Private and public opinions in a model based on the total dissonance function: A simulation study*

Michał Jarema[1][0000−0002−4165−9851] and
Katarzyna Sznajd–Weron[2][0000−0002−1851−8508]

[1] Department of Operations Research and Business Intelligence
[2] Department of Theoretical Physics
Wrocław University of Science and Technology, 50-370 Wrocław, Poland
{michal.jarema,katarzyna.weron}@pwr.edu.pl

Abstract. We study an agent-based model of opinion dynamics in which an agent's private opinion may differ significantly from that expressed publicly. The model is based on the so-called *total dissonance function*. The behavior of the system depends on the competition between the latter and social temperature. We focus on a special case of parental and peer influence on adolescents. In such a case, as the temperature rises, Monte Carlo simulations reveal a sharp transition between a state with and a state without private-public opinion discrepancy. This may have far-reaching consequences for developing marketing strategies.

Keywords: agent-based model · opinion dynamics · private and public opinion · expressed opinion · dissonance · marketing

1 Introduction

As early as the 1980s, it was noted how problematic public opinion polling can be due to *a genuine difference between people's private opinions and their public opinions* [14]. This problem was addressed also within agent-based models in several contexts, such as preference falsification [16], pluralistic ignorance [11, 20], the emperor's dilemma [5, 19], or hypocrisy [8]. Probably the first agent-based model in which agents differ in their beliefs and convictions was proposed in 2005 [5]. However, only recently increased interest in this topic has been observed [2, 6, 10, 12, 17, 21].

To the best of our knowledge, all models dealing with private-public opinion discrepancies are algorithmic, i.e., based on precise rules. However, there is another possibility to model social systems, which is based on the so-called Hamiltonian, or in social terms – the *total dissonance function* [7]. The advantage of this approach, which minimizes a certain global function, is that there is a lot of freedom in choosing how the system is updated, in contrast to models based on dynamical rules [12]. Therefore, in this paper, we propose a model with

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private and *public* (also called *expressed*) opinions based on the Hamiltonian. We study the model on a one-dimensional lattice. We are aware that such a structure is not the best one to describe a social system. However, it is very convenient to visualize temporal–spatial evolution and thus we use it as a zero-level approach. In the future, we plan to investigate the model on various heterogeneous graphs.

2 The model

The system is a chain of N agents, each of them has two binary opinions: public $S_i = \pm 1$ and private $\sigma_i = \pm 1$. We assume here only pairwise interactions, although more complex ones could also be included in the future. Therefore, the Hamiltonian of the model reads

$$\begin{aligned} H = &- J_1 \sum_{i=1}^{N} S_i S_{i+1} - J_2 \sum_{i=1}^{N} S_i S_{i+2} - K_1 \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - K_2 \sum_{i=1}^{N} \sigma_i \sigma_{i+2} \\ &- R_1 \sum_{i=1}^{N} \sigma_i S_{i+1} - R_2 \sum_{i=1}^{N} \sigma_i S_{i+2} - M_0 \sum_{i=1}^{N} \sigma_i S_i, \end{aligned} \tag{1}$$

where periodic boundary conditions are used, i.e. $S_{N+1} \equiv S_1$, $S_{N+2} \equiv S_2$, $\sigma_{N+1} \equiv \sigma_1$, $\sigma_{N+2} \equiv \sigma_2$. The nearest neighbors can be interpreted as family and the next nearest as friends, as in the first and the second social circle [9]. We introduce several restrictions on the model's parameters:

1. $M_0 > 0$, due to *cognitive dissonance*, i.e., the mental conflict that occurs when an agent's public and private opinions do not align;
2. $J_1 < 0$, agents do not agree publicly with their nearest neighbors;
3. $J_2 > 0$, agents agree publicly with their next nearest neighbors;
4. $R_1 > 0$, agents agree privately with public opinion of their nearest neighbors;
5. $R_2 = 0$, private opinion of agents is not influenced by public opinion of their next nearest neighbors;
6. $K_1 = K_2 = 0$, neighbors' private opinions are not known to agents.

Assumptions 1, 5, and 6 are realistic independently of the studied problem. Assumptions 2, 3, and 4 can be treated just as a special case, representing the relation between parents and their teenage children [1, 4].

3 Monte Carlo simulations

In this paper, we use one of the most popular Monte Carlo (MC) simulation algorithms that is used to study Hamiltonian systems, so-called Metropolis algorithm [15]. Within such an algorithm, each MC step (MCS) consists of N updates of randomly selected agents (random sequential updating). The update of agent i consists of two MC trial moves: $S_i \to -S_i$ followed by $\sigma_i \to -\sigma_i$. However, several other updating schemes were used and they all led to the same stationary state, which is the main advantage of the presented approach. The Metropolis algorithm is as follows:

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1. Prepare initial state of S_i and σ_i;
2. Until the requested number of MCS is performed, select randomly an agent and perform a trial move;
3. Calculate $\Delta E = E' - E$, where E', E are the values of H defined by Eq. (1) after and before the trial move respectively;
4. If $\Delta E \leq 0$ accept the trial move and go to step 2; otherwise:
5. Take a pseudorandom number $0 < r < 1$ from uniform distribution;
6. If $r < e^{-\Delta E/T}$, where T is the social temperature [3], accept the trial move and go to 2. Otherwise accept the old configuration again and go to 2.

To describe the limiting behavior of the system, we measure the following quantities: the correlation functions

$$g_S = \frac{1}{N}\sum_{i=1}^{N} S_i S_{i+1}, \quad g_\sigma = \frac{1}{N}\sum_{i=1}^{N} \sigma_i \sigma_{i+1}, \tag{2}$$

which measure the local order and identify the states: consensus (ferromagnetic state in physics) $g_S,\ g_\sigma = 1$, disagreement (antiferromagnetic state) $g_S,\ g_\sigma = -1$, and random (interchangeably referred to as noisy or disordered) state $g_S,\ g_\sigma = 0$.

Additionally, we measure the private-public opinion discrepancy defined as:

$$D = \frac{1}{2} - \frac{1}{2N}\sum_{i=1}^{N} S_i \sigma_i, \tag{3}$$

which takes values in the interval $[0, 1]$. When public and private opinions are the same, then the discrepancy $D = 0$. When they are opposite, then $D = 1$.

4 Results

In the lack of any noise, i.e., at $T = 0$ the system is blocked after a few iterations if $M_0 \neq R_1$. Only for $M_0 = R_1$ private opinions σ are free to change and therefore we focus here on the case $M_0 = R_1$. We introduce the parameter A

$$M_0 = R_1 = |J_1| + A, \tag{4}$$

which measures the competition between two forces – the strength of cognitive dissonance, described by M_0 (note that $M_0 > 0$, so the absolute value is not needed) and the strength of the interactions with the nearest neighbors, given by $|J_1|$. For $|J_1| \geq M_0$ (i.e. $A \leq 0$) the system evolves towards disagreement in the public opinion S and random state in the private opinion σ. For $|J_1| < M_0$ (i.e. $A > 0$) the system evolves towards consensus in both S and σ.

Since in every system there is noise, so the assumption $T = 0$ is not realistic. Therefore, from now we focus on $T > 0$. We present time–space evolution of the system for several values of T in Fig. 1: public opinions are shown in the upper panels, whereas private opinions in the bottom ones. Each row in each plot corresponds to the state of the system at a given time t: white pixels correspond

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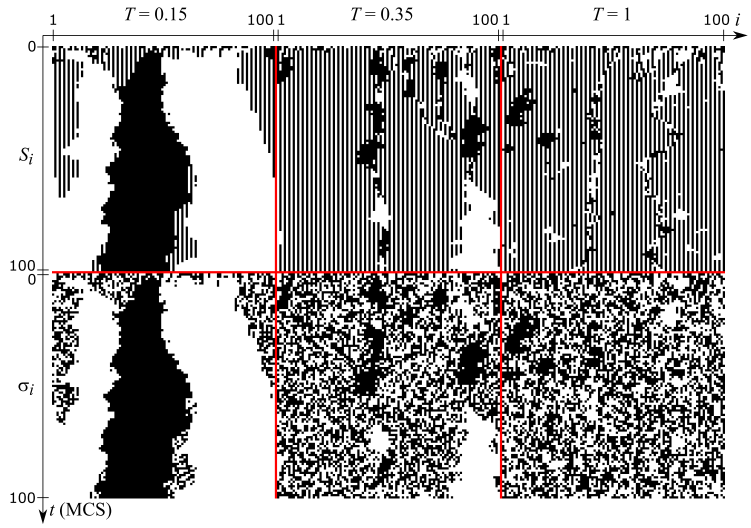

Fig. 1. Time evolution of public S_i (upper row) and private σ_i (bottom row) opinions for random initial conditions at three selected temperatures T indicated above the upper plots and $J_1 = -J_2 = -1$, $A = 0.1$.

to positive opinions, whereas the black pixels to negative ones. Therefore, white domains represent the area of positive consensus, black domains correspond to negative consensus, alternating white with black to disagreement, and randomly mixed white and black to the noisy state.

We see that for low values of T (e.g., $T = 0.15$) the system tends to have large clusters of consensus on both public and private levels. In the finite system, eventually the full consensus is reached, i.e., all agents have the same opinions. However, the time needed to reach such a consensus increases with the system size N and for $N \to \infty$ the system consists of opposite domains forever. Interestingly, these domains are interspersed with areas of disagreement at the public level of opinion and with noisy (random) areas at the private level. For larger values of T ($T = 0.35$ in Fig. 1), the system evolves towards disagreement state in the public opinions S_i and random state in the public opinions σ_i. Only thin consensus inclusions are visible at the boundary between disagreement domains. For $T = 1$ more fluctuations appear and eventually, for very large values of T, the fluctuations destroy the disagreement on the public level.

To analyze the model more systematically and quantitatively, we calculate the average values of correlation functions g_S, g_σ, private-public opinion discrepancy D for different values of T and 3 different types of initial conditions: (r) random, (c) consensus, and (d) disagreement. They are ensemble averaged over 100 configurations, each taken after 1000 MCS, which this was sufficient to reach the stationary state. Corresponding results are shown in Fig. 2.

We see that for high values of T the disordered state is reached independently on the initial conditions. On the other hand, for small T the final state depends

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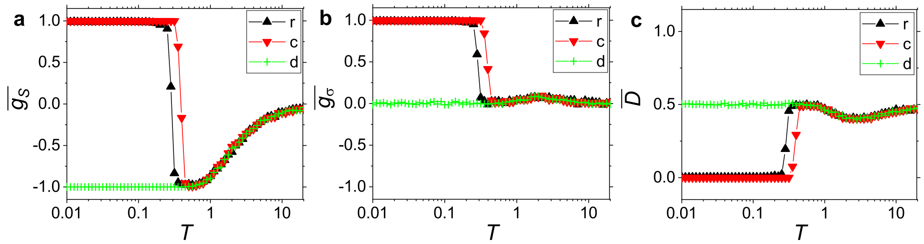

Fig. 2. The ensemble average (over 100 samples) of the correlation functions g_S, g_σ, and private–public opinion discrepancy D as a function of T after 1000 MCS for different types of initial states: r – random, c – consensus, d – disagreement. $J_1 = -J_2 = -1$, $A = 0.1$

on the initial one. The system initialized in the disagreement state at $T < 0.4$ reaches the state with $g_S = -1$, $g_\sigma = 0$ i.e. disagreement in public (S) and disordered in private (σ) opinions. Opinions on both levels are uncorrelated and therefore $D = 0.5$. For two other types of initial conditions (consensus and random), for $T < 0.3$ the system reaches consensus in both public and private opinions ($g_S = g_\sigma = 1$) with no discrepancy between them ($D = 0$). For $0.3 < T < 0.5$ the final state depends on the initial one − this phenomenon is called hysteresis. For $0.5 < T < 1$ we have $g_S = -1$, $g_\sigma = 0$ in the final state, i.e., disagreement is reached on the public level and a disordered state on the private one. To summarize, there are two transitions in the public opinion S: one sharp at $T = T_1$, between consensus and disagreement, and one smooth at $T = T_2$, between disagreement and disorder. In σ there is only one sharp transition at T_1 between consensus and disorder.

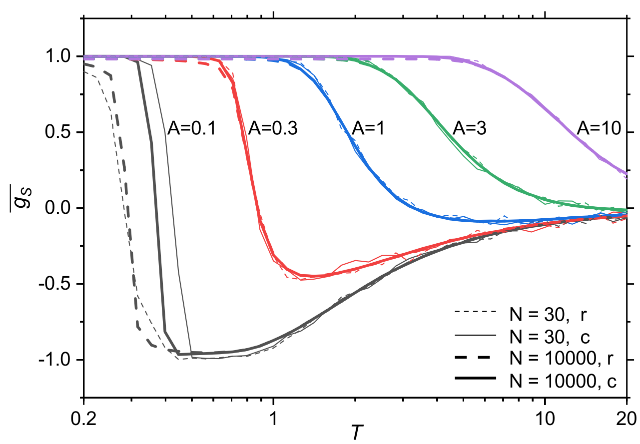

Fig. 3. Dependence of the $\overline{g_S}$ function on the size N of the system and on the type of initialization: r - random, c - consensus.

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Values of T_1, T_2 depends on parameter A, i.e., on the interplay between internal harmony and interactions with others, as shown in Fig. 3. There are two other phenomena shown in Fig. 3. Firstly, the hysteresis exists only for $A = 0.1$ (therefore results for this value are presented in Figs. 1 and 2) and its width decreases with increasing system size. Secondly, apart from hysteresis shrinkage, the results for small ($N = 30$) and large ($N = 10^4$) systems are the same.

5 Conclusions

In this short paper, we conduct Monte Carlo simulations of a relatively simple agent-based model to study opinion formation among parents and their teenage children. The huge spending power and significant influence on family purchase decisions makes adolescents a critical consumer segment. Parents, peers, and media are the three essential factors influencing adolescents' consumer attitudes and purchase behavior [18].

Although we focus on one specific case, the model itself is much more general and allows us to describe many phenomena in which discrepancies between private and public opinions may arise. The novelty with respect to the previous literature on private–public opinion discrepancy is that the model is built around the total dissonance function. This makes it insensitive to the updating scheme, as opposed to many other agent-based models [12].

More specifically, we show that as a result of the competition between different types of interactions (with parents, peers, own private–public opinions) and social temperature T, three interesting phenomena can be observed. Firstly, there is a critical temperature $T = T_1$, below which all agents have the same private and public opinion, so there are no private–public opinion discrepancies. For $T > T_1$ on average half of the agents have different public and private opinions.

Secondly, there is a threshold $T = T_2$, which concerns only the public opinion. For $T_1 < T < T_2$ a disagreement phase appears on the public level: each agent disagrees with the nearest neighbors (interpreted as members of the first social circle – the family), but agrees with the next nearest neighbors (interpreted as the second social circle – peers). Interestingly, for $T_1 < T < T_2$ there is no such phase on the private level – private opinions are just random, which can be interpreted as independent behaviour (not influenced by others). This leads to the third observation: on the private level agents are independent already for $T > T_1$, whereas on the public level the range of independence is smaller and starts at $T = T_2 > T_1$. Identifying and studying such regimes may help marketers formulate appropriate marketing communication strategies that can be effective in resolving parent–child purchase disagreements [13].

Of course, we realize that the one-dimensional case considered here is not very realistic from the social point of view. However, the model based on the total dissonance function can be easily studied on any other type of graph, including actual social networks. This, however, is left for future studies.

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