A review of 3D point clouds parameterization methods

Zaiping Zhu[1], Andres Iglesias[2,3], Lihua You[1] and Jian Jun Zhang[1]

[1] The National Center for Computer Animation, Bournemouth University, UK
[2] Department of Applied Mathematics and Computational Sciences, University of Cantabria, Cantabria 39005, Spain
[3] Department of Information Science, Faculty of Sciences, Toho University, 2-2-1 Miyama, 274-8510 Funabashi, Japan

Abstract. 3D point clouds parameterization is a very important research topic in the fields of computer graphics and computer vision, which has many applications such as texturing, remeshing and morphing, etc. Different from mesh parameterization, point clouds parameterization is a more challenging task in general as there is normally no connectivity information between points. Due to this challenge, the papers on point clouds parameterization are not as many as those on mesh parameterization. To the best of our knowledge, there are no review papers about point clouds parameterization. In this paper, we present a survey of existing methods for parameterizing 3D point clouds. We start by introducing the applications and importance of point clouds parameterization before explaining some relevant concepts. According to the organization of the point clouds, we first divide point cloud parameterization methods into two groups: organized and unorganized ones. Since various methods for unorganized point cloud parameterization have been proposed, we further divide the group of unorganized point cloud parameterization methods into some subgroups based on the technique used for parameterization. The main ideas and properties of each method are discussed aiming to provide an overview of various methods and help with the selection of different methods for various applications.

Keywords: Parameterization, organized point clouds, unorganized point clouds, mesh reconstruction.

1 Introduction

3D point clouds parameterization, also called point clouds mapping, is the process of mapping a 3D point cloud onto a suitable (usually simpler) domain. It has many applications such as object classification, texture mapping and surface reconstruction [1–3]. In many situations, it is computationally expensive or difficult to work with 3D point clouds directly. Therefore, projecting them onto a lower-dimensional space without distorting their shape is necessary. Compared to mesh parameterization, 3D point clouds parameterization is more challenging in general because there is no connectivity information between points, which hinders the direct extension of well-established mesh parameterization algorithms to point cloud parameterization. There are some survey papers on mesh parameterization [4,5]. However, to the best of our knowledge,

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there are no survey papers about point clouds parameterization. In this paper, we will review the methods of parameterizing point clouds. Notice there are also some works on 2D point clouds parameterization. Since 2D point clouds parameterization is different from 3D point clouds parameterization in most cases, this paper will only focus on the methods of 3D point clouds parameterization.

Some methods have been proposed to parameterize point clouds. In this paper, we roughly divide them into two main groups according to whether point clouds are organized or not. For each of the two groups, we further divide it into some subgroups based on the property of the mapping process and review each of the methods.

2 Some concepts

In this section, some concepts related to point clouds will be introduced to help readers understand the problem of point clouds parameterization. Since mesh parameterization has been well investigated in existing work and some ideas of mesh parameterization can be adopted by or adapted to point cloud parameterization, we will also introduce some concepts about mesh parameterization in this section.

1) **Organized and unorganized point clouds:** Generally, point clouds can be divided into organized and unorganized ones. Organized and unorganized point clouds are also called structured and unstructured point clouds, respectively. The division is determined by the way of storing point cloud data. For organized point clouds, the data are stored in a structured manner, while unorganized point cloud data are stored arbitrarily. Specifically, an organized point cloud is similar to a 2-D matrix and its data are divided into rows and columns according to the spatial relationships between the points. Accordingly, the spatial layout represented by the xyz-coordinates of the points in a point cloud decides the memory layout of the organized point cloud. Contrary to organized point clouds, unorganized point clouds are just a collection of 3-D coordinates, each of which denotes a single point.
2) **Global and local parameterization:** To parameterize point clouds, some methods map the whole point set of an underlying structure to a parameterization domain. In contrast, some other methods split the problem into several subproblems, each of which is called a local parameterization. The choice between global and local parameterization has impacts on mapping processes and results. Globally parameterizing the whole point set can guarantee the reconstructed mesh is a perfect manifold, meaning there are no seams, which may exist if the point cloud is partitioned and locally parameterized. However, processing the whole point cloud at the same time may be computationally expensive, especially for large structures.
3) **Topological shapes:** Topological shapes can be grouped based on the number of holes they own. Shapes with no holes such as spheres and bowls are treated as genus-0 shapes. Similarly, genus-1, genus-2 and genus-3 shapes have one, two and three holes in them, respectively, and so on.

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4) **Bijective function:** also called bijection, invertible function, or one-to-one correspondence, pairs each element in one set exactly to one element in the other set, and vice versa.
5) **Isometric, conformal, and equiareal mappings:** Suppose f is a bijective function between a mesh S or a point cloud and a mapping domain S^*, then f is isometric (length preserving) if the length of any arcs on S is preserved on S^*; f is conformal (angle preserving) if the angle of intersection of every pair of intersecting arcs on S is preserved on S^*; f is equiareal (area preserving) if the area of an area element on S is preserved on S^*. Isometric mappings are equiareal and conformal. Any mappings that are equiareal and conformal are isometric mapping.

3 Parameterization methods of organized point clouds

To parameterize an organized point cloud, many methods iteratively obtain a topologically identical 2D triangulation from the underlying 3D triangulation of the point cloud, and the 2D triangulation determines the parameter values of the vertices in the domain plane. Depending on the ways of transforming from 3D to 2D, there are several methods, including Harmonic parameterization [6], Floater's barycentric mappings [7] and the most Isometric parameterization [3]. For Harmonic parameterization in [6], the arc length is regarded as the parameter value of a spline curve, which is used to minimize the integral of the squared curvature with respect to the arc length for fairing the spline curve. With regard to barycentric mappings in [7], a shape-preserving parameterization method is applied for smooth surface fitting; the parameterization that is equivalent to a planar triangulation can be obtained by solving a linear system based on the convex combination. In [3], Hormann and Greiner propose a method to parameterize triangulated point clouds globally, the way of parameterizing inner point set is the same as that of parameterizing boundary point set. However, they ignore the problem of parameterizing triangulated point clouds with holes.

Energy function has also been defined to minimize the metric distortion in the transformation process from 3D to 2D. The methods described in [7, 8] follow the shared approach, which firstly parameterizes the boundary points, and then minimizes the following edge-based energy function for the parameterization of inner points [3]:

$$E = \frac{1}{2} \sum c_{ij} \, ||P_i - P_j||^2 \quad (1)$$

where c_{ij} is the edge coefficient that can be chosen in various ways, P_i and P_j are two points at the same edge.

In order to reconstruct a tensor product B-spline surface from scattered 3D data with specified topology, choosing a suitable way to parameterize the points is crucial in the reconstruction process. The method adopted by Greiner and Hormann in [8] is called the spring model. With this method, the edge of the 3D triangulation is replaced by a spring. Then the boundary points are mapped first onto a plane and stay unchanged. Next, the inner points are mapped onto this plane by minimizing the spring energy. The procedure is repeated to improve the parameters until certain conditions are satisfied.

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The above methods are mainly applicable to structured point clouds. They are not efficient when the number of points increases, and are likely to fail when holes and concave sections exist in the point clouds.

4 Parameterization methods of unorganized point clouds

In comparison with the parameterization of organized point clouds, many more methods have been proposed to parameterize unorganized point clouds. Table 1 lists these methods and gives the information about the category, parameter domain, local or global parameterization, topology, applications and publication year.

Table 1. methods to parameterize unorganized point clouds

Methods	Category	Parameter domain	Local/global parameterization	Topology	Applications	Year
"Simplicial" surface [10]	Base surfaces-based methods	Base surfaces	/	Arbitrary topology	Surface reconstruction	1992
Manually define [9]			Global	/	Least square fitting of B-spline curves and surfaces	1995
Minimizing quadratic function [11]			/	/	B-spline curves and surfaces approximation	2002
Recursive DBS [12]			Global/local	Disk	Efficient parameterization	2005
Recursive subdivision technique [13]			Global/local	Disk (With hole is ok)	Parameterizing point clouds	2007
Floater meshless parameterization [14-17]	Meshless parameterization	Plane	Global	Disk	Surface reconstruction	2000
Meshless parameterization for spherical topology [18]		Planes	Local	Genus-0	Surface reconstruction	2002
As-rigid-as-possible meshless parameterization [19]		Plane	Global	Disk	Denoising and parameterizing point clouds, mesh reconstruction	2010

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Meshless quadrangulation by global parameterization [20]		Plane	Global	Arbitrary genus	Meshless quadrangulation	2011
Spherical embedding [23]	Spherical mapping	Sphere	Global	Genus-0	Mesh reconstruction	2004
3D point clouds parameterization algorithm [22]		Sphere	Global	Relatively simple models	Parameterizing point clouds	2008
Spherical conformal parameterization [21]		Sphere	Global	Genus-0	Mesh reconstruction	2016
Discrete one-forms [24]	Adapt from mesh parameterization	Planes	Local	Genus-1	Mesh reconstruction	2006
Periodic global parameterization [25]		Plane	Global	Arbitrary genus	Direct quad-dominant meshing of point cloud	2011
PDE & SOM [26]	Neural networks-based methods	Adaptive base surface	Global	Complex sculptured surfaces	Surface reconstruction	2001
Adaptive sequential learning RBFnetworks [27]		/	Global	Freeform	Point-cloud surface parameterization	2013
Residual neural network [28]		/	Local	Fixed degree curve	Polynomial curve fitting	2021
A new parameterization method [29]	Other	/	/	/	NURBS surface interpolation	2000
Pointshop 3D [31]		/	/	/	Point-based surface editing	2002
Free-boundary conformal parameterization [30]		/	Global/local	/	Parameterizing point clouds for meshing	2022

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According to the property of the mapping process, we divide the parameterization methods of unorganized point clouds into base surfaces-based methods, meshless parameterization, spherical mapping, methods adapted from mesh parameterization, neural networks-based methods, and other methods.

4.1 Base surfaces-based methods

For parameterization of unorganized point clouds, base surfaces, which approximate the underlying structure of point clouds, have been widely applied to parameterize point clouds. Base surfaces can be a plane, a Coons patch, or a cylinder [2]. The parameter values of each point in a point cloud can be obtained by projecting the point cloud onto a base surface. The projection direction can either be perpendicular to the surface or based on a determined projection vector. According to [9], a base surface should own the following properties:

a) Unique local mapping: The uniqueness implies that any two different points on the underlying surface should be mapped onto two different locations on the mapping domain.
b) Smoothness and closeness of base surface: This indicates that a base surface should be as smooth and simple as possible, while still approximating the underlying surface as much as possible. The balance between these properties should be carefully considered.
c) Parameterization of base surface: This implies that how we parameterize a base surface has a direct effect on the parameterization of the fitting surface. We can choose a more suitable way to parameterize a base surface by referring to the underlying structure of the fitting surface.

To get access to such base surfaces, some approaches have been proposed. For example, Hoppe et al. [10] propose a method to produce so-called "simplicial" surfaces. They first define a function to estimate the signed geometric distance to the underlying surface of the point clouds, then a contouring algorithm is applied to approximate the underlying surface by a "simplicial" surface. Their method is capable of reconstructing a surface with or without boundary from an unorganized point set. However, there is no formal guarantee that the reconstructed result is correct and the space required to store the reconstruction is relatively large. In [9], users can also manually define some section curves and four boundary curves to get a base surface of a point cloud, as some characteristic curves approximating the underlying structure of the point cloud are sufficient in defining a base surface. But it is also necessary to take advantage of the interior characteristic curves when the geometry is complex, even though just four corner points can be used to create a base surface in some cases. A base surface can also be obtained by iteratively minimizing a quadratic objective function [11]. With this method, a linear system of equations is solved in each step. To parameterize unstructured point clouds, Dynamic Base Surfaces (DBS) are also proposed by Azariadis [2]. As its name implies, a BDS is gradually improved regarding its approximation to the underlying structure of a point cloud, and the parameter value of each point in the point cloud is obtained by projecting it orthogonally to the DBS. Different from existing methods, no restrictions are required for the density and the homogeneity of point clouds. The limitation of this method is that it is only applicable to the point clouds where a closed boundary consisting of four curves exists. Azariadis and Sapidis [12]

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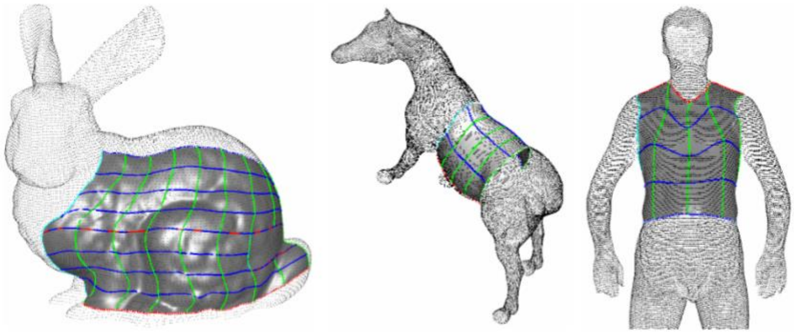
present a method to parameterize a point cloud globally and/or locally using recursive dynamic base surfaces. Their method can handle arbitrary point clouds of disk topology. Figure 1 shows the local parameterization of one subset of several point clouds using this method. The same authors [13] extend the DBS concept and use a recursive subdivision method to improve the accuracy of point clouds parameterization, especially for some small regions of the point clouds, where the approximation error by the DBS is not acceptable. They divide such regions into smaller parts and the points on these parts are approximated by c^0 composite surface based on recursive DBS subdivision to increase the approximation error, then to make the point clouds parameterization more accurately.



(a) (b) (c)

Fig. 1. Local parameterization of: (a) "bunny" point cloud, (b) "horse" point cloud, and (c) "human" point cloud. [12].

4.2 Meshless parameterization

Meshless parameterization, first proposed by Floater and Reimers in [14], is also a widely used method to parameterize and mesh point clouds. As shown in Figure 2, the main idea of meshless parameterization is to map the points in a point cloud onto a plane, where the mapping points are triangulated using an appropriate triangulation method, and then the original point cloud is meshed with the same triangulation edge structure as the mapping points. In order to make sure the reconstructed mesh has high quality, the mapping points should preserve the local structure of the original point clouds as much as possible. Therefore, the shape distortion ought to be minimized in the parameterization process. This is formulated as the problem of solving a sparse linear system [14, 15]. Since the mapping does not depend on the topological structure of point clouds, this method is called meshless parameterization. After the projection, the corresponding triangulation of the point clouds before mapping can be obtained by triangulating the projecting points in the planar parameter domain. This method has some limitations. First of all, solving a large linear system using their method is not efficient. Secondly, the reconstructed 3D triangles may distort and intersect each other due to the artificial convex boundary, which is also a problem when there are concave holes and the convex combination is not well defined along the concave parts of the hole boundary. To improve the efficiency of solving the linear system more efficiently, Volodine et al. [16] show that it can be done by an appropriate reordering of the matrix,

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which enables the linear system to be solved efficiently by deploying a direct sparse solver. To overcome the second problem, the same authors [17] extend the method to avoid distortion in the vicinity of concave boundaries by inserting virtual points to the concave neighbourhood, which can make sure the convex combination mapping is always defined. The methods described in [17] are only applicable to disk shape point clouds. To make the method presented in [17] more general, Hormann and Reimers [18] present an algorithm that can handle genus-0 topology as well by dividing the problem of triangulating point clouds into subproblems, each of which can be solved using the method in [17]. To improve the reconstructed result, Zhang et al. [19] apply an "as-rigid-as-possible" meshless parameterization method to parameterize a disk topology point cloud onto a plane while denoising the point cloud. Since their method can preserve local distances in the point cloud, a more regular 3D mesh can be obtained. Li et al. [20] present a meshless global parameterization method to parameterize point clouds and use the obtained parameterization to mesh the point clouds automatically.

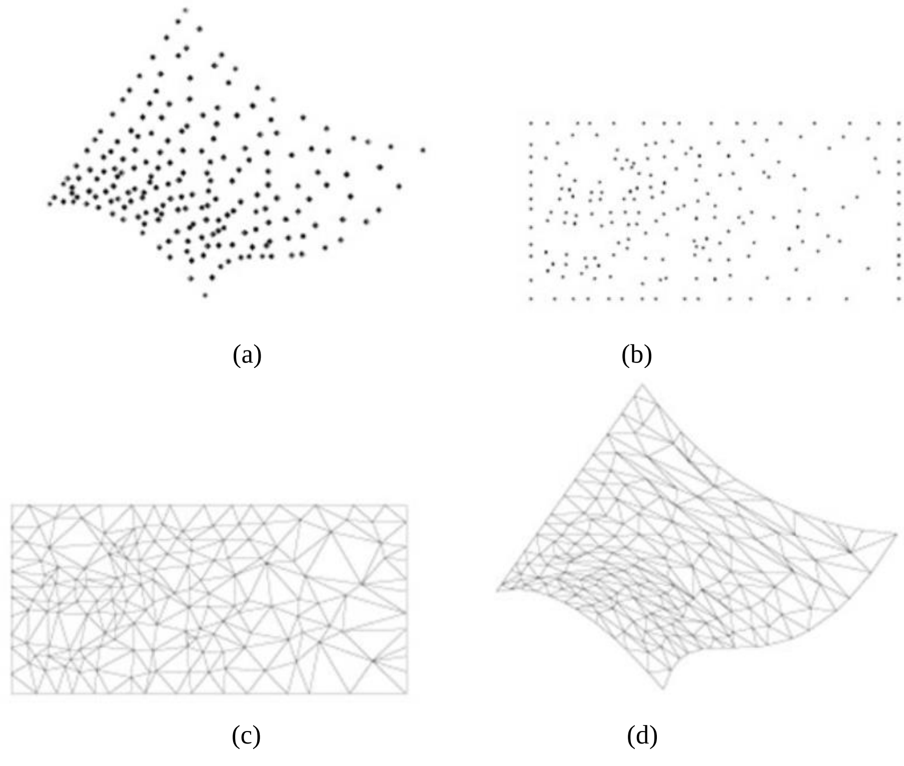

Fig. 2. (a) Point set. (b) meshless parameterization. (c) Delaunay triangulation of the mapping points. (d) surface triangulation [15].

4.3 Spherical mapping

When the underlying structure of the point clouds is closed, which means there are no boundaries of the structure, "spherical mapping" is normally applied to parameterize

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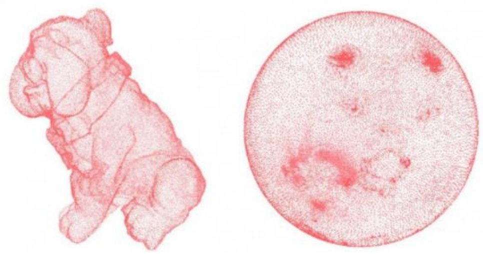
the point clouds. The reason why "spherical mapping" is applied under such conditions can be partly explained by the uniform theorem [21], which states that every genus-0 closed surface is conformally equivalent to S^2. Thus, mapping from a genus-0 surface to the unit sphere is natural. The same idea is also applied to genus-0 point clouds. One such example is shown in Figure 3. The problem of forming a spherical mapping given a point cloud model P can be formulated as [22]:

$$s = o + r_s \frac{p-o}{||p-o||} \tag{2}$$

where s are the spherical mapping points, p is the original point set, o is the centre of the original point set and r_s represents the largest distance between the original point set and the centre with the radius of the sphere.

Spherical parameterization is mostly used to mesh point clouds. For example, Zwicker and Gotsman [23] present a method to reconstruct a manifold genus-0 mesh from a 3D point cloud by using spherical embedding of a k-nearest neighbourhood graph of a point cloud. Then the embedded points are triangulated and the reconstructed mesh structure is used to mesh the original point cloud. The main advantage of this method is that it can guarantee a closed manifold genus-0 mesh, even the input point cloud is noisy. However, its drawbacks are that pre-processing and post-processing may be required for the input point clouds and the output mesh, respectively. In [21], Choi et al. extend a state-of-the-art spherical conformal parameterization algorithm used to parameterize genus-0 meshes to the case of point clouds, which are achieved by using an improved approximation of the Laplace-Beltrami operator on the point cloud and a scheme named the north-south reiteration for the meshing of point clouds. The reason why they apply the method of spherical conformal parameterization method to reconstruct meshes from point clouds is mainly that directly triangulating a point cloud is challenging, especially for complex geometry, which can be achieved more easily with the aid of spherical conformal parameterization. Specifically, instead of directly triangulating a point cloud, the points on the unit sphere after mapping are triangulated using the spherical Delaunay triangulation algorithm. Then triangulation of the original point cloud can be obtained from the triangulation on the spherical point cloud as these two point clouds have a one-to-one correspondence.



(a) (b)

Fig. 3. (a) A bulldog point cloud. (b) the spherical conformal parameterization of the bulldog point cloud [21].

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4.4 Methods adapted from mesh parameterization

There are also some methods that are adapted from parameterizing meshes to parameterizing point clouds. For example, Tewari et al. modify the harmonic one-form method used in parameterizing manifold meshes to parameterize genus-1 point clouds that are sampled from such meshes [24]. They locally parameterize the subsets of a point cloud and the way they parameterize the point cloud can guarantee the consistency between the pieces. Even though the reconstructed results using their method are not much better than other reconstruction techniques, their method presents some new tools to the surface reconstruction problem and is very simple to implement. Li et al. [25] present a new method to reconstruct quad-dominant mesh from unorganized point clouds using the adapted periodic global parameterization method, which is modified from the periodic global parameterization method that is used to parameterize a triangle mesh. The local Delaunay triangulation is used to design the parameterization of the point cloud. Their method can be used to deal with noisy point clouds without global connectivity. But it suffers from close-by structures because topological errors may be raised from the local Delaunay triangulation method by connecting two nearby surfaces.

4.5 Neural networks-based methods

With the rapid development of neural network techniques, they have been applied to three main tasks of point cloud processing, i. e., 3D shape classification, 3D object detection and tracking, and 3D point cloud segmentation [26]. Besides their applications in the three main tasks, some researchers have investigated neural network-based point cloud parameterization. For example, Barhak and Fischer [27] adopt a self-organizing map (SOM) for the parameterization of small sets of clean points with low-frequency spatial variations, which can be used to reconstruct smooth surfaces. There are mainly two steps in the parameterization process: In the first step, Partial Differential Equation (PDE) and SOM are applied where the former technique can yield a parametric grid without self-intersection and the latter one makes sure all the sampled points have an impact on the grid, which guarantees the uniformity and smoothness of the reconstructed surface. In the second step, an adaptively modified 3D base surface is created for point clouds parameterization. Meng et al. [28] proposed a method to parameterize larger, noisy and unoriented point clouds by using adaptive sequential learning RBFnetworks. The network adopts a dynamic structure by adaptive learning and the neurons are adjustable regarding their locations, widths and weights, thus making it more powerful compared to other methods that apply RBFs at determined locations and scales. What is more, multi-level parameterization and multiple level-of-details (LODs) can be achieved in two ways. When multiple LODs meshes are required, parameterizing the point clouds with the best resolution and the points and surfaces can be computed at degrading sampling level to get the required LODs. In the second case where only one downgraded LOD is required, downgraded parameterization can be applied to obtain the result. Scholz and Juttler [29] apply residual deep neural networks to parameterize point clouds for polynomial curve fitting. Since the network approximates the function that assigns a suitable parameter value to a sequence of data points, optimal curve reconstruction from point clouds can be obtained. However, their method is only applicable to a small number of sample points and the proposed neural networks

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do not consider discrete surface point data. Figure 4 shows the layout of their proposed residual neural network.

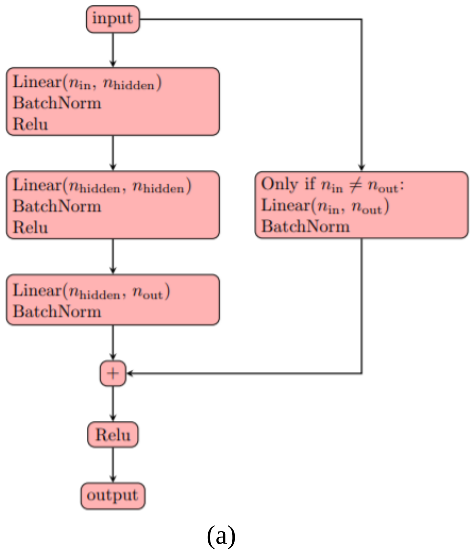

Fig. 4. (a) The layout of a building block. (b) the layout of the whole residual neural network [29].

4.6 Other methods

Some other methods cannot be easily grouped. Therefore, we refer to them as other methods in this subsection and review them below.

As Ma and Kruth discuss in [9], three methods are usually adopted to parameterize digitized points for performing least squares fitting of B-spline curves and surfaces. These three methods are uniform parameters, cumulative chord length parameters and centripetal parameterization parameters. Since all these methods assume that the points are scattered in a special pattern, like chain points for curves and grid points for surfaces, these methods are very likely to fail when the points are irregularly spaced. To address this issue, Ma and Kruth [9] propose a simple technique, which parameterizes the irregularly spaced points by projecting them onto a base surface and obtaining their parameters from the parameters of the projected points. Jung and Kim [30] propose a new method to parameterize data points for NURBS surface interpolation, which is more powerful than the existing point clouds parameterization methods. With this method, the parameter value at the maximum of each rational B-spline basis function is treated as the parameter value of the corresponding data point. The empirical results show that their method outperforms the other methods as aforementioned in [10] regarding interpolation surfaces. In addition, many works consider mapping them onto a simple domain with a fixed boundary shape such as a sphere, a circle or a rectangle. However, some undesirable distortion may occur during the parameterization process due to the fixed boundary shape. To overcome such a problem, Choi et al. [31] develop a free-boundary conformal parameterization technique to parameterize disk-shape

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point clouds, which leads to high quality of the reconstructed mesh. By free boundary, it means that the positions of only two boundary points are fixed, and the left boundary points are parameterized to a suitable location automatically based on the structure of the original point clouds. To make the parameterization of point clouds more flexible, Zwicker et al. [32] present a system in which interactively parameterizing point clouds can be done. During the mapping process, an objective function is applied to minimize distortions automatically. Furthermore, the user can adjust the mapping intuitively at the same time.

5 Conclusion

In this paper, we have reviewed various methods used to parameterize 3D point clouds. These methods are grouped into organized point parameterization and unorganized point cloud parameterization ones and unorganized point cloud parameterization methods are further divided into some subgroups according to the property of the point clouds and the mapping technique. We discussed each of these methods.

It should be pointed out that there is no "best" parameterization method applicable to all point clouds, as one method may succeed in parameterizing some point clouds but fail in parameterizing other point clouds. Therefore, for a given point cloud, it is necessary to choose a suitable method to parameterize the 3D point cloud according to the desirable properties of low distortion and high computing efficiency in parameterizing the point cloud.

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References

1. Meng, T.W.; Choi, G.P.-T.; Lui, L.M. Tempo: Feature-Endowed Teichmuller Extremal Mappings of Point Clouds. *SIAM Journal on Imaging Sciences* **2016**, *9*, 1922–1962.
2. Azariadis, P.N. Parameterization of Clouds of Unorganized Points Using Dynamic Base Surfaces. *Computer-Aided Design* **2004**, *36*, 607–623.
3. Hormann, K.; Greiner, G. MIPS: An Efficient Global Parametrization Method. *Curve and Surface Design*: Saint-Malo, 2000, 153-162.
4. Floater, M.S.; Hormann, K. Surface Parameterization: A Tutorial and Survey. *Advances in multiresolution for geometric modelling* **2005**, 157–186.

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To cite this paper please use the final published version:
DOI: 10.1007/978-3-031-08757-8_57

5. Sheffer, A.; Hormann, K.; Levy, B.; Desbrun, M.; Zhou, K.; Praun, E.; Hoppe, H. Mesh Parameterization: Theory and Practice. *ACM SIGGRAPPH, course notes* **2007**, *10*.
6. Eck, M.; Hadenfeld, J. Local Energy Fairing of B-Spline Curves. In Proceedings of the Geometric Modelling; Hagen, H., Farin, G., Noltemeier, H., Eds.; Springer: Vienna, 1995; pp. 129–147.
7. Floater, M.S. Parametrization and Smooth Approximation of Surface Triangulations. Computer Aided Geometric Design 1997, 14(3), 231-250.
8. Greiner, G.; Hormann, K. Interpolating and Approximating Scattered 3D-Data with Hierarchical Tensor Product B-Splines. In Proceedings of the in Surface Fitting and Multiresolution Methods; Vanderbilt University Press, 1997; pp. 163–172.
9. Ma, W.; Kruth, J.-P. Parameterization of Randomly Measured Points for Least Squares Fitting of B-Spline Curves and Surfaces. *Computer-Aided Design* **1995**, *27*, 663–675.
10. Surface Reconstruction from Unorganized Points | Proceedings of the 19th Annual Conference on Computer Graphics and Interactive Techniques Available online: https://dl.acm.org/doi/abs/10.1145/133994.134011 (accessed on 19 February 2022).
11. Pottmann, H.; Leopoldseder, S.; Hofer, M. Approximation with Active B-Spline Curves and Surfaces. In Proceedings of the 10th Pacific Conference on Computer Graphics and Applications, 2002. Proceedings.; IEEE, 2002; pp. 8–25.
12. Azariadis, P.; Sapidis, N. Efficient Parameterization of 3D Point-Sets Using Recursive Dynamic Base Surfaces. In Panhellenic Conference on Informatics, pp. 296-306. Springer, Berlin, Heidelberg (2005).
13. Azariadis, P.; Sapidis, N. Product Design Using Point-Cloud Surfaces: A Recursive Subdivision Technique for Point Parameterization. *Computers in Industry* **2007**, *58*, 832–843, doi:10.1016/j.compind.2007.03.001.
14. Floater, M.S.; Reimers, M. Meshless Parameterization and Surface Reconstruction. *Computer Aided Geometric Design 18*, 77–92.
15. Floater, M.S. Meshless Parameterization and B-Spline Surface Approximation. In Proceedings of the The Mathematics of Surfaces IX; Springer, 2000; pp. 1–18.
16. Volodine, T.; Roose, D.; Vanderstraeten, D.; Volodine, T.; Roose, D.; Vanderstraeten, D. 65F05, 65M50. Efficient Triangulation of Point Clouds Using Floater Parameterization 2004.
17. Volodine, T.; Vanderstraeten, D.; Roose, D. Experiments on the Parameterization of Point Clouds with Holes. *TW Reports, volume TW432* **2005**, *12*.
18. Hormann, K.; Reimers, M. Triangulating Point Clouds with Spherical Topology. Curve and Surface Design: Saint-Malo (2002), pp. 215-224.
19. Zhang, L.; Liu, L.; Gotsman, C.; Huang, H. Mesh Reconstruction by Meshless Denoising and Parameterization. *Computers & Graphics* **2010**, *34*, 198–208.
20. Li, E. Meshless Quadrangulation by Global Parameterization. Computer and Graphics 35(5), 992-1000 (**2011)**.

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21. Choi, G.P.-T.; Ho, K.T.; Lui, L.M. Spherical Conformal Parameterization of Genus-0 Point Clouds for Meshing. *arXiv:1508.07569 [cs, math]* **2016**, doi:10.1137/15M1037561.
22. 3D Point Clouds Parameterization Alogrithm | IEEE Conference Publication | IEEE Xplore Available online: https://ieeexplore.ieee.org/abstract/document/4697396 (accessed on 19 February 2022).
23. Zwicker, M.; Gotsman, C. Meshing Point Clouds Using Spherical Parameterization. In PBG, PP. 173-180 (2004).
24. Tewari, G.; Gotsman, C.; Gortler, S.J. Meshing Genus-1 Point Clouds Using Discrete One-Forms. *Computers & Graphics* **2006**, *30*, 917–926, doi:10.1016/j.cag.2006.08.019.
25. Li, E.; Che, W.; Zhang, X.; Zhang, Y.-K.; Xu, B. Direct Quad-Dominant Meshing of Point Cloud via Global Parameterization. *Computers & Graphics* **2011**, *35*, 452–460, doi:10.1016/j.cag.2011.03.021.
26. Guo, Y.; Wang, H.; Hu, Q.; Liu, H.; Liu, L.; Bennamoun, M. Deep learning for 3D point clouds: a survey. IEEE Transactions on Pattern Analysis and Machine Intelligence 2021, 43, 4338-4364.
27. Barhak, J.; Fischer, A. Parameterization and Reconstruction from 3D Scattered Points Based on Neural Network and PDE Techniques. IEEE Transactions on visualization and computer graphics 2001, 7, 1–16.
28. Meng, Q.; Li, B.; Holstein, H.; Liu, Y. Parameterization of Point-Cloud Freeform Surfaces Using Adaptive Sequential Learning RBFnetworks. Pattern Recognition 2013, 46, 2361–2375, doi:10.1016/j.patcog.2013.01.017.
29. Scholz, F.; Jüttler, B. Parameterization for Polynomial Curve Approximation via Residual Deep Neural Networks. Computer Aided Geometric Design 2021, 85, 101977, doi:10.1016/j.cagd.2021.101977.
30. Jung, H.B.; Kim, K. A New Parameterisation Method for NURBS Surface Interpolation. The International Journal of Advanced Manufacturing Technology 2000, 16, 784–790, doi:10.1007/s001700070012.
31. Choi, G.P.T.; Liu, Y.; Lui, L.M. Free-Boundary Conformal Parameterization of Point Clouds. arXiv:2010.15399 [cs, math] 2021.
32. Zwicker, M.; Pauly, M.; Knoll, O.; Gross, M. Pointshop 3D: An Interactive System for Point-Based Surface Editing. ACM Transactions on Graphics (TOG) 2002, 21, 322–329.

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