Experiment-driven quantum error reduction

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Abstract. Error correction is wide and well elaborated area of quantum information theory. Those methods, however, demand additional resources, like quantum gates, qubits or time. We have observed, in statistical sense, that the qubit's error in real quantum computers, once calibrated doesn't change much until next one. Then being so, for quantum sampling based computations, one can determine the correction experimentally and use it until the next calibration, without a need of utilize additional resources. In this work we present the method of determining such a correction and applying it to practical quantum-sampling algorithms.

Quantum sampling is the method, which we deliberately decline to obtain one deterministic result of one-shot computation in. Instead of that, we provide a number of same experiments. Then we observe the probability distribution function (PDF) thus obtained, which is considered as the final result of computation. We have observed and experimentally proved in this work, that error of this probability distribution is correlated with the local quantum phase of qubits involved in computations. Hence we are able to create a *Phase Distortion Unraveling* (PDU) function for each qubit and for whole system as well, that depends on this phase. Briefly, the final result after correction is the sum of PDF and PDU.

Keywords: quantum computing, quantum error correction, quantum sampling, quantum information theory, NISQ era

1 Introduction

We introduce the novel method of experiment-based error correction in quantum sampling computation process: *Phase Distortion Unraveling* PDU. Initially, the *determined errors* set ε_d is obtained as the difference between the experimental results and expectation values for each input binary string d representing eigen state $|x_d\rangle$. Then the *Phase Distortion Unraveling Function* is computed as the function interpolated by set of points (x_d, ε_d). Once obtained, it can be applied

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to the results of quantum sampling until the next physical calibration. In current work we present theoretical and experimental proofs of correctness of the PDU method: we will show that the PDU improves quantum calculation results statistically significant. Moreover, we will show that PDU is temporary stable, which we mean that the final error doesn't change significantly over time from the last physical calibration.

Our work is motivated by necessity of disposing the practically applicable error correction in the era of NISQ (Noisy, Intermediate Quantum) computers without requirement of involving extra qubits or gates. Due to limitation in QV (quantum volume), the methods that can correct errors in the evolution phase decreasing QV by grabbing qubits or gates simultaneously is not practically applicablePDU allows to proceed the computations utilizing whole QV and perform the correction procedure on the results. We are aware that PDU utility in beyond-NISQ era is problematic, however in the next ten-fifteen years it avails to proceed with experiments involving QV approaching the current maximum.

Current works in the area of error correction run towards of stabilizer [1] codes, the surface codes, cyclic codes and other less common. In the area of stabilizer codes two works of Nguyen and Kim [8, 9] are interesting. In the first one, they shows the stabilizer codes generated from Hermitian self-orthogonal ones and in the second - based on the binary formalization. Lv et al. in [6] shows another conversion od quaneum error codes: from quasi-cyclic to stabilizer codes. Ryan-Anderson et al. in [10] proposed the real-time method based on stabilized codes implemented on 10-qubits ion-trapped computer. Dymarsky and Shapere in [3] published the theoretical consideration about stabilizer codes in the perspective of CFT (Conformal Field Theory). Bravyi et al. in [2] describes the method of correction of coherent errors using surface codes. Litinski in [5] describe the interesting issue - he discuss the strategies of surface codes applied to different scale quantum computers.

There are methods of error mitigation [4, 7], which is basically the process of creating the additional operator applied to the given gate, that represents the inverse of an error that has been determined experimentally beforehand. This methods utilize the notion of expectation value or gate or detector tomography. Our method treats the quantum circuit as a black box containing an algorithm for solution of one specific problem, through the quantum sampling procedure. It is problem-oriented (not gate-oriented) method, hence for each problem and quantum computer the PDU should be designated. On the one hand it is the limitation, but on the other hand it is much simpler then mitigation methods, since it relies on observing the difference between experimentally obtained and expected results so it doesn't need any complex computation.

In this work we use PDU on the Quantum Cosine Series Sampling operator [11], which generally describes the method of quantum computing based on interpretation of outputs eigen state appearance normalized histograms as the function that can be mapped into the sine-cosine Fourier series. The result is interpreted in the context of problem to be solved, like image processing.

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2 Materials and methods

Let's consider „one half" of first component of QCoSamp $\nu_1(x)$ where parameters $r_1 = s_1 = 0$. We will use it for determining an error in resulting PDF, therefore we will call it reference function, which we formally define as: The *reference function* is the function $\gamma : \mathbb{R} \cap [-1, 1] \longrightarrow [0, 1] \cap \mathbb{R}$ for which for some set $\{x_0, ..., x_K\}$ there are known expected values $\{\gamma(x_0), ..., \gamma(x_K)\}$, e.g. determined classically. For each argument x_k that we can encode, we conduct sufficient[1] number of experiments realizing the reference function $\gamma = \nu_1$. Because we measure the last qubit, the histogram of outputs has two beams for $|0\rangle$ and $|1\rangle$. We normalize it, take the value for $|0\rangle$ and we denote it $\tilde{\gamma}(x_k)$. In that manner we construct the values of a function $\tilde{\gamma}(x)$ which is unknown in general, however we have obtained its values for 2^X arguments x_k, from experiments. On the other hand we can say that, for each $x_k : \tilde{\gamma}(x_k) = \gamma(x_k) + \varepsilon_k$, where ε_k is considered as error. It is different for different k, most probably. Therefore we can say, similarly, that there exists a function $\varepsilon(x) = \tilde{\gamma}(x) - \gamma(x)$, which we don't know, but we know some of its values: $\varepsilon_k = \varepsilon(x_k)$. Hence we can interpolate this function and use as the correction for the next computations. This interpolated function we call *Phase Distortion Unraveling Function* PDU. The PDU function has to be used for the same setup of qubits it was determined for. It is because the PDU contains both the errors of specific qubits themselves, and the errors connected with relations between qubits.

Experimental protocol In the experiments we have proved that it is possible to once experimentally designated correction function basing on one component $\frac{1}{2}(1 + cos(x))$, will improve significantly the results of subsequent $\frac{1}{2}(1 + cos(nx + r))$ components, until next physical calibration of the system. All experiments were done using library Qiskit for Python. This library was developed by IBM, and allowed Us to run Our experiments on real quantum computers (backends). Backends available to us were IBMQ: Lima, Manila, Bogota, Belem, Quito and Santiago.

Temporal stability experiment This experiment was aimed to examine if an error on IBMQ quantum computers is temporal stable. For this purpose we proceed with the same circuit in different times after declared calibration. Then we determine an error and compute the trendline over the time. For this experiment we run whole set of circuits - consisting of circuits for 16 different input values on a selected quantum computer, multiple times. After each run we were collecting actual values returned by quantum computer, along side expected results, and storing them in a file.

Significance of the PDU method. As the second experiment we examined if the PDU procedure of error correction is statistically significant. For this

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[1] In ideal world we should make statistical analysis what does the word „sufficient" means in the reality, however in real world we are limited to the offer of NISQ computers suppliers. Hence in this work we consider that 8.192 repetitions of the experiment, which is maximal number we can make on IBM Q Experience, is sufficient

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purpose we determine the PDU function for the given quantum computer. Then we run the circuits representing the same $\frac{1}{2}(1+cos(x))$ and different function $\frac{1}{2}(1+cos(2x))$ for inputs from the range $[-\pi,\pi]$ with the resolution $\frac{1}{8}\pi$ since we had disposed 6 qubits computers. The experiments was run in different time after the process of physical calibration. We denoted the results, MSE (Mean Square Error) according to the result expected and the time of an experiment. After we have collected the data we computed the statistical measures like average, standard deviation and Pearson correlation coefficient for the whole dataset containing results from all computers and for each computer separatelly.

3 Results

Temporal stability experiment. For this experiment, we collected data from various quantum computers and circuits shown in 2. Data were collected between 05.2021 and 01.2022. Data we collected showed us general boundaries of quantum errors - especially quantum noise.

We observed that on some quantum computers error values were more concise - like IMBQ Manila, where 83% of measured error values differed no more than 20% from an average, and standard error deviation for error was 0.033. On others - like IBMQ Santiago - error values were spread out. Over 56% of error values exceeded the average by more then 20%. Hence calculating error correction factors needs to be measured/done every time quantum computer is being calibrated. During Our tests we discovered, that despite information provided by IBM regarding last calibration time for certain quantum computers - declared error values on gates changed once per day. We decided to run the same tests for hours since initial calibration - which set error values on gates. Finally, we proceed with the main experiment in this task: to prove that errors are on the stable level over time. We use Athens, Belem, Bogota, Lima, Manila, Quito and Santiago. The trendlines for each of them are shown on the fig. 1 We see that 4 of computers has downward trend, which is quite surprising, two of them - stable and only one has the upward trend. However we expected stable trends, the difference of MSE over time doesn't exceed the threshold of 0.02 in the perspective of 20h. Which we recognize as promising for our method stability and usability.

Significance of the PDU method. On fig. 2 there is shown example how PDU influences on the final result. We can see, that the correction is distinct visually. Numerically, in the cases shown there, we observed mean square error to drop from 0.0190 to 0.0016, and standard deviation from 0.1333 to 0.0401 - which was consistent with the other results we have obtained during our experiments.

The PDU process increase the value of Pearson correlation coefficient value in general (see table 1), which means that the correlation between results corrected by PDU and reference expected ones is better then before this process. The increase is smallest for IBMQ Athens (0.0032), Quito (0.0298) and Santiago (0.099). For those computers that we disposed datasets consisting of over 1000 tuples the increase of Pearson correlation factor is obvious and is in the

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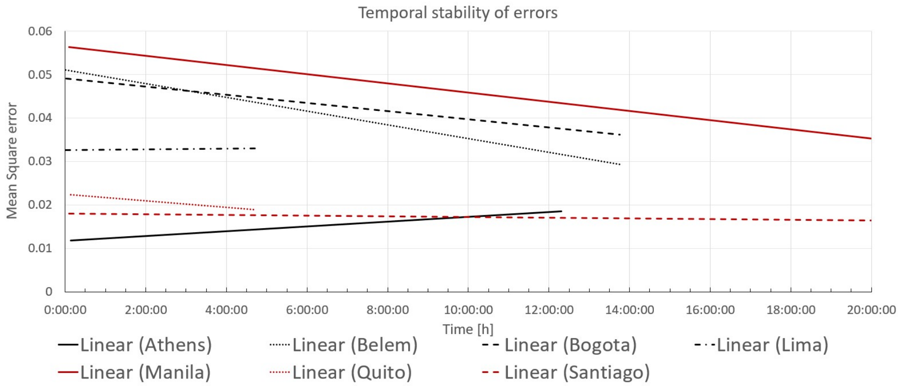

Fig. 1: Temporal stability trendlines for Athens, Belem, Bogota, Lima, Manila, Quito and Santiago computers in time period of 0-20 hours after physical calibration

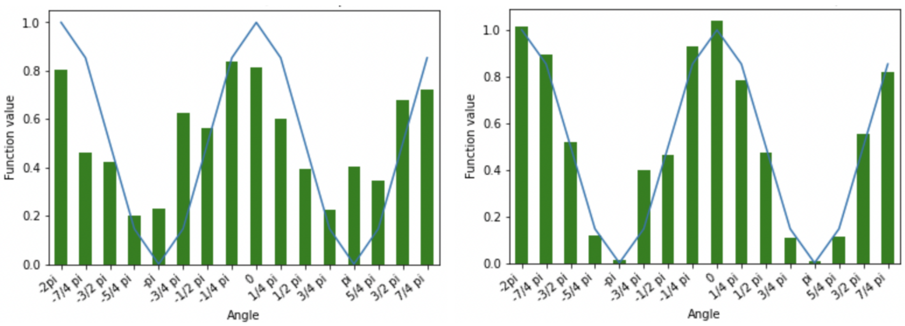

Fig. 2: Results for function $\frac{1}{2}(1 + cos(2x))$ collected on backend IBMQ Belem before (left) and after (right) applying error correction factors. Blue lines means the reference plot of the given function. Beans are the results obtained from quantum computer before (left) and after (right) PDU correction process.

range 0.0728 (Lima) to 0.1438 (Manila). Overall factors are: 0.876 for the original results and 0.9707 after PDU, which gives 0.0947 increase. Moreover, the coefficients value are high, sometimes very close to 1 with small p-values, which means that observed dependence between reference and corrected values is not accidental. Which is experimental prove of statistical significance of results we obtained.

4 Conclusions

In this paper we have presented PDU, the new, practical method for quantum error correction suitable for quantum sampling protocols of quantum computing, made without any additional involvement of quantum gates or qubits, made after measurement. We show that our approach, laying in one time determination

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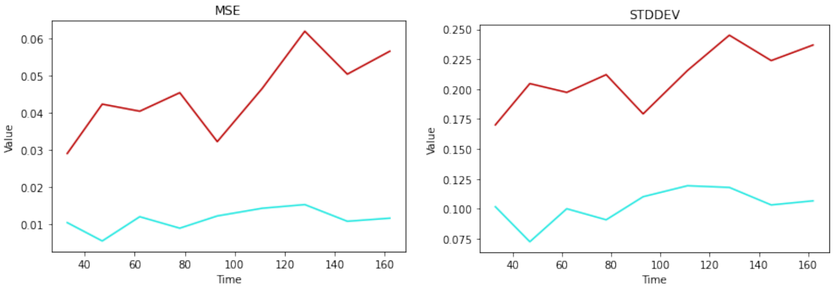

Fig. 3: Values of mean square error and standard deviation on error for functions $\frac{1+cos(x)}{4} + \frac{1+cos(2x)}{4}$ collected on IBMQ Lima. The red line shows the quantities (MSE, STDDEV) before PDU correction and blue ones- after PDU correction

computer	samples	Pearson		Kendall's		Spearman	
name	number	score	p-value	τ score	p-value	ρ score	p-value
Athens	100	0.98	2.2e-16	0.91	2.2e-16	0.98	2.2e-16
Belem	1360	0.96	2.2e-16	0.85	2.2e-16	0.96	2.2e-16
Bogota	1856	0.97	2.2e-16	0.85	2.2e-16	0.96	2.2e-16
Lima	990	0.97	2.2e-16	0.84	2.2e-16	0.96	2.2e-16
Manila	1027	0.96	2.2e-16	0.84	2.2e-16	0.95	2.2e-16
Quito	154	0.98	2.2e-16	0.89	2.2e-16	0.97	2.2e-16
Santiago	519	0.99	2.2e-16	0.91	2.2e-16	0.98	2.2e-16

Table 1: the correlation coefficients: Pearson product - moment, Kendall's τ and Spearmans ρ with p-values generated for examination of simplicity for result after PDU correction vs reference (ideal) results.

of PDU and applying it many times until the next physical calibration of the given computer is proper, due to temporal stability of errors and the correction process, which was proved in sections: 3, 3. Moreover, we have shown the single results of PDU correction on two real quantum computers 2. Finally, we have presented that our method increase significantly (by ca. 10% overall) the level of Pearson correlation coefficient, which confirms experimentally effectiveness of our method.

Surprisingly, from results obtained follows that temporal stability, meaning as the horizontal trend of error volatility over time, is less important for the increase of correlation, which was our initial assumption. Looking on the figures: 1 and table 1, we see that e.g., Santiago is temporal stability better then Manila, but increase is smaller. We can also observe that for computers that has higher increase (Belem, Bogota, Manila), the volatility trend is decreasing. Maybe there is some rule in this observation, but it need further investigations.

In conclusion we can say that PDU, the new method of quantum correction is suitable for use in the era of NISQ computers and we prove experimentally significant increase of correlation with reference values of the results corrected with PDU.

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