martino, M., Gunning, C.E., Selden, J.A.: The rise of partisanship and super-cooperators in the u.s. house of representatives. PLoS One **10**(4), 127–144 (2016). https://doi.org/doi.org/10.1371/journal.pone.0123507
3. Atzmueller, M.: Towards socio-technical design of explicative systems: Transparent, interpretable and explainable analytics and its perspectives in social interaction contexts information. In: Nalepa, G.J., Ferrández, J.M., Palma-Méndez, J.T., Julián, V. (eds.) Proceedings of the 3rd Workshop on Affective Computing and Context Awareness in Ambient Intelligence (AfCAI 2019), Universidad Politécnica de Cartagena, Spain, November 11-12, 2019. CEUR Workshop Proceedings, vol. 2609. CEUR-WS.org (2019)
4. Barrat, A., Barthelemy, M., Pastor-Satorras, R., Vespignani, A.: The architecture of complex weighted networks. Proceedings of the national academy of sciences **101**(11), 3747–3752 (2004)
5. Borgatti, S.P., Everett, M.G.: A graph-theoretic perspective on centrality. Soc. Networks **28**(4), 466–484 (2006). https://doi.org/10.1016/j.socnet.2005.11.005
6. Borgatti, S.P., Halgin, D.S.: Analyzing Affiliation Networks. In: The SAGE Handbook of Social Network Analysis, pp. 417–433. SAGE Publications Ltd, 1 Oliver's Yard, 55 City Road, London EC1Y 1SP United Kingdom (2014). https://doi.org/10.4135/9781446294413.n28
7. Bratton, K.A., Rouse, S.M.: Networks in the legislative arena: how group dynamics affect cosponsorship. Legislative Studies Quarterly **36** (2011). https://doi.org/doi-org.ez108.periodicos.capes.gov.br/10.1111/j.1939-9162.2011.00021.x
8. Brito, A.C.M., Silva, F.N., Amancio, D.R.: A complex network approach to political analysis: Application to the brazilian chamber of deputies. PLOS ONE **15**(3), 1–21 (03 2020). https://doi.org/10.1371/journal.pone.0229928
9. Bursztyn, V.S., Nunes, M.G., Figueiredo, D.R.: How brazilian congressmen connect: homophily and cohesion in voting and donation networks. J. Complex Networks **8**(1) (2020). https://doi.org/10.1093/comnet/cnaa006

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

10. Cardoso, D.O., França, F.M.G., Gama, J.: WCDS: A Two-Phase Weightless Neural System for Data Stream Clustering. New Generation Computing **35**(4), 391–416 (Oct 2017). https://doi.org/10.1007/s00354-017-0018-y, https://doi.org/10.1007/s00354-017-0018-y
11. Dal Maso, C., Pompa, G., Puliga, M., Riotta, G., Chessa, A.: Voting behavior, coalitions and government strength through a complex network analysis. PLOS ONE **9**(12), 1–13 (12 2015). https://doi.org/10.1371/journal.pone.0116046
12. Das, K., Samanta, S., Pal, M.: Study on centrality measures in social networks: a survey. Soc. Netw. Anal. Min. **8**(1), 13 (2018). https://doi.org/10.1007/s13278-018-0493-2
13. Dhar, V.: Data science and prediction. Commun. ACM **56**(12), 64–73 (2013). https://doi.org/10.1145/2500499
14. Fowler, J.H.: Connecting the congress: A study of cosponsorship networks. Political Analysis **14**(4), 456–487 (2006). https://doi.org/10.1093/pan/mpl002
15. Fowler, J.H.: Legislative cosponsorship networks in the US house and senate. Soc. Networks **28**(4), 454–465 (2006). https://doi.org/10.1016/j.socnet.2005.11.003
16. Frank, H.: Shortest Paths in Probabilistic Graphs. Operations Research **17**(4), 583–599 (Aug 1969). https://doi.org/10.1287/opre.17.4.583, publisher: INFORMS
17. Freeman, L.C.: Centrality in social networks conceptual clarification. Social networks **1**(3), 215–239 (1978)
18. Fushimi, T., Saito, K., Ikeda, T., Kazama, K.: A new group centrality measure for maximizing the connectedness of network under uncertain connectivity. In: Aiello, L.M., Cherifi, C., Cherifi, H., Lambiotte, R., Lió, P., Rocha, L.M. (eds.) Complex Networks and Their Applications VII - Volume 1 Proceedings The 7th International Conference on Complex Networks and Their Applications COMPLEX NETWORKS 2018, Cambridge, UK, December 11-13, 2018. Studies in Computational Intelligence, vol. 812, pp. 3–14. Springer (2018). https://doi.org/10.1007/978-3-030-05411-3_1
19. Kaveh, A., Magnani, M., Rohner, C.: Defining and measuring probabilistic ego networks. Soc. Netw. Anal. Min. **11**(1), 2 (2021). https://doi.org/10.1007/s13278-020-00708-w
20. Kirkland, J.H., Gross, J.H.: Measurement and theory in legislative networks: the evolving topology of congressional collaboration. Social Networks **36** (2014). https://doi.org/dx.doi.org/10.1016/j.socnet.2012.11.001
21. Kolaczyk, E.D.: Statistical Analysis of Network Data: Methods and Models. 1 edn. (2009). https://doi.org/10.1007/978-0-387-88146-1
22. Lee, B., Bearman, P.: Political isolation in america. Network Science p. 1–23 (02 2020). https://doi.org/10.1017/nws.2020.9
23. Lee, L.: Measures of distributional similarity. In: Proceedings of the 37th annual meeting of the Association for Computational Linguistics on Computational Linguistics. pp. 25–32. ACL '99, Association for Computational Linguistics, USA (Jun 1999). https://doi.org/10.3115/1034678.1034693, https://doi.org/10.3115/1034678.1034693
24. Lee, S.H., Magallanes, J.M., Porter, M.A.: Time-dependent community structure in legislation cosponsorship networks in the congress of the republic of peru. Journal of Complex Networks **5**(1), 127–144 (2016). https://doi.org/doi-org.ez108.periodicos.capes.gov.br/10.1093/comnet/cnw004
25. Murdoch, W.J., Singh, C., Kumbier, K., Abbasi-Asl, R., Yu, B.: Definitions, methods, and applications in interpretable machine learning. Proceedings of the National Academy of Sciences **116**(44), 22071–22080 (2019). https://doi.org/10.1073/pnas.1900654116

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7

26. Neal, Z.: The backbone of bipartite projections: Inferring relationships from co-authorship, co-sponsorship, co-attendance and other co-behaviors. Soc. Networks **39**, 84–97 (2014). https://doi.org/10.1016/j.socnet.2014.06.001
27. Neal, Z.P.: A sign of the times? weak and strong polarization in the U.S. congress, 1973-2016. Soc. Networks **60**, 103–112 (2020). https://doi.org/10.1016/j.socnet.2018.07.007
28. Newman, M.E.: Scientific collaboration networks. ii. shortest paths, weighted networks, and centrality. Physical review E **64**(1), 016132 (2001)
29. Peng, T.Q., Liu, M., Wu, Y., Liu, S.: Follower-followee network, communication networks, and vote agreement of the u.s. members of congress. Communication Research **43**(7), 996–1024 (2014). https://doi.org/doi-org.ez108.periodicos.capes.gov.br/10.1177/0093650214559601
30. Potamias, M., Bonchi, F., Gionis, A., Kollios, G.: k-nearest neighbors in uncertain graphs. Proc. VLDB Endow. **3**(1), 997–1008 (2010). https://doi.org/10.14778/1920841.1920967
31. Sevon, P., Eronen, L., Hintsanen, P., Kulovesi, K., Toivonen, H.: Link Discovery in Graphs Derived from Biological Databases. In: Data Integration in the Life Sciences. pp. 35–49. Springer, Berlin, Heidelberg (Jul 2006). https://doi.org/10.1007/11799511_5
32. Tsur, O., Lazer, D.: On the interpretability of thresholded social networks. In: Proceedings of the Eleventh International Conference on Web and Social Media, ICWSM 2017, Montréal, Québec, Canada, May 15-18, 2017. pp. 680–683. AAAI Press (2017)
33. Ward, M.D., Stovel, K., Sacks, A.: Network Analysis and Political Science. Annual Review of Political Science **14**(1), 245–264 (2011). https://doi.org/10.1146/annurev.polisci.12.040907.115949
34. Zenil, H., Kiani, N.A., Tegnér, J.: Numerical investigation of graph spectra and information interpretability of eigenvalues. In: Guzman, F.M.O., Rojas, I. (eds.) Bioinformatics and Biomedical Engineering - Third International Conference, IWBBIO 2015, Granada, Spain, April 15-17, 2015. Proceedings, Part II. Lecture Notes in Computer Science, vol. 9044, pp. 395–405. Springer (2015). https://doi.org/10.1007/978-3-319-16480-9_39
35. Zhang, X., Wang, H., Yu, J., Chen, C., Wang, X., Zhang, W.: Polarity-based graph neural network for sign prediction in signed bipartite graphs. World Wide Web **25**(2), 471–487 (Mar 2022). https://doi.org/10.1007/s11280-022-01015-4, https://link.springer.com/10.1007/s11280-022-01015-4

ICCS Camera Ready Version 2023
To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-36024-4_7