Continuous-to-continuous data model vs. discrete-to-discrete data model for the statistical iterative reconstruction method*

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Abstract. The article presents a comparison of two statistical approaches to the problem of image reconstruction from projections: the worldwide known concept based on a discrete-to-discrete data model and our original idea based on a continuous-to-continuous data model. Both reconstruction approaches are formulated taking into account the statistical properties of signals obtained by CT scanners. The main goal of this strategy is significantly improving the quality of the reconstructed images, so allowing a reduction in the x-ray dose absorbed by a patient during CT examinations. In the concept proposed by us, the reconstruction problem is formulated as a shift-invariant system. In consequence, that significantly improves the quality of the subsequently reconstructed images, and it allows to reduce the computational complexity compared to the reference method. The performed by us experiments have shown that our original reconstruction method outperforms the referential approach regarding the image quality obtained and the time of necessary calculations.

Keywords: Iterative reconstruction algorithms · Computed tomography · Statistical methods.

1 Introduction

1.1 Motivation

Although computed tomography was invented many years ago, it continues to be a very appealing field of research. Every new generation of CT devices stimulates the development of reconstruction algorithms adapted for the new design. Put simply, we can say that since the first design made by Godfrey Newbold Hounsfield in 1971 all the most significant reconstruction algorithms have used one of two basic approaches, depending on the signal processing methodology used in them: these are analytical methods (continuous-to-continuous data

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model), especially those based on convolution and back-projection [1], and methods based on the strategy called the algebraic strategy (discrete-to-discrete data model) [2]. It is worth underlining that apart from in the first scanner designed by Hounsfield, the EMI Mark I, and the prospective use of the newest concepts in reconstruction algorithms in high definition computed tomography (HDCT) devices, all other CT designs have been equipped with analytical reconstruction algorithms. The use of the algebraic method (ART - Algebraic Reconstruction Technique) in the first historical CT apparatus was presumably because there was no alternative at the time. After this "early mistake", the next generation of CT systems used only reconstruction algorithms based on analytical image processing methods. The main reason for this was the huge size of the matrices which appear in the algebraic reconstruction problem and the calculation complexity of the reconstruction method based on this methodology that this caused. The analytical (or transformation) methodology drastically simplifies the number of calculations needed and so is more appealing. It has been proven (e.g. [3]) that the frequency of cancerous diseases for patients who had had a CT scan (at least one year after the scan) is about 24% higher than in the case of patients who had not had the scan. Due to the enormous prevalence of CT scans, any action aimed at reducing this impact are of fundamental importance, assuming of course, the further existence of this popular, cheap and effective diagnostics technique. For these reasons, but also for both social and commercial ones, manufacturers began a kind of competition to develop methods of reducing the X-ray dose absorbed by patients. The seemingly obvious solution, to simply reduce the radiation dose given during a scan, cannot be applied. This is because the required radiation dose is determined by the SNR, which defines the image quality. Thus, if the image quality is to remain high, the radiation intensity should stay at a defined level (which has a direct impact on the absorbed radiation dose during the scan).

1.2 Contribution

It is possible to improve the resistance of tomographic images to the measurement noise which occurs during image reconstruction by using appropriate statistical signal processing. This means that it is possible to decrease the radiation intensity applied, and so decrease the dose absorbed by patients. Recently, some commercial solutions of such systems have been developed, which perform reconstruction processing iteratively to decrease the noise in the images. These systems take into consideration the probabilistic conditions present in the measurement systems of CT scanners in order to limit the influence of noise on the images obtained from the measurements. The most interesting approach, called MBIR (Model-Based Iterative Reconstruction), is presented in such papers as [4] [5], where a statistical model of the measurements is derived analytically, and, based on this, a statistical iterative reconstruction algorithm is formulated. The reconstruction problem formulated algebraically plays a crucial part in this approach. Indeed, the algebraic approach to the image reconstruction from projections problem is being intensively explored once more. This is because of

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one obvious reason - the measurement noises in it can be modelled relatively easily, because each measurement is considered separately. The reconstruction idea presented in the above publications is based on the maximum likelihood (ML) approach and a development of this concept - the maximum *a posteriori* probability (MAP) estimation approach (the iterative coordinate descent (ICD) algorithm described comprehensively in [6]) implements the MAP approach). Consequently, in 2013, this development had its debut under its commercial name Veo - CT Model-Based Iterative Reconstruction. This application of the algebraic reconstruction method, however, presents some significant technical difficulties in its practical realization, namely: the difficulty in establishing the coefficients of the forward model for 3D spiral cone-beam scanners [7], [6]. The huge number of these coefficients in this model means that it is impossible to keep all of them in memory at the same time and the requirement for the simultaneous calculation of all voxels in the range of the reconstructed 3D image make the reconstruction problem extremely complex. Although, there have been attempts to decrease the calculation complexity of this approach, as presented for example in the paper [8], they have, as yet, only met with limited results. Moreover, this system uses a reconstruction problem model that has been shown to be extremely ill-conditioned. One can say that there are many solutions on the market in this area, but they are still insufficient when it comes to the limiting the radiation dose. Therefore, there is still room for improvement of such systems. It would be interesting to formulate a statistical reconstruction method which would take into consideration the statistical conditions of the measurement physics, as in the ICD algorithm, thereby eliminating most of the disadvantages of the algebraic scheme of signal processing methodology. We could avoid the above mentioned difficulties connected with using an algebraic methodology by using an analytical strategy for the reconstructed image processing. In previous papers, we have shown how to formulate the analytical reconstruction problem consistent with the ML methodology for scanners with parallel geometry [9], [10], for fan-beams [11], and finally we have proposed a scheme of reconstruction method for the spiral cone-beam scanner [12]. Our approach has some significant advantages compared with algebraic methodology. Firstly, in our method, we establish certain coefficients, but this is performed in a much easier way than in comparable methods. Secondly, we perform the reconstruction process in only one plane in 2D space, greatly simplifying the problem. In this way, the reconstruction process can be performed for every cross-section image separately. After this, it is possible to reconstruct the whole 3D volume image from the set of previously reconstructed 2D images. And finally, because of the analytical methodology of the reconstruction process, we can perform most of the computationally expensive operations in the frequency domain (2D convolutions). Because it is a very much less computationally demanding approach, by using FFT, we make our reconstruction method independent of the dimensions of the reconstructed image, to an acceptable degree. This approach also outperforms the algebraic method regarding the better condition number at the level of problem formulation. This makes our method really competitive

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in terms of its resistance to the influence of measurement noise and errors in the forward model. The main motivation for this paper is to present a comparison between these two model-based approaches to the statistical reconstruction problem. In particular, we will present considerations and computer simulations that correspond with the optimization of the computational complexity in the case of the continuous-to-continuous method designed by us. We will also show how this optimization achieved by problem reformulation can impact the time of calculations obtained under actual conditions.

2 Statistical Reconstruction Approaches

We below present two approaches that can be directly applied to parallel beam tomography, but it is possible in an easy way to adapt them for a majority of all existing geometries of the CT scanners.

Let function $\mu(x, y)$ denote the unknown image representing a cross-section of an examined object (in medicine, a human body). Image $\mu(x, y)$ will be calculated using projections obtained by using the Radon transform. A diagram of a single projection measurement is depicted in Fig. 1.

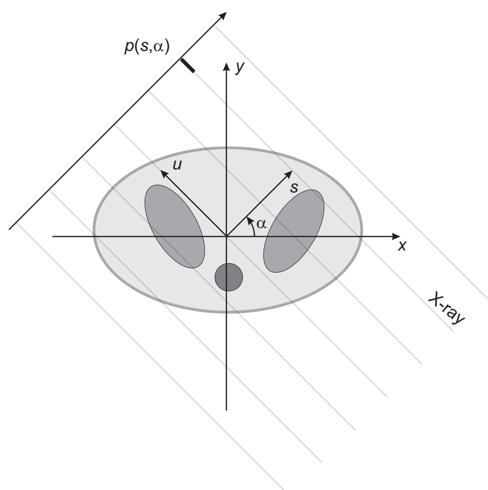

Fig. 1. The geometry of the projection system

The function $p(s, \alpha)$ is the result of a measurement carried out at a distance s from the origin when a projection is made at a specific angle α. This is called the Radon transform and is written mathematically as

$$p(s, \alpha) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) \cdot \delta(xcos\alpha + ysin\alpha - s)\, dxdy. \tag{1}$$

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Both reconstruction approaches can only make use of projections obtained at certain angles and measured only at particular points on the screen. Therefore, beams of x-rays reaches the individual detectors at points $l = -L/2, \ldots, L/2$, where L is a number of detectors placed on a screen. Values $s_l = l \cdot \Delta_s$ denote the distances on the screen between each ray and the origin, and Δ_s denotes the interval between detectors. In turn, parameters α_ψ denote discrete values of the projection angles indexed by the variable ψ, where $\psi = 0, \ldots, \Psi - 1$, where Ψ is the number of projections. Subsequent projections are carried out after a rotation by Δ_α. Following a discrete nature of available measurements, we will consider the discrete form of the image $\mu(i,j)$ as well, where $i = 1, \ldots, I$, $j = 1, \ldots, I$.

2.1 A statistical approach based on the discrete-to-discrete data model

First, we consider a referential approach to the image reconstruction problem, in which a model-based iterative reconstruction method is based on a discrete-to-discrete data model. A forward model (a system of linear equations) of this approach can be presented as follows:

$$\mathbf{p} = \mathbf{A}\mu, \tag{2}$$

where: $\mathbf{p} = [p_m]$ is the projection vector with $m = 1, \ldots, L \cdot \Psi$; $\mathbf{A} = [a_{mn}]$ is a system matrix with dimensions $1, \ldots L \cdot \Psi \times 1, \ldots I^2$; $\mu = [\mu_n]$ is a vector representing a reconstructed image with dimension $n = 1, \ldots, I^2$. Practically, the elements a_{mn} can be interpreted as the contribution of a given image block (pixel) with parameters n to the formation of the projection value p_m, measured at the screen.

Defined above forward model was applied to formulate, according to statistical considerations (see e.g. [4] or [5]), the following iterative reconstruction method which is based on Maximum Likelihood estimation of the reconstructed image:

$$\mu_0 = \arg\min_{\mu} \left(\frac{1}{2} (\mathbf{p} - \mathbf{A}\mu)^{\mathrm{T}} \mathbf{D} (\mathbf{p} - \mathbf{A}\mu) \right), \tag{3}$$

where $\mathbf{D}$ is a diagonal matrix:

$$\mathbf{D} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{L\cdot\Psi} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{L\cdot\Psi} \end{bmatrix}, \tag{4}$$

wherein (see [13]):

$$d_m \cong \frac{1}{\sigma_{p_m}}, \tag{5}$$

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where σ_{p_m} are the variances of the projection measurements p_m.

It is worth noting that formula (3) represents a Weighted Least Squares (WLS) problem and it can be solved using any gradient descent method. However, the huge number of a_{mn} coefficients in this model means that it is impossible to keep all of them in memory at the same time and the requirement for the simultaneous calculation of all voxels in the range of the reconstructed 3D image make the reconstruction problem extremely complex. Moreover, the computational complexity of the problem is approximately proportional to J^2, where J is the number of voxels in the reconstructed 3D image, and the iterative reconstruction procedure based on this conception necessitates simultaneous calculations for all the voxels in the range of this image. For a 3D geometry of the scanner (e.g. spiral cone-beam geometry), it means that the reconstruction for all $J = I^2 \times Z$ voxels is performed simultaneously, where Z is a number of examined cross-sections of a body. Therefore, the computational complexity of this approach is evaluated as $O\left(Z^2 I^4\right)$. The diagram of a basic form of this reconstruction algorithm is depicted in Fig. 2.

It is well-known that the form of the ML methodology expressed by (3) is ill-conditioned and, as described in the literature [5], is unstable in practice. That is why regularizing *a priori* terms are standardly introduced into the loss function. On the other hand, these additional terms cause an increase in the calculation demands during the optimization process, and lead to smoothing of the reconstructed image. It would be very appealing to use a reconstruction methodology based on a pure ML scheme, without any regularizing *a priori* term and so avoid these instabilities of the reconstruction process and the smoothing effect. It was proposed a modification of the loss function (3), in the following manner:

$$\mu_0 = \arg\min_{\mu} \left(\frac{1}{2} \left(\mathbf{p} - \mathbf{A}\mu\right)^{\mathrm{T}} \mathbf{D} \left(\mathbf{p} - \mathbf{A}\mu\right) \right) + \beta f\left(\mu\right), \tag{6}$$

where $f\left(\mu\right)$ is some scalar regularization term, whose introduction has the aim of penalizing local differences between elements of the reconstructed image; β is a constant coefficient. This regularization term may take different forms, however, an interesting approach to this method is the total variation (TV) regularization [14].

It is possible to implement this approach in practice, mainly thanks to the attempts to decrease the calculation complexity of this approach (for details of the ICD algorithm see [8]). Consequently, in 2013, this development had its debut under its commercial name Veo - CT Model-Based Iterative Reconstruction (GE Medical Systems).

2.2 A statistical approach based on the continuous-to-continuous data model

Our reconstruction method also is based on the well-known maximum-likelihood (ML) estimation, where an optimization formula is consistent with the C-C data model, as follows:

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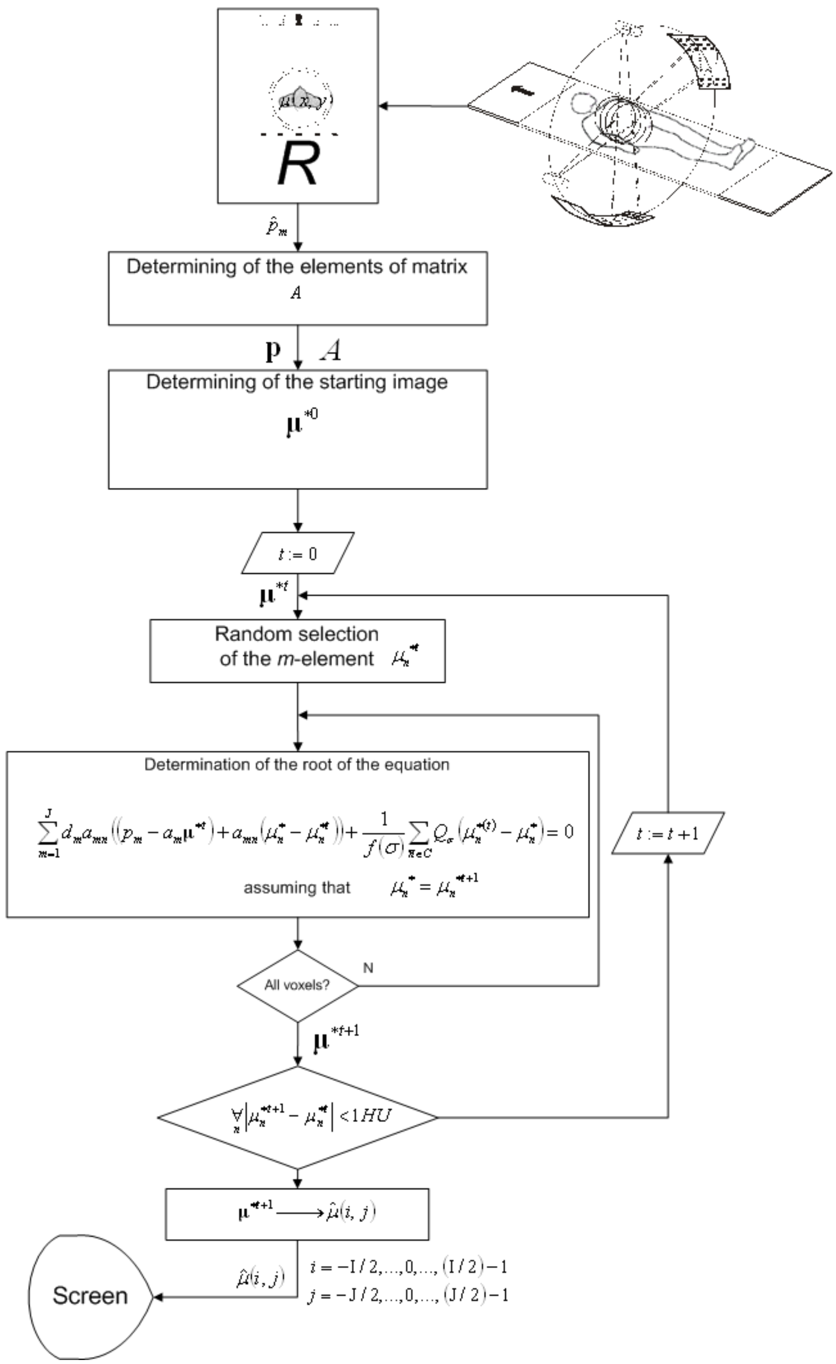

Fig. 2. The diagram of the approach based on the D-D data model

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$$\mu_{\min} = \arg\min_{\mu} \left(\int_x \int_y \left(\int_{\bar{x}} \int_{\bar{y}} \mu\left(\bar{x}, \bar{y}\right) \cdot h_{\Delta x, \Delta y} d\bar{x} d\bar{y} - \tilde{\mu}\left(x, y\right) \right)^2 dxdy \right), \quad (7)$$

where $\tilde{\mu}(x,y)$ depicts an image obtained using a back-projection operation, theoretically in the following way:

$$\tilde{\mu}\left(x, y\right) \approxeq \int_0^{2\pi} \int_{-\infty}^{\infty} p\left(s, \alpha\right) int_L\left(\Delta s\right) d\beta d\alpha, \quad (8)$$

wherein $p(s,\alpha)$ are measurements carried out using a scanner, and the coefficients $h_{\Delta i, \Delta j}$ can be precalculated according to the following relation:

$$h_{\Delta x, \Delta y} = \int_0^{2\pi} int\left(\Delta x \cos\alpha + \Delta y \sin\alpha\right) d\alpha, \quad (9)$$

and $int(\Delta s)$ is a linear interpolation function.

According to the originally formulated by us iterative approach to the reconstruction problem, described by Eqs (7)-(9), it is possible to present a practical model-based statistical method of image reconstruction, as follows:

$$\mu_{\min} = \arg\min_{\mu} \left(\sum_{i=1}^{\mathrm{I}} \sum_{j=1}^{\mathrm{J}} \left(\sum_{\bar{i}} \sum_{\bar{j}} \mu^*\left(x_{\bar{i}}, y_{\bar{j}}\right) \cdot h_{\Delta i, \Delta j} - \tilde{\mu}\left(x_i, y_j\right) \right)^2 \right), \quad (10)$$

and $\tilde{\mu}(i,j)$ is an image obtained by way of a back-projection operation, in the following way:

$$\tilde{\mu}\left(x_i, y_j\right) = \Delta_\alpha \sum_{\theta} \dot{p}\left(s_{ij}, \alpha_\psi\right). \quad (11)$$

It is necessary to use an interpolation to evaluate projections at points s_{ij} based on the measured projections $p(s_l, \alpha_\psi)$. We can obtain an approximations of these projections as follows:

$$\dot{p}\left(s_{ij}, \alpha_\psi\right) = \sum_l p\left(s_l, \alpha_\psi\right) int\left(s_{ij} - l\Delta_s\right), \quad (12)$$

where $int(\Delta s)$ is the interpolation functions, i.e. in the simplest case, linear interpolations:

$$int\left(s\right) = \begin{cases} \frac{1}{\Delta_s}\left(1 - \frac{|s|}{\Delta_s}\right) & \text{for } |s| \leq \Delta_s \\ 0 & \text{for } |s| \geq \Delta_s \end{cases}. \quad (13)$$

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In turn, the coefficients $h_{\Delta i,\Delta j}$ are determined according to the following formula:

$$h_{\Delta i,\Delta j} = \Delta_\alpha \sum_{\psi=0}^{\Psi-1} int\left(\Delta i \cos\psi\Delta_\alpha + \Delta j \sin\psi\Delta_\alpha\right), \quad (14)$$

wherein $int\left(\Delta s\right)$ is the same interpolation function as was used in the back-projection operation.

Same as before, the optimization problem (10) can be solved using any gradient descent method. Basically, in this case, the computational complexity of this problem is approximately proportional to J^2, where $J = I^2$ is the number of pixels in the reconstructed 2D image, where I is the image resolution, and the iterative reconstruction procedure based on this conception necessitates simultaneous calculations for all the pixels in the reconstructed 2D image, despite the geometry of a scanner. However, a shift-invariant system in the optimization problem (10) means that it is possible to transpose the most demanding computations into a frequency domain.

Therefore, it is necessary to transform two times a processed vector into a frequency domain, decreasing the computational complexity of the convolution from $O\left(I^4\right)$ to $O\left(I^2\right)$. Of course, each FFT costs $O\left(2log_2 I^2\right)$ operations, and we have to invert this transform every time. In total, that gives $O\left(8log_2 4I^2\right)$ operations per one iteration of the iterative reconstruction procedure (dimension of the image has to be doubled for the FFT processing). Figure 3 depicts this algorithm after discretization and implementation of the FFT that significantly accelerates the calculations.

Actually, this statistical reconstruction method consists of two steps, namely: a back-projection operation described by relation (8) and an iterative reconstruction procedure according to formula (7). In this case, the back-projection operation is not computationally demanding because there is no filtration during this operation, and it has a marginal influence on the real reconstruction time.

Although we have to establish certain coefficients in our method, this can be performed more easily than in a referential approach and the matrix containing these coefficients has relatively small dimensions, thus allowing it to be precalculated. Moreover, these coefficients can be transformed into the frequency domain and saved in memory in this form for further processing. In should be noted that this system is much better conditioned than the WLS problem present in the referential approach.

3 Experimental results

We divided our experiments into two phases: first, we will try to show that the C-C data model gives better quality of the reconstructed images, and then, we will perform original tomographic data to evaluate both approaches regarding the time of calculations.

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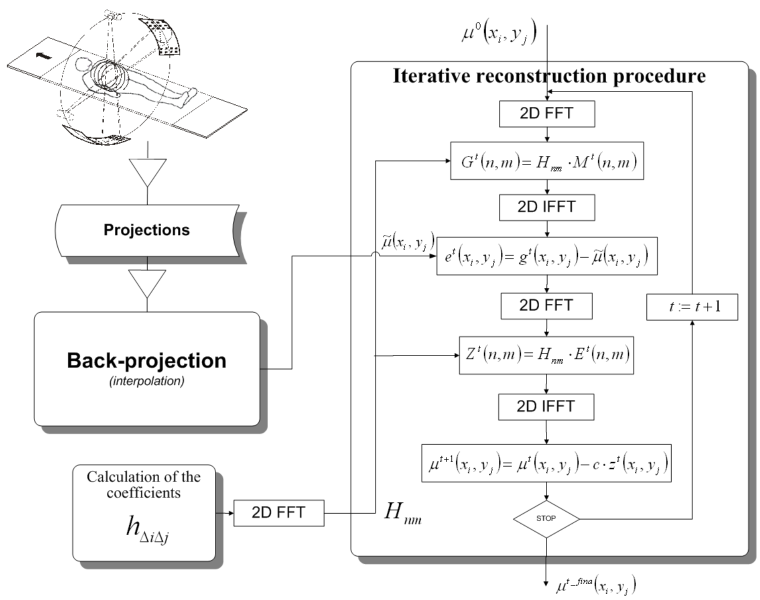

Fig. 3. The diagram of the approach based on the C-C data model

In this phase of our experiments, we have adapted the FORBILD[1], a mathematical phantom of the head. All the values of the attenuation coefficients placed in the original model were divided by a factor 10^{-3} in order to facilitate the calculations. This model was used to generate projections with noise with a Poisson probability distribution. During the simulations, we fixed L = 1024 measurement points (detectors) on the screen. The number of projections was chosen as $\Psi = 3220$ rotation angles per full-rotation and the size of the processed image was fixed at $\mathrm{I} \times \mathrm{I} = 1024 \times 1024$ pixels.

It was convenient to establish coefficients $h_{\Delta i,\Delta j}$ using relation (9) before we started the reconstruction process and these coefficients were fixed for the subsequent processing.

Having obtained the coefficients $h_{\Delta i,\Delta j}$, we can start the actual reconstruction procedure and perform the back-projection operation using relationships (8) to get a blurred image of the x-ray attenuation distribution in a given cross-section of the investigated object. We must use the linear interpolation function.

Evaluating a reconstruction procedure based only on a view of the reconstructed image is very subjective. For that reason the quality of the reconstructed image has been evaluated by an error measure defined as follows

$$MSE = \frac{1}{I^2} \sum_{i=1}^{\mathrm{I}} \sum_{j=1}^{\mathrm{J}} \left(\mu^{(t)}\left(i,j\right) - \mu\left(i,j\right) \right)^2 , \tag{15}$$

[1] http://www.imp.uni-erlangen.de/forbild/deutsch/results/head/head.html

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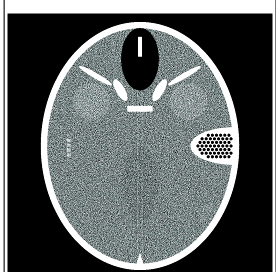
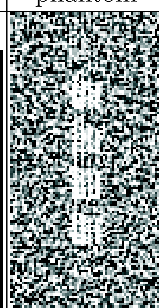
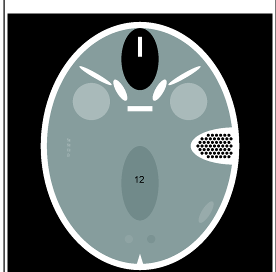
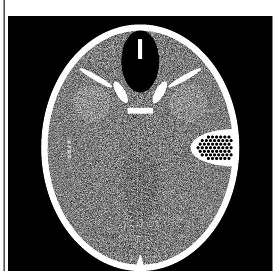
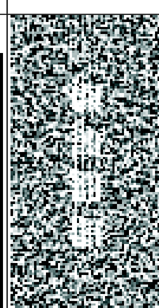
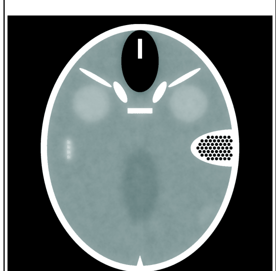
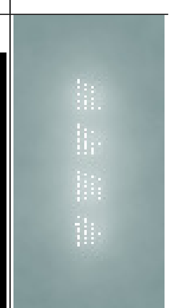
where: $\mu^{*(t)}(i,j)$ is the reconstructed image after t iterations and $\mu(i,j)$ is the original image of the FORBILD phantom.

At this time, we have taken into account one form of regularization: the total variation (TV) prior [14]. The result obtained are shown in Table 1 (for all reconstructions, the starting image is a flat image $\mu^{*0} = 0.005$). For comparison, the original phantom image (Table 1.B), the image reconstructed by a standard FBP reconstruction method (Table 1.A) and by the referential ICD algorithm (Table 1.C) are also presented.

Table 1. Views of the images: reconstructed image using the standard FBP method with Shepp-Logan kernel (A); original image (B); reconstructed image using the D-D method described in the paper [8] (after 15 iterations) (C); reconstructed image using the C-C method described in this paper (after $t = 10^3$ iterations) (D).

	Image	Resolution phantom		Image	Resolution phantom
A	 $MSE = 4.31 \cdot 10^{-9}$		E		
B	 $MSE = 3.37 \cdot 10^{-9}$		F	 $MSE = 3.86 \cdot 10^{-9}$	

Experiments in the next phase were carried out using projections obtained from a Somatom Definition AS+ scanner with parameters, as follows: reference tube potential at the level 120kVp, quality reference effective at the level 200mAs.

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The size of the reconstructed image was fixed at 512×512 pixels. A discrete representation of the matrix $h_{\Delta x,\Delta y}$ was established in a computational way before the reconstruction process was started. These coefficients were fixed (transformed into the frequency domain) and used for the whole iterative reconstruction procedure. A result of an FBP reconstruction method was chosen as the starting point of the iterative reconstruction process.

A crucial parameter for the practical implementation of a reconstruction method is the actual computation time of the reconstruction procedure. We have implemented our iterative reconstruction procedure using some hardware configurations, namely: a computer with 10 cores, (Intel i9-7900X BOX/3800MHz processor), using different GPUs (see 3). It is worth noting that our iterative procedure was implemented at assembler level. In Table 2, we show time result for application which is working only on CPU which is develop in Assembler (special vector registers AVX 512 used). In turn, in table 3, we present time result for application which is working only on GPU accelerators. There are compared those accelerators. It is worth noting that it is very stable time, because deviation is extremely small and that application is very susceptible to parallelisation, because time for one iteration it is getting smaller with on more CUDA Cores assembled in GPU Accelerator.

Table 2. Results of reconstruction on multi threading, i.e. CPU Intel i9-7900X (10-cores, 20-treads).

Threads:	4	8	10	16	20
Avg. time 30000[ms]	63 724	33 571	29 836	30 532	27 905
Avg. time 20000[ms]	42 482	22 380	19 890	20 354	18 603
Avg. time 10000[ms]:	21 241	11 190	9 945	10 177	9 301
Time 1 iteration [ms]:	2,1241	1,1190	0,9945	1,0177	0,9301
HT effectiveness:	-	-	-	0,9094	0,9352
Median for 30000:	63 694	33 542	29 800	30 566	27 854
Deviation std.:	135,69	117,32	217,58	193,88	391,76

Table 3. Results of reconstruction on different models of GPUs accelerator.

GPU:	MSI GTX 1050	ASUS GTX 1080 Ti	nVidia Titan V
Avg. time 30000[ms]	2 562 175,10	49 699,71	28 858,40
Avg. time 20000[ms]	170 845,28	33 132,52	19 224,48
Avg. time 10000[ms]:	85 467,24	16 593,00	9 616,75
Time 1 iteration [ms]:	8,540583	1,656657	0,961947
Median for 30000:	256 229,55	49 703,68	28 861,24
Deviation std.:	0,160806	0,310476	0,010239

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According to an assessment of the quality of the obtained images by a radiologist, 7000 iterations are enough to provide an acceptable image for medical purposes. One can compare the results obtained by assessing the view of the reconstructed image in Figure 4, where where the quarter-dose projections were used.

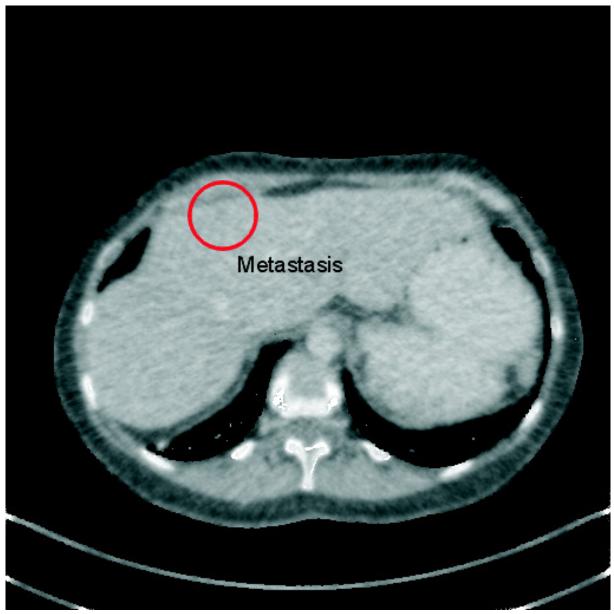

Fig. 4. Obtained image (a case with relative small pathological change in the liver) using quarter-dose projections with application of the statistical method presented in this paper.

4 Conclusion

Comprehensive experiments have been performed, which prove that our reconstruction method is relatively fast (thanks to the use of FFT algorithms) and gives satisfactory results with suppressed noise. It should be noted that approximately the same results were achieved for both hardware implementations: the iterative reconstruction procedure takes less than 7s, mainly thanks to the use of an FFT algorithm in the iterative reconstruction procedure and to the use of the efficient programming techniques. These are rewarding results regarding possibilities of the commercial Veo system (referential MBIR technique), where reconstruction times range between 10 to 90 minutes depending on the number

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of reconstructed slices [15]. It means an unacceptable delay between data acquisition and availability for interpretation for emergent indications. Additionally, the hardware used by us is relatively cheap (about 5000 USD) compared to the price of the equipment necessary for the referential solution. It should be emphasized that the designed by us statistical approach (formerly formulated for CT scanner with parallel beam geometry) can be adapted for helical scanners with various geometries, e.g. with cone-beams or with x-tube with flying focal spot.

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