A new statistical approach to image reconstruction with rebinning for the x-ray CT scanners with flying focal spot tube

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Abstract. This paper presents an original approach to the image reconstruction problem for spiral CT scanners where the FFS (Flying Focal Spot) technology in an x-ray tube is implemented. The geometry of those scanners causes problems for CT systems based on traditional (FDK) reconstruction methods. Therefore, we propose rebinning strategy, i.e. a scheme with abstract parallel geometry of x-rays, where these problems do not occur. As a consequence, we can reconstruct an image from projections with a non-equiangular distribution, present in the Flying Focal Spot technology. Our method is based on statistical model-based iterative reconstruction (MBIR), where the reconstruction problem is formulated as a shift-invariant system. The statistical fundamentals of the proposed method allow for a reduction of the x-ray dose absorbed by patients during examinations. Performed simulations showed that our method overcomes the traditional approach regarding the quality of the obtained images and an x-ray dose needed to complete an examination procedure.

Keywords: X-ray computed tomography · flying focal spot · statistical method · image reconstruction from projections· rebinning.

1 Introduction

At the beginning of the previous century, the spiral scanner with an X-ray tube with a flying focal spot (FFS) has been introduced [3]. This new technique aims at increasing the density of simultaneously acquired slices in the longitudinal direction and the sampling density of the integral lines in the reconstruction planes. This technique allows for view-by-view deflections of the focal spot in the rotational α-direction (αFFS) and in the longitudinal z-direction (zFFS). Thanks to this, the quality of the reconstructed images can be improved, mainly by reducing the windmill artifacts in z-direction (zFFS) and by decreasing the influence of the aliasing effect in the reconstruction plane(αFFS). The FFS implementation excludes the usage of traditional methods to reconstruct an image from projections. So, in practice, manufacturers decided primarily to modify the

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DOI: 10.1007/978-3-031-36021-3_64

adaptive multiple plane reconstruction (AMPR) method in this case (see, e.g. [4]). The AMPR technique is a nutating reconstruction method and, specifically, is an evolution of the advanced single slice rebinning (ASSR) algorithm [1, 5]. Generally, nutating methods have several serious drawbacks, particularly equispaced resolution of the slices in z-direction is very difficult to obtain due to the constant change in the position of successive reconstruction planes.

Meanwhile, CT scanner manufacturers began a challenge to develop methods of reducing the X-ray dose absorbed by patients during their examinations. Generally recognized strategy in this direction is to keep the image quality high, when radiation intensity is kept at a defined low level. It is possible by using appropriately formulated methods that are able to suppress noise which appears at the decreased intensity of x-rays. The most advanced conceptually are approaches based on a probabilistic model of the individual measurements in which the reconstruction problem is reformulated into an optimization problem (MBIR - Model-Based Iterative Reconstruction methods).

Unfortunately, the MBIR methods used commercially [8], have some problems with computational complexity because it is approximately proportional to N^4, where N is the image resolution. That approach is based on discrete-to-discrete (D-D) data model, i.e. both the reconstructed image and the measurements during reconstruction problem formulation in discrete forms are considered. Our approach has some significant advantages over the methods based on the D-D model. Firstly, the forward model is formulated here as a shift-invariant system. That allows for implementing an FFT algorithm in the most computationally demanding parts of the reconstruction algorithm. Secondly, our approach reduces the scheme of transforming the X-rays from the cone-beam geometry of the scanner to a parallel geometry, where the problem of not equiangular x-ray naturally does not occur. Finally, the reconstruction process can be carried out in only one plane in 2D space, thus greatly simplifying the reconstruction problem. Additionally, every scan of the body can be obtained separately.

The analysis of the method described in this article is strongly based on the construction principles of a CT scanner with a multifocal x-ray tube.

2 Flying Focus Spot Technique

The FFS technique utilizes the specific design of an X-ray tube, where it is possible to deflect the electron beam (using an electric field) before it hits the anode of the X-ray tube. This mechanism allows for view-by-view deflections of the focal spot for X-rays emitted from that anode. As a result of the FFS, it is possible to obtain the greater density of lines of rays used in the reconstruction process, both in the plane of the reconstructed image and along the z axis around which the projection system rotates. These geometrical conditions are shown in Figures 1 and 2.

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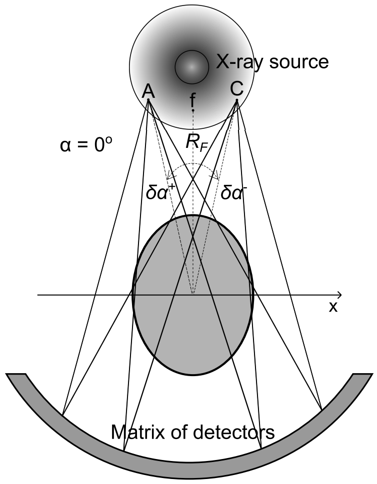

Fig. 1: Scheme of densification of rays in the reconstruction plane (αFFS).

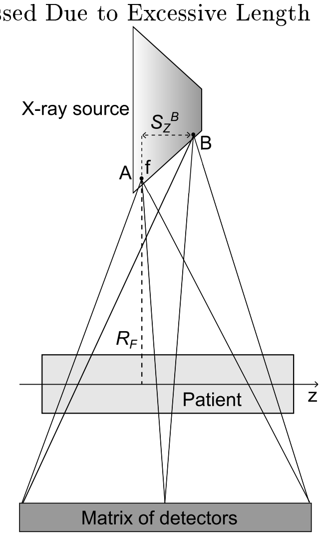

Fig. 2: Geometry of X-rays along the z axis (zFFS).

3 Reconstruction Algorithm

The reconstruction method dedicated for the flying focal spot technique proposed by us in this paper, is based on the maximum-likelihood (ML) estimation (see e.g. [7]). Usually, the objective of this kind of approach to the reconstruction problem is formulated according to a discrete-to-discrete (D-D) data model. In opposition to that approach, we have designed an optimization method. According to the originally formulated by us statistical approach to the reconstruction problem, it is possible to present a practical model-based iterative reconstruction procedure, as follows:

$$\mu_{\min} = \arg\min_{\mu} \left(\sum_{i=1}^{\mathrm{I}} \sum_{j=1}^{\mathrm{J}} \left(\sum_{\bar{i}} \sum_{\bar{j}} \mu^* \left(x_{\bar{i}}, y_{\bar{j}} \right) \cdot h_{\Delta i, \Delta j} - \tilde{\mu} \left(x_i, y_j \right) \right)^2 \right), \quad (1)$$

and $\tilde{\mu}(i,j)$ is an image obtained by way of a back-projection operation, in the following way:

$$\tilde{\mu}(x_i, y_j) = \Delta_\alpha \sum_{\theta} \dot{p}(s_{ij}, \alpha_\psi), \quad (2)$$

where $\dot{p}(s_{ij}, \alpha_\psi)$ are measurements performed using parallel beams, and:

$$h_{\Delta i, \Delta j} = \Delta_\alpha \sum_{\psi=0}^{\Psi-1} int \left(\Delta i \cos\psi \Delta_\alpha + \Delta j \sin\psi \Delta_\alpha \right), \quad (3)$$

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and $int\,(\Delta s)$ is an interpolation function (we used the linear interpolation function).

In our reconstruction procedure one can distinguish two parts. In the first one, we prepare and perform rebinning of all cone beam X-ray raw data from CT scanners with FFS to X-rays of parallel geometry. This operation is described in detail in subsection 3.1. After rebinning we determine a set of parallel projections which are ready to use for back-projection operation (see relation 2). Secondly, we use a statistical iterative procedure for 2D images, which computational complexity is $8log_2 4N^2$, where N is the dimension of the reconstruction image. Finally set of 2D views can be successfully used to obtain a 3D image.

3.1 Rebinning Operation

During the rebinning process we will describe every X-ray from cone-beam by elementary parameters, namely following specific points: Focus, SemiIsocenter, Detector [6]. Each of these parameters have coordinates in 3D space. This way, it is possible to change the description of equiangular X-rays using a set of three points in 3D. Next, we can compute all necessary calculations based on only those elementary components. This redefining can be described as follows:

$$xray(F_x^A, F_y^A, F_z^A, F_x^B, F_y^B, F_z^B, Q_x, Q_y, Q_z, D_x, D_y, D_z, p, s_m, \alpha_\psi), \quad (4)$$

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where points F determining Focus, points Q determining SemiIsocenter, points D determining Detector, all of those points are represented in 3D (x,y,z), p is value of projection, s_m and α_ψ described the same X-ray but in parallel projection. The parameters are depicted in Figure 3.

After the above calculations, the virtual X-rays have to be chosen as the most suitable real X-ray. This ray is chosen in a comparing procedure. For this purpose, we chose the best Focus in the following way:

$$f_x = -(R_F + \Delta R_F^T) \cdot \sin(\alpha + \Delta\alpha^T); \quad (6)$$

$$f_y = (R_F + \Delta R_F^T) \cdot \cos(\alpha + \Delta\alpha^T); \quad (7)$$

$$f_z = z_0 + \Delta z^T; \quad (8)$$

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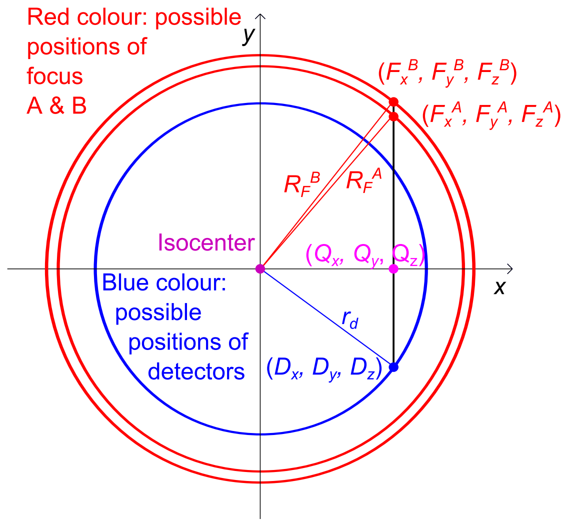

Fig. 3: Scheme of determined X-ray points.

where the T is one of two focuses, R_F is the distance between the focus and isocenter. The comparing procedure is based on calculating the ζ angle between real focus and virtual focus with connection to SemiIsocenter:

$$\zeta = \arccos\left(\frac{\hat{w}_x \cdot \hat{v}_x + \hat{w}_y \cdot \hat{v}_y}{\sqrt{\hat{w}_x^2 + \hat{w}_y^2} \cdot \sqrt{\hat{v}_x^2 + \hat{v}_y^2}}\right), \tag{9}$$

where:

$$\hat{v}_x = f_x^T - Q_x; \hat{v}_y = f_y^T - Q_y; \hat{w}_x = F_x^T - Q_x; \hat{w}_y = F_y^T - Q_y. \tag{10}$$

After choosing the two nearest focuses we try to determine the best detectors. There are some problems regarding the fact that we can not use the square function because its definition is unsuitable for the line parallel to y-axis. That is why we use a more complicated formula for the collinearity of three points in space. Thanks to this formula we can calculate the real detector position that is being searched for (D_x, D_y):

$$\begin{cases} (D_x - f_{x_0})^2 + (D_y - f_{y_0})^2 = R_{FD}^2 \\ (Q_x - f_x)(D_y - f_y) - (Q_y - f_y)(D_x - f_x) = 0 \end{cases}, \tag{11}$$

After finding this detector, we can calculate the value for a virtual detector using the three-linear interpolation function. Finally, the determined virtual parallel X-ray could be used for back-projection operation to create an image for an iterative reconstruction procedure (described in the next subsection). These virtual measurements can be used for filtered back-projection and the creation of a starting image for the iterative reconstruction procedure (see e.g. [2]). As a result

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of the transfer of the most demanding calculations into the frequency domain, it is possible to reduce the computational complexity from N^4 to $8log_2 4N^2$ where N is the dimension of the reconstruction image.

4 Results

This section is devoted to presentation of the results of our research based in the first part on mathematical simulation data and second part for the physical tomographic data that the authors of the work obtained by participating in the Low-Dose Grand Challenge. The physical data has been prepared artificially to simulate a quarter-dose of the X-ray radiation.

Our primary experiments are based on the *Shepp-Logan* mathematical data model. That model consists of ellipses or ellipsoids, depending on the two-dimensional or three-dimensional geometric system under consideration. They

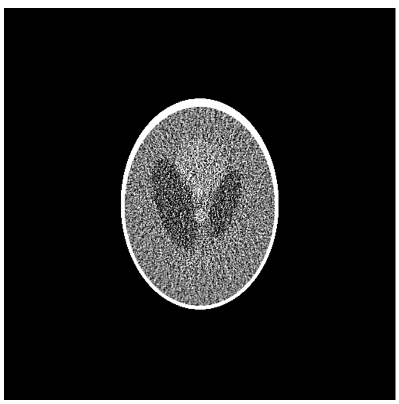

Fig. 4: Traditional reconstructed image; Noise: large; MSE: 766.691; NRMSE: 0.108585.

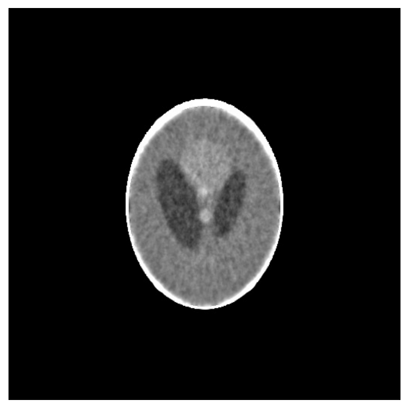

Fig. 5: Iterative reconstructed image; Noise: large; Iteration: 5000; MSE: 246.7224; NRMSE: 0.06159765.

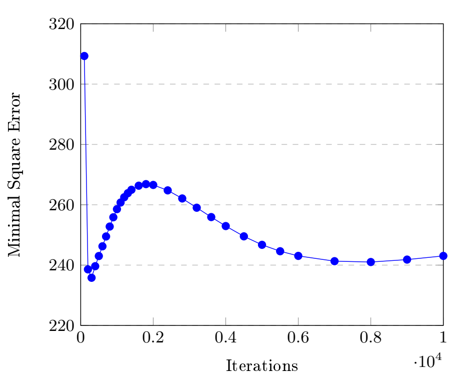

Fig. 6: MSE plot for large noise

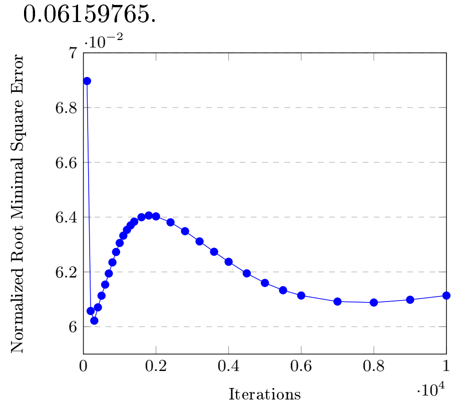

Fig. 7: NRMSE plot for large noise

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are the most simple geometric figures used to build mathematical phantoms. The ellipse/ellipsoid phantom allows for obtaining projection values for all rays at any projection angle. If simulating projection systems use a conical beam of radiation spiralling around the patient, it is necessary to define a mathematical phantom in 3D. It means that the equations for ellipses (flat figures) should be replaced with the equation for an ellipsoid (three-dimensional solid) using the following relations. These 3D mathematical model of head in our experiments was used.

Figures 4 and 5 show a visual comparison of the reconstructed mathematical sections of the mathematical model for $z = 0$. Figure 4 was made using the traditional method, while 5 was reconstructed using an iterative-statistical method, both with a high level of noise. We also presented the Minimal Square Error (MSE) plot - Figure 6 and Normalised Root Minimal Square Error (NRMSE) plot - Figure 7 booth are for iterative method.

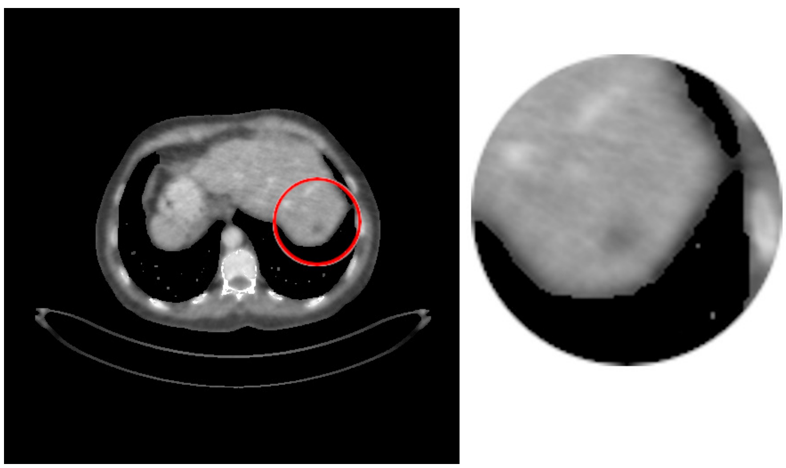

Fig. 8: Reconstruction view of the first patient. Z 109; Iterations 5000; Dose 1/4.

An important issue here is the subjective visual quality improvement of the reconstructed images for high level of noise presented in input data. Obtained results indicate that this method can be dedicated to increasing the usefulness of diagnostic images using low-dose computed tomography.

Next, we conducted reconstruction experiments using data from a commercial CT device with the simulated quarter-dose intensity of the X-rays. We present the reconstruction image (Figure 8) at the simulated quarter dose. This reconstructed image is obtained after 5 000 iteration. It is usually enough to recognize all the details in each view. This reconstructed image contains a form of lesion (recognized earlier by radiologists). The places of the presence of this pathology is marked by red circle on this pictures.

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5 Conclusion

An original complete statistical iterative reconstruction method with a rebinning method has been presented. The proposed by us algorithm can be applied in scanners with a flying focal spot here. Experiments conducted by us have proved that our reconstruction approach is relatively fast (about 5s for all operations in middle efficient GPU), mainly thanks to applying of an FFT algorithm during the most demanding calculations in the iterative reconstruction procedure.

6 Acknowledgements

The project financed under the program of the Polish Minister of Science and Higher Education under the name "Regional Initiative of Excellence" in the years 2019 - 2023 project number 020/RID/2018/19 the amount of financing PLN 12,000,000.

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DOI: 10.1007/978-3-031-36021-3_64