

Modeling mechanisms of school segregation and policy interventions: a complexity perspective

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Abstract. We revisit literature about school choice and school segregation from the perspective of complexity theory. This paper argues that commonly found features of complex systems are all present in the mechanisms of school segregation. These features emerge from the interdependence between households, their interactions with school attributes and the institutional contexts in which they reside. We propose that a social complexity perspective can add to providing new generative explanations of resilient patterns of school segregation and may help identifying policies towards robust school integration. This requires a combination of theoretically informed computational modeling with empirical data about specific social and institutional contexts. We argue that this combination is missing in currently employed methodologies in the field. Pathways and challenges for developing it are discussed and examples are presented demonstrating how new insights and possible policies countering it can be obtained for the cases of primary school segregation in the city of Amsterdam.

Keywords: complex systems · policy · school segregation · agent-based modeling

1 Introduction

In many educational systems, levels of school segregation are still substantial to high. Meaning that children with different characteristics such as race [43], ethnicity [10], income levels [24], educational attainment [7] and ability [50] cluster together in different schools, which is widely associated with existing inequalities and their reproduction [51]. Even despite the wealth of knowledge about the problem and the many policy interventions that have been proposed to counteract it, segregation continues to plague educational systems globally and hence proves a hard problem to solve [8].

Research in the field of (social) complexity has highlighted how interactions between components of a system, can cause an unanticipated and possibly unintended social outcome such as segregation at the macro-level due to choices individuals make [11]. Anticipating such dynamics and developing policies that

could prevent or mitigate them is a notoriously hard problem to solve. Moreover, if the focus of research lies primarily on isolated individual choices or on macro-level characteristics, these explanations miss potentially important effects of interactions [34]. Similarly for empirical studies in the field of school segregation, the focus has mainly been on individual-level decision making or macro-level patterns. For example through parent interviews [2], discrete choice analysis [14], changes in levels and trends of segregation [43], or associations of segregation levels with characteristics of neighborhoods or municipalities [33]. Additionally, the factors in school segregation are often treated as individual variables that do not affect each other [42]. Ignoring such interactions can lead to a lack of understanding of the mechanisms underlying school segregation and could result in ineffective policies. This could be an explanation of why school segregation is still a robust and resilient phenomenon in society even with counteracting policies. This is substantiated by recent theoretical work, utilizing complexity-inspired methodologies, that substantial school segregation can even result from relatively tolerant individuals due to their interactions in the system [48, 45, 20]. However, these stylized models have limited applicability to reality and methodologies inspired by complexity have been, in our view, underused in the field of school segregation.

Therefore, this paper argues that the dynamics of school choice constitute a complex system not only in principle, but also in light of theoretical and empirical research in the field. The dynamics of school choice at the micro-level (e.g., household) on the one hand, and macro-level contextual factors (e.g., school compositions, quality, institutional aspects) on the other, are interdependent and interacting elements of a broader complex system. These interactions result in the emergent patterns of school segregation at the macro-level. This system, we contend, is characterized by the adaptive behavior of individual actors who respond to changing contextual conditions and simultaneously influence these conditions by the choices they make. Therefore, the behavior of the system as a whole (e.g., school segregation) is difficult to infer from the analysis of individual components in isolation [34]. Common methodologies in this field often take a reductionist approach, treating the macro-level as the sum of its isolated individual parts. We elaborate and exemplify how Agent-Based Models (ABMs) can overcome these limitations, allowing for the explicit modeling of interactions between the components conditional on their specific educational context [17, 40]. We identify pathways and challenges for how to incorporate ABM in future empirical research and show examples of how it can lead to new insights and possible policies countering primary school segregation in the city of Amsterdam.

2 Features of complex systems

Complex systems consist of numerous, typically heterogeneous, elements operating within an environment, whose behavior is defined by a set of local rules. Interactions can occur directly between components or via their environment

and are not restricted to be physical, but can also be thought of as exchanging energy or information. The state of an element can be influenced by the current and/or previous states of possibly all other elements in the system. Although there is no universal definition of complexity, there is more consensus on features arising in complex systems, which are described in the next sections.

2.1 Emergence

A complex system shows emergent behavior in the sense that the system exhibits properties and dynamics that are not observed in the individual elements of the system. Typically, the quantity of interest is some system level emergent property, which might be lost by isolating the individual elements. Emergence makes complex systems hard to reduce and study from the isolated components, the interactions of the elements are crucial to the system dynamics [1]. The interactions is what establish the link between the micro- and macro-level and it is the understanding of this relationship that is often of importance. Examples of emergent phenomena are the forming of coalitions in societies [17] and swarming behavior in animals [37].

2.2 Self-organization, adaptation and robustness

Where emergence is focused on new, sometimes unexpected, measures and structures that arise from the interaction of individual components, self-organization stresses adaptive and dynamic behavior that leads to more structure or order without the need for external control. The resulting organization is decentralized and is formed by the collection of individual components. These elements are able to adapt their behavior by learning and react to the environment and other elements [16]. This typically creates robust structures that are able to adapt and repair and can make it difficult to control complex systems using traditional global steering mechanisms. Self-organization and emergence can exist in isolation, yet a combination is often present in complex systems [19]. The idea of robustness is the ability of a system to resist perturbations or its ability to function with a variety of different inputs/stimuli. The closely related concept of resilience, a specific form of adaptation, is the ability of the system to continue to function by adapting or changing its behavior in the face of perturbations.

2.3 Feedback, non-linearity and tipping points

The interactions between the elements and the environment and adaptation imply that the components can respond to and thereby change the properties of the system. This generally leads to feedback loops, which can create complex causality mechanisms and nonlinear responses [30]. These feedback loops can have (de)stabilizing effects [36] and can take place at different time scales. Non-linear systems have the property that the change in the output of the system does not scale proportionally with the change of input of the system. This

makes it hard to predict the behavior of the system, the same small perturbation that caused small responses may at some point lead to drastic changes at the macro-level. Such non-linearity can be triggered by feedback and other mechanisms [39], which might even lead to tipping points where systems can shift—abruptly—from one state to another, such as virus outbreaks, stock market crashes, and collapsing states [46]. In the case of segregation, the relocation of one individual or a small group can increase the proportion of the majority in the new neighborhood/school. This in turn could lead to individuals of the minority group to leave, which decreases their number even more, resulting in more moves and so on. This can trigger a whole cascade where a small effect can tip a neighborhood/school to become homogeneous [15, 45].

2.4 Path-dependency

Path-dependency explains how decisions at a given point in time are constrained by the decisions made in the past or by events that occurred leading up to that particular moment, although past events may no longer be relevant. This implies that once a decision has been made, by simply making that decision, you make it hard/impossible to change or re-make that decision in the future. A form of path-dependency may appear in self-fulfilling prophecies [13] or in politics, where a certain sequence or timing of political decisions, once introduced, can be almost impossible to reverse.

3 Complexity in the mechanisms of school segregation

This section extracts general mechanisms underlying school segregation from the existing literature and connects them to the previously discussed features of complex systems. Although the factors in school segregation are highly context-specific, different educational systems exhibit similarities through which the dynamics of school segregation operate similarly [8]. These factors are discussed in separate sections, however, it should be stressed that they are intrinsically linked and interact with each other. The described dynamics are found in most, if not all, educational systems, but relevant importance of mechanisms can differ substantially depending on the context, which is crucial for eventual understanding.

3.1 Distance

Numerous studies find that, irrespective of household characteristics or educational system, most children attend a school close to their home [8]. Hence, residential segregation (partially) portrays itself in the population of schools. However, socio-economically advantaged subgroups of parents are found to be more willing/able to travel further or even move neighborhoods for more favorable school characteristics [14], increasing school segregation and complicating the dynamics. The latter might be more prominent in educational systems with geographic assignment mechanisms, forcing households to either accept

their neighborhood schools or *adapt* and opt-out of the system. This establishes a mutual relationship (*feedback*) between school characteristics and residential segregation.

3.2 School profile

In various educational systems, schools can differ on their religious foundation, pedagogical principles, curricula, or status. These profiles are important in choice of school, but they might attract only certain groups of parents and thereby increasing segregation between schools [10]. For example, in the Netherlands, highly-educated parents are found to be more attracted to schools with a certain pedagogy than their counterparts [5]. Moreover, existing residential segregation might make schools more inclined to adopt a certain profile [25], connecting the location of a school and the profile, resulting in possible interactions between these factors.

3.3 School quality

Although an ambiguous concept, quality could refer to a school's added value to academic achievement, but also to its climate, order, and discipline [9]. Nevertheless, parents have a strong preference for proxies of quality, such as the academic performance of schools, and even more so if the family is advantaged (e.g., education, income) [26]. However, academic performance might say more about a school its ability to attract a better performing student population via profile, gatekeeping or its location, rather than measuring its "added value" or actual quality [10]. Perceived high-quality schools could increase house prices in the neighborhood, making it more likely that only high-income households can afford living close [41], affecting residential segregation and school segregation in turn (via distance preferences).

3.4 School assignment

Many cities employ school allocation mechanisms. If geographically restricted, these could strengthen the link between residential and school segregation [8], but can also lead to specific types of households opting out and attend non-public schools for example. Additionally, advantaged households are suggested to be better aware of registration deadlines, accompanying assignment mechanisms and their consequences [23]. This can lead to disadvantaged parents registering late more often, resulting in less choice or lower rankings. Another form of assignment is early ability tracking, where most European countries, the US and the UK group students together based on ability [50] or use selection criteria. Although implementations differ, [47] find that tracking and selecting increase inequality with respect to socio-economic status. Moreover, grouping children based on ability is itself already segregation, but also strongly overlaps with ethnicity and social class.

3.5 Gatekeeping

Gatekeeping or cream skimming can be summarized as the (un)intentional and (in)formal use of selective criteria by schools. High tuition fees, voucher systems, waiting lists, catchment areas, advising children about different schools or organizing education in such a way that particular children do not feel at home, are all reported gatekeeping practices [28]. Hence, schools can also *adapt* their behavior by reacting to the system, creating additional interactions between households and schools. Many systems offer alternatives to public education. Private schools, exclusively funded by school fees, hybrid schools, where the state also partly funds the privately organized schools and charter/magnet schools also play a substantial role and hence should be mentioned here [44]. However, their effect on school segregation can be attributed to already described factors. Schools in this sector, depending on funding, often have substantially more autonomy. This could lead to highly specialized school profiles,—perceived—high quality (attracting a high-ability population), gatekeeping practices (e.g., high fees) and allocation mechanisms [8].

3.6 Social network

The decision of which school to attend could also be influenced by interactions through social networks, preschool groups or school visits for example [6]. Social networks can provide (mis)information on school choice and act as a platform for social comparison, where people trust the opinions of high-status individuals in their choice of school [2]. As one tends to be friends with similar people, these networks tend to be socially and geographically structured [35]. Hence, opinions or factual information about schools are not equally accessible to everyone in the network and could enforce certain group-specific preferences [31], leading to *feedback loops*, *non-linearity* and *robustness*. For example, high-status schools could be more accessible to those with better connections and, in turn, can offer status and connections to those who attend them [27].

3.7 School composition

Another important determinant, irrespective of context, is that people are more attracted to schools attended by larger proportions of their own group and avoid schools with large shares of other groups [8]. This behavior has obvious effects on school segregation and could induce self-reinforcing processes. Empirically, this is demonstrated in phenomena such as White flight, where households of a particular group affect each other and opt out of schools and/or neighborhoods to avoid undesirable school composition/quality [18]. If schools are already segregated, a preference for attending a school with similar pupils will reinforce future patterns of school segregation. These *feedback mechanisms* make segregation both *robust* and *path-dependent* (i.e., historical school segregation affects future school choices). Furthermore, school closure/founding might lead to *non-linear* effects, where the new choices of the movers induce children at other schools to

move. This can result in some systems going from an integrated to a segregated state (*tipping*). Homophily, the tendency to associate with similar others [35] can be an explanation for this mechanism, but the composition of a school could also emerge from the profile it propagates, as particular types of people might be attracted to certain profiles, or due to the projection of existing residential segregation in schools. Another way in which school composition affects school choice is because parents, apart from official, “cold knowledge”, also rely on social network-based information, “hot knowledge”, [2] to assess school quality. It has been demonstrated that (perceptions of) school composition, circulated by parents is used as a proxy for quality of a school [3].

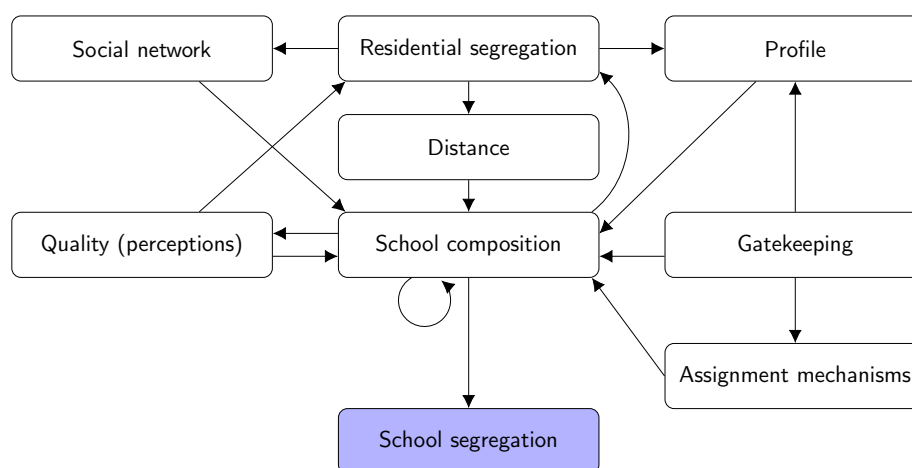


Fig. 1: Mechanisms underlying school segregation and their interactions.

4 Modeling school segregation dynamics

We have argued that school choice and school composition form a complex system that is hard to understand, let alone predict or be influenced by policy without a good understanding of the interactions between its various levels and components. From a complexity perspective, a suitable model of school choice dynamics thus needs to contain descriptions of the micro- and macro-level, as well as of the interactions between levels and their components. Interviews, document analysis, and surveys are all methods used to study school choice and are useful for obtaining an overview of the factors decision makers and experts consider important in a concrete case. However, it is often hard to quantify the relative importance of factors and how these vary across different types of people in the system under study. Complementary, using observed choices, discrete choice models are able to quantify the effect that individual, school and institutional characteristics have on individual choice behavior [34]. Nevertheless,

common assumptions are that decision makers operate independently of each other, have full knowledge of the system and are rationally maximizing their utility. These methods, separately or combined, provide valuable and necessary information, as one needs to have input for how to model the individual elements in the system. However, a remaining challenge is to model the interactions within and between the micro- and macro-level simultaneously [40].

4.1 Agent-based models

Agent-Based Models (ABM) have long been used to model complex systems, but recently also to model the dynamics of school choice. ABMs are algorithmic, aiming to describe dynamics of systems in terms of the processes or algorithms through which each of the individual elements of the systems changes its state (e.g., school choice), responding to perceived inputs from its local environment. At the simplest level, an ABM consists of a system of agents, an environment and the relationships between them [4]. These agents are often autonomous, yet interdependent, and behave according to certain rules, where these rules can vary from simplistic to very sophisticated. Individual level decision rules can be simple heuristics, be specified by discrete choice models, neural networks or evolutionary algorithms as behavioral processes. This allows agents, interactions between them, or with the environment to be modeled explicitly, including assumptions describing their learning and adaptation to changing situational conditions. This approach becomes more important if analyzing average behavior is not enough, because a system is composed of heterogeneous substructures such as in clustered social networks or heterogeneous spatial patterns shaping the spatial distribution and accessibility of schools and the composition of their residential environments. Further, *adaptation* can occur when schools open or close, or households move due to changing demographics or residential patterns [13]. Moreover, *feedback effects* and *adaptation* potentially *emerge* from the rules and *interactions* governing agents' (inter)actions. ABM also allow to simulate multiple generations choosing schools, which could be used in analyzing *robustness*, *resilience*, *path-dependency* and *tipping points* with respect to school segregation. Also, agents can all be different to incorporate heterogeneity (i.e., varying household preferences or resources) and the environment allows for the explicit modeling of space, such as cities with infrastructure. These models have been applied in a stylized manner for school choice already, to link theoretical behavior rules to aggregate patterns [48, 45, 20] and more data-driven models to test system interventions or component behavior [38, 21, 49]. However, their usage in the field have been limited so far. To emphasize what ABM can add to existing methodologies, two examples are provided on how both stylized and more empirically calibrated ABM can improve our understanding of school segregation.

4.2 An alternative explanation for excess school- relative to residential segregation

In various educational systems, the level of school segregation is consistently higher than that of residential segregation [51, 8]. One hypothesis is that parents might want to live in a diverse neighborhood, but when it comes to their children, they are less tolerant with respect to school compositions leading to less diverse schools [18]. However, residential and school segregation are not distinct emergent processes, they are intertwined as school choices are partially dependent on residential segregation (i.e., via distance preferences). Hence, this phenomenon can maybe also emerge, not because people really want it (intolerance), but due to *feedback loops* and *nonlinear* effects of interacting processes.

To study this, [20] create a stylized ABM where households are assumed to belong to one of two groups and choose neighborhoods based on composition and schools based on a trade-off between composition and distance. For composition preferences, households are assumed to have an optimal fraction of their own group ($t_i \in [0, 1]$) in a school/neighborhood, giving a utility of 1 (linearly increasing from 0) and receive only ($M \in [0, 1]$) if it is completely homogeneous (Figure 2b), as research suggests many parents actually want some diversity in schools [9]. Schools are chosen by weighting ($\alpha \in [0, 1]$) distance and school composition, where 0 means only distance matters and 1 means only composition. Households strive to maximize their residential utility first and after that their school utility (given residential location). Importantly, households have the same composition preferences for neighborhoods as for schools, to test the alternative scenario. For a more detailed description of the model, the reader is referred to [20].

Unexpectedly, the level of residential- and school segregation increase when the optimal fraction increases or the penalty for homogeneity decreases (Figure 3). However, given the exact same composition preference for neighborhoods as for schools, it is surprising that school segregation is consistently higher than residential segregation. This provides an alternative explanation of why schools are often more segregated than neighborhoods, not because of extra intolerance but due to compounding *feedback* and *nonlinear* effects. First, neighborhoods segregate more than expected based on individual preferences and, given distance preferences, households would prefer to attend the nearest school. However, some schools have a composition that likely deviates from the optimal one, due to residential segregation. Thus, at least one group starts to move out and travel further for a more favorable composition, triggering a whole cascade and increasing the level of school segregation above the level that would be expected without residential patterns.

4.3 An ABM of Amsterdam primary school choice

However, this stylized example has limited applicability to reality. In an more empirically calibrated model, [21] approximate residential locations of low- (blue) or high-income (red) households (Figure 4). In Amsterdam, you get priority if

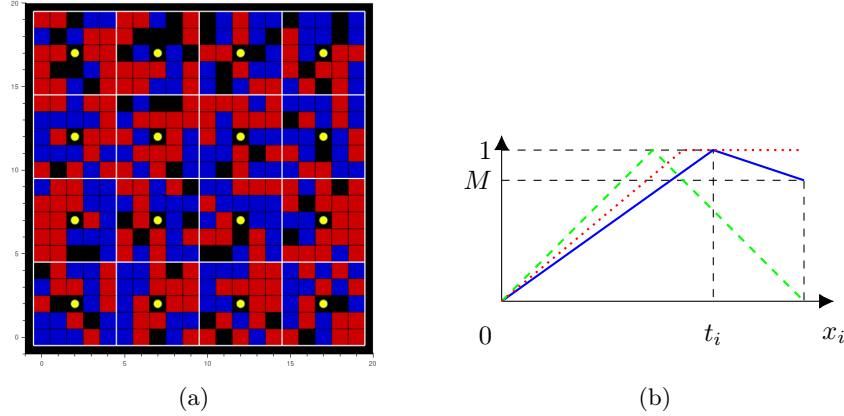


Fig. 2: (a) The environment of the ABM of [20], only 16 neighborhoods (white) and schools (yellow) are portrayed for visualization purposes. Households belong to one of two groups, either blue or red. (b) Single-peaked utility function. For the blue line, the agent obtains the maximum utility at t_i and only M if the neighborhood or school is homogeneous with respect to their own group. The green (dashed) and red line (dotted) show alternative values for t_i and M .

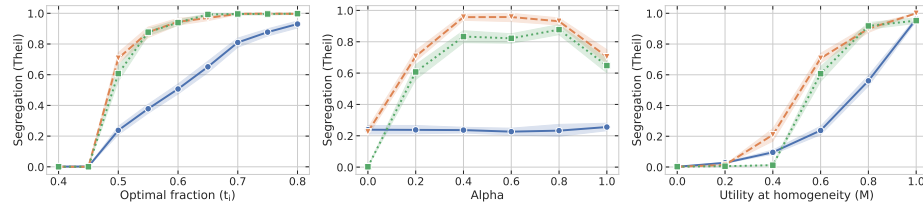


Fig. 3: The level of residential segregation (blue), school segregation with residential patterns (orange, dashed) and without (green, dotted). The plotted level of segregation is the average over 30 model runs and the bands represent the 2.5% and 97.5% percentiles. When the optimal fraction (t_i) is varied, $\alpha = 0.2$, $M = 0.6$, otherwise $t_i = 0.5$, fixed values of other parameters that are not varied can be found in Table 1 of [20].

you apply at one of you eight closest primary schools, which around 86% of pupils ultimately does [12]. As this percentage is quite high, this experiment models this priority-scheme as a strict geographic assignment mechanism: households are only allowed to consider their $n \in \{1, 2, 4, 8\}$ closest schools. Within this subset, households are assumed to be indifferent from a distance perspective and only composition matters (following the same function as in 2b). While this might be a strong assumption, 50% of both groups still have their eight closest schools within 1.9 kilometers [21]. Hence, within their feasible choice set, households strive to find schools with their “optimal” composition.

Figure 5 indicates that school segregation is shown to be affected by both residential segregation and limiting the amount of schools households are allowed to consider. If everyone is assigned their closest school, residential segregation is fully determining school choices. Interestingly, as soon as households are given more choice (2, 4 or 8 schools) and they require at least a slight majority of their own group ($t \geq 0.5$), school segregation increases drastically. Segregation almost exclusively rises when the number of schools is increased, even for moderate preferences. Hence, in this model, more school choice leads to more segregation, which is also a consistent finding in empirical research [51, 8]. Moreover, at some point more choice does not mean anything anymore (schools are fully segregated). Note that in the part of the parameter space where households strive to be a minority ($t < 0.5$, $M < 0.4$), although very small, more schools actually allow for less segregation to emerge compared to when households only consider the closest schools, resonating earlier results from stylized models [45]. This experiment suggests the final segregation that emerges is a delicate balance between preferred and locally available compositions (i.e., within “reasonable” distance). Hence, understanding how households consider distance and composition is vitally important to understand the potential for segregation in the city and also offering the number of priority schools can have significant effects on the resulting segregation levels. If one prefers more own group and residential segregation is strong: more schools might not help to desegregate, but if one prefers a smaller fraction of similar households and residential segregation is strong, then more choice could lead to less segregation. Note that context is still important, in an additional experiment with five income groups instead of two, no group will (realistically) have more than 50% in a school, hence the transitions in similar plots already happen at lower values values of the optimal fraction. This also stresses the importance of finding out what groups households feel they belong to and what they consider to be others. Also, constraining households to their closest school might have the adverse affect of inducing particular households (high-income) to move neighborhoods, increasing residential- and school segregation, which is not modeled here.

5 Challenges

While ABM are theoretically able to model the described underlying mechanisms of school segregation and can simulate (hypothetical) what-if scenarios,

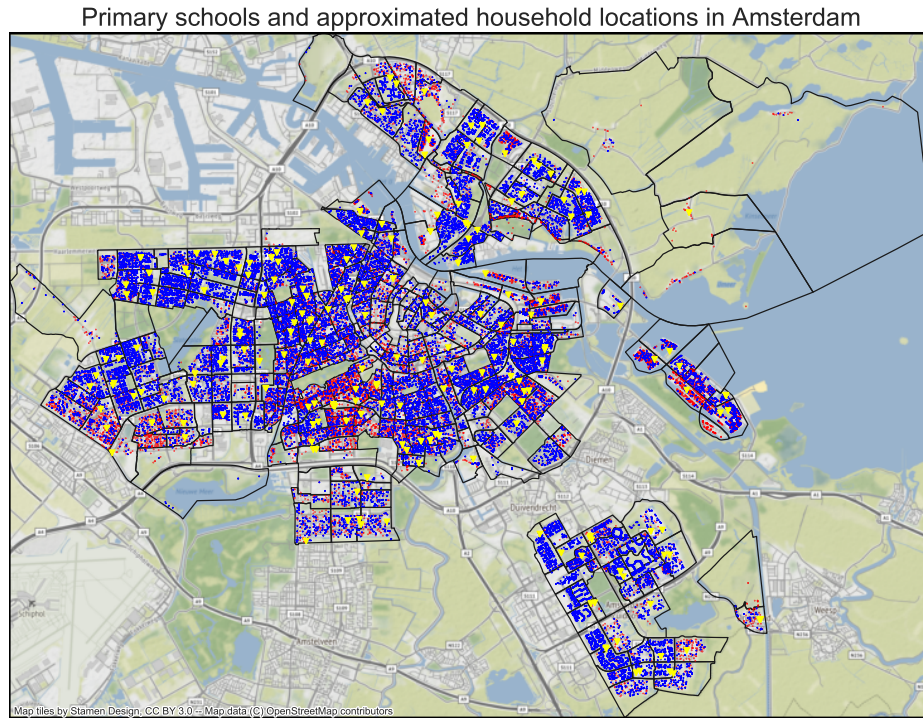


Fig. 4: Q1 households are in blue, Q5 in red, yellow triangles are the schools.

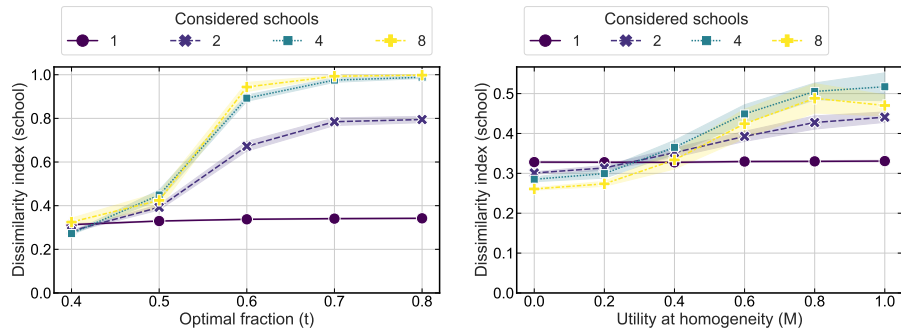


Fig. 5: The level of school segregation when varying the optimal fraction (t) and utility at homogeneity (M). $t = 0.5$, $M = 0.6$, when they are not varied. Estimates are the means of 10 model runs, error bands are one standard deviation.

they also present several challenges. One major challenge is the difficulty in determining the appropriate level of detail and granularity for agent behavior and interactions. Specifying these rules at a fine-grained level can result in models that are computationally intractable and difficult to interpret, while overly simplified rules may not accurately reflect the complexity of real-world social phenomena [13]. Moreover, obtaining data to calibrate and validate the model can pose issues. Data on the micro-level (household) can be hard to gather or raise privacy and ethical concerns for example. Additionally, for actual decisions and/or policy interventions it is necessary that model assumptions, mechanisms and outcomes correspond with empirical observations, allowing validation and possible refutation of hypotheses generated by an ABM [40]. Specifying interactions between agents may require making assumptions about how information is transmitted and interpreted, which can be difficult to validate empirically. Moreover, traditional estimation/calibration methods often require a specification of the likelihood function, which are almost impossible to write down for ABM. Existing studies have estimated parameters using different methodologies, before incorporating them in their ABM. However, likelihood-free inference provides techniques to calibrate ABM directly, but these can be computationally very expensive [22]. Finally, due to the nonlinear, stochastic, and dynamic nature of ABM, model output may be highly sensitive to initial conditions, and small changes in parameters can produce vastly different outcomes. Hence uncertainty quantification is an important aspect as well. However, these caveats should not be mistaken as argument that ABM is impractical to use in empirical research. It only shows that using ABM fruitfully requires adherence to methodologies and best practices for model analysis, calibration and validation which are increasingly developed in the field of computational social science [32].

6 Conclusion

This paper has argued that school choice exhibits all features commonly found in complex systems and therefore qualifies as one [29]. Households base their choice on information/perception of school properties (e.g., distance, composition, profile, quality). As their actual decision can change these properties it possibly influences choices of current/future generations (e.g., *feedback*, *path dependence*). Not only do they interact with schools, but also with their geographical/institutional environment (e.g., residential patterns, assignment mechanisms) and each other (e.g., school compositions, social network). Schools are players as well, depending on their level of autonomy they can impact affordability, school profiles, and their own admission policies. This indicates *feedback mechanisms* that can possibly lead to *non-linear* effects and *tipping points* that are hard to anticipate from the analysis of isolated individual components. School segregation might even be a sub-optimal outcome, as there is evidence of parents preferring more diverse schools than they actually attend [9].

Hence, to model the mechanisms behind school segregation one needs to model the individual households in the system, but also how they interact, in-

fluence macro-level components and feedback into individual choices. We argue that ABM can adhere to this need [40]. Depending on the specific question, these models can take a more stylized approach and allow to test how (theoretical) behavioral rules could be related to particular levels of school segregation or to provide alternative explanations for observed phenomena. The models can also be more data-driven, for example how primary school segregation in Amsterdam can emerge from relatively tolerant households with distance constraints and residential segregation. This can be extended by augmenting them with large-scale empirical data about socio-geographic contexts and properties of individual households and schools. This is important to validate model outcomes about household behavior (i.e., using micro-level data) and for testing whether the assumptions about which mechanisms lead to observed levels of school segregation explain real-life behavior and macro-level patterns [13, 32]. However, determining the appropriate level of detail for these models involve a trade-off with computational tractability. Additionally ABM can be very sensitivity to initial conditions where small changes can lead to very different outcomes.

Keeping these challenges in mind, future modeling attempts in the domain of school choice and school segregation, could make more use of a social complexity approach such as ABM [21, 49]. We believe that this offers a way forward to provide generative explanations of the mechanisms that lead to resilient patterns of school segregation and a pathway towards understanding policies that can help achieve more robust levels of school integration [13, 40, 32].

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