The use of type-2 fuzzy sets to assess delays in the implementation of the daily operation plan for the operating theatre

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Abstract. In the paper we present a critical time analysis of the project, in which there is a risk of delay in commencing project activities. We assume that activity times are type-2 fuzzy numbers. When experts estimate shapes of membership functions of times of activities, they take into account both situations when particular activities of the project start on time and situations when they start with a delay. We also suggest a method of a sensitivity analysis of these delays to meeting the project deadline. We present a case study in which the critical tome analysis was used to analyse processes implemented in the operating ward of a selected hospital in the South of Poland. Data for the empirical study was collected in the operating theatre of this hospital. This made it possible to identify non-procedural activities at the operating ward that have a significant impact on the duration of the entire operating process. In the hospital selected for testing implementation of the daily plan of surgeries was at risk every day. The research shows that the expected delay in performing the typical daily plan - two surgeries in one operating room – could be about 1 hour. That may result in significant costs of overtime. Additionally, the consequence may also include extension of the queue of patients waiting for their surgeries. We show that elimination of occurrence of surgery activity delays allows for execution of the typical daily plan of surgeries within a working day in the studied hospital.

Keywords: Operating Theatre, Project Critical Time, Scheduling, Type-2 Fuzzy Number.

1 Introduction

The literature has numerous proposals as to the models of optimization of schedules on operating wards in hospitals. A review of the methods may be found in the work by Gür and Eren [1]. Here, we will present models in which authors used the fuzzy sets

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theory to define the schedules. Al-Refaie et. al. [2] propose multiple-period fuzzy optimization models for scheduling and sequencing of patients in operating theatres. In their model, the surgery time is deterministic. The goals are fuzzy, the criterion functions are: minimization of undertime and overtime and maximization of patients' satisfaction. Dexter et. al. [3] proposes an algorithm to schedule add-on elective cases that maximizes operating room suite utilization. Surgery times are given as real numbers. In the model they use the fuzzy constrain theory. Nasiri et. al. [4] consider three-criterion surgery scheduling problem. The criteria are maximizing the number of surgeries that can be done using given fixed resources, minimizing the total fixed costs and overtime costs and minimizing the cost of completion time. They assume stochastic times of surgeries and fuzzy constraints for resources and overtime. Rachuba and Werners [5] propose algorithms of determining robust schedules where they assume stochastic surgery times and randomly arriving emergency demand. For target functions they assume a degree of satisfaction of patients with a short time of waiting for their surgeries, minimization of rejected requests and minimization of overtime. Gűl et. al. [6] for the time analysis of flow in an emergency room apply the fuzzy CPM and fuzzy PERT. Times of activities are described here as type-1 fuzzy numbers. Nazif and Makis [7] solve the problem of determining times for commencing particular surgeries in the operating ward, taking into account availability of resources. The scheduling issue is modelled as a fuzzy flexible flow shop scheduling problem, assuming that times of activities are type-1 fuzzy variables. To determine the schedules they use simulation and heuristics methods (Ant colony optimization algorithms). Lahijanian [8] proposes a mixed-integer programming model for scheduling operating theatres. Surgery times are given as triangular type-1 fuzzy numbers. The target function is to minimize the total weighted time. Wang and Xu [9] use the hybrid intelligent algorithm to determine the schedule of surgeries with limited resources. Times of surgeries are given as type-1 fuzzy variables. As the criterion function they assumed minimization of overtime and undertime costs. Behmanesh, Zandieh and Hadji Molana [10] as target functions adopt minimization of makespain, Behmanesh and Zandieh [11] used minimization of makespain and minimization of the number of unserved patients. Times of surgeries are type-1 fuzzy numbers. The optimal solution is determined with the Fuzzy Pareto Envelope-based Selection Ant System algorithm.

In this work we present a method of time analysis of a project, in which there is a risk of delay in starting project activities. We also suggest a method of analysing sensitivity of these delays to meeting the project deadline. We assume that activity times are given as type-2 fuzzy numbers. When experts estimate shapes of membership functions of times of activities, they take into account both situations when particular activities of the project start on time, and situations when they start with a delay. Therefore, this method of estimating fuzzy times of particular phases of a surgery indirectly takes into account availability and readiness of human resources. We are also presenting a case study in which we implement the critical time analysis method to the time analysis of the process of surgery in an operating ward of a hospital in Poland.

The paper is organized as follows. Next Section presents main terms from the fuzzy theory. In Section 3 we present a method of time analysis of a project, in which there is a risk of delay in starting project activities. We assume that activity times are type-2

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fuzzy numbers. We also suggest a method of sensitivity analysis of these delays to meeting the project deadline. In Section 4 we present a case study in which our method was used to analyse processes implemented in the operating ward of a hospital in Poland. Sections 5 and section 6 are Discussion and Conclusions.

2 Basic notions

In this section we are presenting a basic notion from the fuzzy sets theory.

An interval number $\bar{A}$ is an closed interval $\bar{A} = [\underline{a}, \overline{a}] = \{x \in \Re: \underline{a} \leq x \leq \overline{a}\}$. Values $\underline{a} = -\infty$ and $\overline{a} = +\infty$ are allowed. The interval number $\bar{A} = [\underline{a}, \overline{a}]$, means unknown realization x which may take values from the interval$[\underline{a}, \overline{a}]$.

Let $\bar{A} = [\underline{a}, \overline{a}]$ and $\bar{B} = [\underline{b}, \overline{b}]$ be two interval numbers. The sum of the two interval numbers $\bar{A} = [\underline{a}, \overline{a}]$ and $\bar{B} = [\underline{b}, \overline{b}]$ is the interval number of the form $\bar{A} + \bar{B} = [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$. Maximum of two interval numbers $\bar{A} = [\underline{a}, \overline{a}]$ and $\bar{B} = [\underline{b}, \overline{b}]$ is the interval number of the form $max\{\bar{A}, \bar{B}\} = [max\{\underline{a}, \underline{b}\}, max\{\overline{a}, \overline{b}\}]$ [12]. The degree to which the number $\bar{A}$ is greater than the number $\bar{B}$ is defined as follows:

$$degree(\bar{A} \geq \bar{B}) = |\{x: x \in \bar{A} \; and \wedge \; y \in \bar{B}, x \geq y\}| / |\bar{A}| \tag{1}$$

In 1965 Zadeh proposed his concept of possibility theory [13] We will present the basic notions of this theory. First, we will present the concept of a fuzzy number (type-1 fuzzy number). Let $\tilde{X}$ be a single valued variable whose value is not precisely known. The membership for $\tilde{X}$ is a normal, quasi concave and upper semi continuous function $\mu_X: \mathcal{R} \to [0,1]$, see [14], [15]. The value $\mu_X(x)$ for $x \in \mathcal{R}$ denotes the possibility of the event that the fuzzy variable $\tilde{X}$ takes the value of x. We denote this as follows $\mu(x) = Pos(\tilde{X} = x)$. For a given fuzzy number $\tilde{X}$ and a given λ, the λ-level is defined to be the closed interval $[\tilde{X}]_\lambda = \{x: \mu(x) \geq \lambda\} = [\underline{x}(\lambda), \overline{x}(\lambda)]$.

An interval type-1 fuzzy number $\tilde{X}$ is called an $L - R$ fuzzy number if its membership function takes the form of [14]:

$$\mu_X(x) = \begin{cases} L\left(\frac{\mathrm{m}-x}{\alpha}\right) & for \quad x < \underline{m} \\ \mu_m & for \quad \underline{m} \leq x \leq \overline{m} \\ R\left(\frac{x-\overline{\mathrm{m}}}{\beta}\right) & for \quad x > \overline{m} \end{cases} \tag{2}$$

where: $L(x)$, $R(x)$ - continuous non-increasing functions x; $\alpha, \beta > 0$.

Functions $L(x)$. $R(x)$ are called shape functions of a fuzzy number. The most commonly used shape functions are: $\max\{0, 1 - x^p\}$ and $\exp(-x^p)$, $x \in [0, +\infty)$, $p \geq 1$. An interval-valued fuzzy number for which $L(x)$= $R(x) = \max\{0, 1 - x^p\}$ and $p = 1$ is called a trapezoid fuzzy number, which we denote as $(\underline{x}, \underline{m}, \overline{m}, \overline{x})$. A trapezoid fuzzy number for which $\underline{m} = \overline{m} = m$ is called a triangular fuzzy number. A type-2 fuzzy set (T2FS) $\tilde{A} \in \mathcal{F}_2(\mathrm{X})$ is an ordered pair $\tilde{A} = \{(x, u), J_x, f_x(u)/x \in X; u \in J_x \subseteq [0,1]$, where $\tilde{A}$ represents uncertainty around the word A, J_x the *primary membership* function of x, u is the domain of uncertainty, and $\mathcal{F}_2(\mathrm{X})$ is a class of type-2 fuzzy sets [16, 17]:

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$$\tilde{A}: X \to [0,1]$$
$$\tilde{A} = \int_{x\epsilon X} \int_{u\epsilon J_x} 1/(x,u) \quad , J_{x\epsilon[0,1]} \tag{3}$$

An interval type-2 fuzzy number is a simplification of a T2FS. Its secondary membership function is assumed to be 1:

$$\tilde{A} = \int_{x\epsilon X} \int_{u\epsilon J_x} 1/(x,u) = \int_{x\epsilon X} \left[\int_{u\epsilon J_x} 1/u \right]/x \,, \tag{4}$$

where x, u are primary and secondary variables, and $f_x(u)/u = 1$ is the secondary membership function.

The footprint of uncertainty of the interval type-2 fuzzy number $\tilde{A}$ is bound by two functions: an upper membership function and lower membership function. For trapezoid type-2 fuzzy number we will use the following notion $\tilde{X} = \left(\left(\underline{x}^U, \underline{m}^U, \overline{m}^U, \overline{x}^U ; \mu_m^U\right), \left(\underline{x}^L, \underline{m}^L, \overline{m}^L, \overline{x}^L ; \mu_m^L\right)\right)$, where both upper and lower memberships functions have the shapes given by formula (2). When upper and lower membership functions are the same (equal to each other) type-2 fuzzy number is type-1 fuzzy number.

In arithmetic of fuzzy numbers we will apply Zadeh's extension principle [13] $\mu_Z(z) = \sup\limits_{z=f(x_1,\dots,x_n)} min\{\mu_{X_1}(x_1), \dots, \mu_{X_n}(x_n)\}$ extended to type-2 fuzzy numbers [18]:

$$\mu_Z(z) = \left(\left(\sup_{z=f(x_1,\dots,x_n)} min\{\mu_{X_1}^U(x_1), \dots, \mu_{X_n}^U(x_n)\}\right), \left(\sup_{z=f(x_1,\dots,x_n)} min\{\mu_{X_1}^L(x_1), \dots, \mu_{X_n}^L(x_n)\}\right)\right) \tag{5}$$

The interval possibility that the realization of type-2 fuzzy number $\tilde{X}$ will be greater or equal to the realization of type-2 fuzzy number $\tilde{Y}$ is equal [19]:

$$\overline{Pos(\tilde{X} \geq \tilde{Y})} = \left[sup_{x\geq y}\left(min\left(\mu_X^L(x), \mu_Y^L(y)\right)\right), sup_{x\geq y}\left(min\left(\mu_X^U(x), \mu_Y^U(y)\right)\right)\right] \tag{6}$$

Those index is an extension of index of relations of majority of those proposed by Dubois and Prade [14].
The interval expected value of type-2 fuzzy variable is [19]:

$$\overline{E(\tilde{X})} = \left[min\left\{\int_0^{\mu_m^L} \frac{1}{2}\left(\underline{x}^L(\lambda) + \overline{x}^L(\lambda)\right) d\lambda, \int_0^{\mu_m^U} \frac{1}{2}\left(\underline{x}^U(\lambda) + \overline{x}^U(\lambda)\right) d\lambda\right\}, max\left\{\int_0^{\mu_m^L} \frac{1}{2}\left(\underline{x}^L(\lambda) + \overline{x}^L(\lambda)\right) d\lambda, \int_0^{\mu_m^U} \frac{1}{2}\left(\underline{x}^U(\lambda) + \overline{x}^U(\lambda)\right) d\lambda\right\}\right] \tag{7}$$

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3 Method of time analysis of the project

In this section we present a method of time analysis of a project, in which there is a risk of delay in starting project activities. We also suggest a method of the sensitivity analysis of these delays to meeting the project deadline.

Let a project be represented as an acyclic network $G(N, \mathcal{A}, \tilde{T})$, where $N = \{1, \dots, n\}$ is the set of nodes (events), $\mathcal{A} \subset N \times N$ is the set of arcs (activities), and $\tilde{T}: A \to \mathcal{F}^+$ – a function representing the type-2 fuzzy durations of these activities. For each activities $(i,j) \in \mathcal{A}$, the experts determine the optimistic duration t_{ij}^{opt}, the most possible duration t_{ij} and the pessimistic one t_{ij}^{pes} for stable (normal, most typical, scenarios) conditions (circumstances) for the realization of a project. The experts also judge which activities could start later because of poor organizational reason and determine the optimistic duration τ_{ij}^{opt}, the most possible duration τ_{ij} and the pessimistic one τ_{ij}^{pes} for such conditions and give the possibility μ_{ij} of occurrence of the most possible lateness τ_{ij}. Based on that data we estimate the type-2 fuzzy membership function of time of realization of those activities. Upper membership function represents a possibility that time of realization of activities will be realized in standard conditions, namely in conditions which may entail a lateness in commencement of activity realization (i,j). Lower membership function defines a possibility of activity realization (i,j) in optimal conditions, namely in conditions where an activity commences on time.

Times of realization of activities for which there is no risk that activities will not be commenced on time we estimate with trapezoid type-2 fuzzy numbers in a form of:

$$\tilde{T}_{ij} = \left(\left(t_{ij}^{opt}, t_{ij}, t_{ij}\, t_{ij}^{pes};\ 1\right), \left(\tau_{ij}^{opt}, \tau_{ij}, \tau_{ij}, \tau_{ij}^{pes};\ \mu_{ij}\right)\right) \tag{8}$$

Let assume that we have due date d for our project. We will now present a critical time analysis method where we will apply arithmetic based on the Zadeh's extension principle (4) and modified interval index of majority relation (5).

Algorithm 1

Step 1. Number nodes of network (events of project) $i \in N$ ascendingly starting with the initial node: $i = 1, 2, \dots, n$.

Step 2. Set $\tilde{T}_1 = 0$.

Step 3. For $i = 2, \dots, n$:

Find the earliest time of the event i implying Zadeh's extension principle (5[1]):

$$\tilde{T}_i = \max_{(j,i) \in P_i} \tilde{T}_j + \tilde{T}_{ji}$$

where P_i – the set of predecessors of i.

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[1] In order to find the sum and the maximum of interval fuzzy numbers one may apply the rule of dividing a part [0, 1] of the value of membership functions into intervals and to find approximate value of the maximum, applying interval arithmetic for those intervals.

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Step 4. Find the critical time $\tilde{T}^{crit} = \tilde{T}_n$.

Step 5. Find using equations (6) interval possibility that due date d is greater or equal than critical time $\overline{Pos\left(d \geq \tilde{T}^{crit}\right)}$.

Step 6. Using the Zadeh's extension principle (5) and equation (7), find project lateness $\tilde{L} = \tilde{T}_{crit} - d$ and the interval expected value of the project lateness $\overline{E(\tilde{L})}$.

We will now propose the method of analysing the impact of particular delays of activities on the total project time.

Sensitivity analysis

For a given activity $(i, j) \in \mathcal{A}$, for which there is a risk of lateness of its commencement, assume that this lateness will not occur, so the time of realization of that activity is in a form of:

$$\tilde{T}_{ij} = \left(\left(t_{ij}^{opt}, m_{ij}, m_{ij}, \ t_{ij}^{pes}; \ 1 \right), \left(t_{ij}^{opt}, m_{ij}, m_{ij} \ t_{ij}^{pes}; \ 1 \right) \right) \tag{9}$$

Step 1. Find the critical time $\tilde{T}_{crit}$ of the project using Algorithm 1.

Step 2. Find using equation (5) interval possibility that due date d is greater or equal than critical time $\overline{Pos_{Int}\left(d \geq \ \tilde{T}_{crit}\right)}$.

Step 3. Using the Zadeh's extension principle (5) and equation (7), find project lateness $\tilde{L} = \tilde{T}_{crit} - d$ and the interval expected value of project lateness $\overline{E(\tilde{L})}$.

It should be emphasized that in the arithmetic applied in the algorithm based on the Zadeh's extension principle the maximum value (lower, upper) of the membership function of the critical time is equal to the minimum value of membership functions of particular times of the project activities. Instead of the Zadeh's arithmetic, one may apply another arithmetic for type-2 fuzzy numbers, e.g. arithmetic proposed by Hu et. al. [20]. Then, for projects composed of numerous activities the maximum value (lower, upper) of the membership function of the project critical time will equal approximately 1.

4 Case study

In this section we will apply Algorithm 1 to analyse the time of a surgery in an operating ward. The research was conducted in a large hospital in the south of Poland. There, the operating theater is a separate department and consists of 10 operating theaters. In the subject unit of a selected hospital, the main problem regarded untimely realization of the operating schedule. As shown in the analysis, that was caused with occurring of latenesses in commencement of subsequent activities of the process occurring in the operating ward.

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In the selected hospital work in the operating ward was planned one day ahead. The daily schedule of surgeries is established based on the queue of patients awaiting surgeries and expert knowledge of a person authorized to prepare the schedule. It contains the sequence of surgeries and numbers of operating theatres assigned to given surgeries. It does not define the expected duration of the surgery as it is difficult to determine. However, the person preparing the schedule consults experts (surgeon - operator) in order to determine estimated duration of a given surgery. This knowledge to a large extent facilitates planning of work in each operating theatre.

The operating ward is a specific organizational unit of a hospital whose structure is very complex and at the same time dependent on numerous aspects of work of the whole hospital. The analysed process of performing surgeries composes of many activities that may be divided into three groups: pre-surgery activities, the surgery and post-surgery activities. They take into account both the place of performing the surgery and the required medical personnel taking part in a given activity. Medical personnel creates a team. The team composes of various professional groups, among others those are nurses, anaesthetists or operators of various specializations, and supporting personnel such as e.g. cleaner's staff. For the process to run smoothly it is necessary that all people involved in the process cooperate closely. It is also necessary to coordinate efficiently and plan work of the whole team. Each lateness in realization of an activity results in extension of the duration of the whole process, and latenesses in realization of particular processes have consequences in a form of failure to realize the daily schedule of surgeries. Correct planning of that schedule has very large impact on problems with realization of the planned surgeries. It has to take into account the estimated duration of each surgery. That duration is defined based on the type of surgery based on expert knowledge of the ward head and operators.

The subject hospital operating theatres in the operating ward it function from Monday to Friday from 7:25 am to 3 pm. Additionally, shall that be the case, one operating theatre is available also on Sundays and bank holidays, as well as on weekdays afternoon and night (from 3 pm to 7:25 am). All that accounts for theoretical availability of the operating ward calculated to 303 hours and 20 minutes within one week. The procedures assume that in each operating theatre within one working day there should be three surgeries. If a planned surgery is not realized on the planned date and time, it has adverse consequences for the whole hospital. A person that should already be after the surgery returns to the original ward, is planned to another date which results in postponing of next surgeries and, in consequence, drastically extends the queue. It has adverse impact both on patients and on functioning of the hospital itself. The former still have health issues, their health is not improving, what is more – they lose their confidence in public health care. And for the hospital this situation generates additional costs related with return of the same patient to their wards, as well as reputational damage.

The person responsible for planning of the schedule must very precisely both select types of surgeries to a given theatre, and specify the estimated duration of each of them. Information on duration of each surgery is key so that the schedule of work in one theatre of an operating ward is prepared in the most reliable way. Improper selection of surgeries may lead to overtime of the operating ward personnel. For the hospital this is not a demanded situation due to economic reasons.

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According to the initial analysis of the situation in the operating ward, duration of the process in a given operating theatre was the most significant factor influencing timeliness of work of the whole ward. Therefore, one needed to determine durations of particular activities composing the whole surgery process. That process was defined as one surgery with pre-surgery activities and post-surgery activities. In order to determine durations of particular activities of the process there were studies conducted in the operating ward including observation of a working day of the whole medical personnel.

There was a snapshot taken of the working day using an original form. It included all procedural activities that are performed in the process occurring in an operating ward. Description of all activities are presented in Table 1. Those are so called procedural activities, namely those which are described in work procedures applicable in the operating ward of a given hospital. These activities are mandatory and the manner of their realization is precisely defined.

Table 1. Network's structure and activities' times.

Activity	Predecessor	Description of activities	Duration [min]
A	-------	Taking over the patient to the operating ward by operating room nurses and taking him to the waiting room	((2, 3, 3, 27; 1), (2 , 3, 3, 6; 0.93))
D	-------	Preparing the operating room by operating room nurses	((5, 7, 7, 18; 1), (5, 7, 7, 18; 1))
E	-------	Preparing instruments in the operating room by the operating room nurse	((6, 7, 7, 38 ; 1), (6, 7, 7, 38; 1))
B	A	Preparing the patient by the anaesthesiologist team	((9, 10, 10, 46; 1), (9, 10, 10, 25; 0.92))
C	A	Preparing the patient by the nurse in the preparatory room	((6, 7, 7, 19;1), (6, 7, 7, 19; 1))
F	B, C, D	Taking in the patient to the operating room (time between the patient's readiness in the waiting room and taking the patient to the operating room)	((2, 2, 2, 28; 1), (2, 2, 2, 7; 0.89))
G	E, F	Anaesthesia	((4, 5, 5, 24; 1), (4, 5, 5, 24; 1))
H	G	Performing the surgery (duration is counted from cutting the patient until suturing)	((54, 95, 95, 405; 1), (54, 95, 95,375; 0.88))
I	H	Filling in documentation by the anaesthesiologist	((5, 7, 7, 17; 1), (5, 7, 7, 17; 1))
J	G	Filling in documentation by the operating room nurse	((6, 8, 8, 12; 1), (6, 8, 8, 12; 1))
K	H	Postoperative activities with the patient performed in the operating room	((4, 5, 5, 44; 1), (4, 5, 5, 44; 1))
L	K	Taking the patient to the recovery room	((2, 2, 2, 17; 1), (2, 2, 2, 17; 1))

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M	K	Postoperative activities performed by operating room nurses in the operating room	((5, 5, 5, 31; 1), (5, 5, 5, 31; 1))
N	H	Preparing instruments for sterilization by operating room nurses	((4, 5, 5, 26; 1), (4, 5, 5, 26; 1))
O	I, J, L, M, N	Cleaning of the operating room by the cleaning personnel	((6, 7, 7, 57; 1), (6, 7, 7, 35; 0.91))

During the research it appeared that duration of the whole process is significantly impacted by latenesses in commencement of some activities procedural. Additional activities have been identified. Those are not procedural activities, they do not contribute any added value for the process, only significantly extend its duration. They were classified, it was determined commencement of which procedural activities is disturbed that way (see Table 2). Table 2 includes also example reasons for latenesses. During the studies duration of procedural and non-procedural activities were registered.

Table 2. Lateness – "non-productive" observed in an operating room of an operating ward.

Activity	Description of the "waiting" time of activity	Employee performing the activity
A	Patient waiting to be taken in the waiting room reason: e.g. lack of patient's documents	Operating room nurses
B	Waiting for the first anaesthesiologist reason: occurs only before the first surgery as anaesthetists have the morning briefing at 7:30 am	The anaesthesiologist team
F	'Prepared' patient's waiting to be taken in the operating room (time between completion of anaesthetic preparation and taking the patient in the operating room), change of surgeries in the daily plan, related with e.g. change of instruments reason: e.g. unprepared room, unprepared instruments, no instructions	Operating room nurses
H	Waiting for the operator reason: e.g. the operator is in the ward, fills in documentation	Operator
O	Waiting for the cleaning service reason: e.g. cleaning rooms as they are located, not according to priorities	Cleaning service

It needs to be emphasized that it was possible to identify latenesses thanks to observation conducted by the researchers when taking the snapshot of a working day in an operating ward. If data on duration of surgeries was taken from the data bases serviced in that ward, it would not be possible to observe their occurrence. As emphasized by experts from the subject hospital, one should aim at minimizing durations of those latenesses but their total elimination is not possible, among other due to financial policy of the hospital and the state.

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There was a network of activities prepared for a process occurring in one selected operating theatre. Its correctness was consulted with experts (operators, manager and director of the operating ward, and theatre nurses).

The subject of this case study is to check whether and how elimination of the reasons for latenesses will impact the total time of the process of surgeries performed during the day in the operating ward.

Based on the time analysis of 107 surgeries performed in the operating ward, fuzzy times were approximated $\tilde{T}_{ij} = \left(\left(t_{ij}^{opt}, t_{ij}, t_{ij}^{pes}; 1\right), \left(\tau_{ij}^{opt}, \tau_{ij}, \tau_{ij}^{pes}; \mu_{ij}\right)\right)$ of particular activities, $(i, j) \in \mathcal{A} = \{A, B, C, \dots, O\}$. For activities $(i, j) = A, B, F, H, O$ it was adopted that t_{ij}^{opt}, t_{ij}, t_{ij}^{pes} are correspondingly: the shortest time, the most possible time and the longest time of realization of activity (i, j) together with the time of non-procedural activity corresponding to that activity. And times $\tau_{ij}^{opt}, \tau_{ij}, \tau_{ij}^{pes}$ are correspondingly: the shortest time, the most possible time and the longest time of realization of a procedural activity, whereas $\mu_{ij} = 1 - p_{ij}$, where p_{ij} is the frequency of occurrence of latenesses in activities (i, j). For other activities $(i, j) = C, D, E, G, I, J, K, L, M, N$, for which there were no latenesses observed, it was assumed that $\tau_{ij}^{opt} = t_{ij}^{opt}$ tis the shortest, $\tau_{ij} = t_{ij}$ – is the most possible and $\tau_{ij}^{pes} = t_{ij}^{pes}$ – the longest time of realization of activity (i, j) and $\mu_{ij} = 1$.

Let us now perform a time analysis for a working day (from 7:25 am to 3 pm) in one operating theatre, assuming that there are two surgeries planned in that theatre. In the analysed hospital, in one operating theatre, one to three surgeries are performed during a working day. Most often, in about 50% of cases, these are two surgeries. Therefore, we will carry out our time analysis for a daily plan of 2 surgeries. We will determine in line with the Algorithm 1 the critical time of the process of 2 surgeries and a possibility that we will perform two surgeries by 3 pm (working time $d = 455\ min$). Let us assume that all non-procedural activities may (but do not have to) occur. Then, the critical time for 2 surgeries is equal to $\tilde{T}_{crit} = ((172, 264, 264, 1324;\ 1)\ (172, 263.3, 387.7, 1094;\ 0.88))$ and the possibility that we will finish all surgeries by 3 pm ranges from 0.88 to 1 (see Table 3).

Table. 3. Critical time and possibility of realization 2 surgeries on time, and expected lateness.

Type of lateness	Critical time	Possibility of finish on time	Expected lateness
Possible each lateness – current status	((172, 264, 264, 1324; 1), (172, 263.3, 287.7, 1094; 0.88))	[0.88, 1]	[-55, 51]
No lateness in commencement of activity A	((172, 264, 264, 1282; 1), (172, 263.1, 288.1, 1094; 0.88))	[0.88, 1]	[-55, 41]
No lateness in commencement of activity B	((172, 264, 264, 1282; 1), (172, 263.1, 290.0, 1094; 0.88)	[0.88, 1]	[-55, 41]
No lateness in commencement of activity F	((172, 264, 264, 1282; 1), (172, 263.2, 288.8, 1094; 0.88))	[0.88, 1]	[-55, 41]

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No lateness in commencement of activity H	((172, 264, 264, 1264; 1), (172, 254.3, 346.5, 1094; 0.89))	[0.89, 1]	[-38, 36]
No lateness in commencement of activity O	((172, 264, 264, 1280; 1), (172, 263.1, 292.6, 1094; 0.88))	[0.88, 1]	[-54, 40]
No lateness	((172, 264, 264, 1094; 1), (172, 264, 264, 1094; 1))	1	-7

We will now perform a sensitivity analysis of the critical time of 2 surgeries with the assumption that we will eliminate particular latenesses, see Table 3. The possibility of performing 2 surgeries during a business day ranges from 0.88 to 1. The largest will be when we eliminate lateness of the activity *Performing the surgery* – according to equation (1) $degree([0.89, 1] \geq [0.88, 1]) = 0.09$. Therefore, the bottleneck here are non-procedural activities causing lateness of commencement of the procedural activity *Performing the surgery,* such as waiting for the operator caused by the fact that the operator is in the ward, fills in documentation, etc. When we eliminate any other one non-procedural activity, then the possibility of completing 2 surgeries by 3 pm ranges from 0.88 to 1. If we eliminate all non-procedural activities, i.e. all activities will commence on time, then the expected possibility of performing 2 surgeries during a working day will be equal to 1.

5 Discussion

The conducted analysis of critical time of two surgeries (a typical daily surgery plan) showed that with the current organization of work in the subject operating ward the expected delay goes from half an hour to around one hour (from 36 to 51 minutes) (see Table 3). In the case there is no delay in the activity *Performing the surgery* (activity H), the time reserve is the lowest and amounts to 36 minutes. Elimination of delay in commencing this activity may reduce the expected time of delay to around ½ hours. Elimination of reasons for delay of each remaining activity of the surgery process could result in reduction of the working day by around 10 minutes. It needs to be emphasized that as a result of the analysis it was proven that there is a possibility to perform two surgeries within less than 7 hours and 35 minutes. Optimistic time reserve is equal to around 1 hour. Such a situation will occur e.g. if the performed surgeries are not complicated surgeries, i.e. do not require large outlay of time or human resources or/and if particular activities of the surgery process start on time. If in the surgery process there are no non-procedural activities, then the possibility to perform 2 surgeries within one working day is equal to 1, and the expected time reserve equals 7 minutes.

In the considered process of surgery, in the selected hospital in Poland, the bottleneck is the delay in starting the activity Performing the surgery. The reason for this, namely not commencing the activity *Performing the surgery* on time, is among others lack of doctors in Poland. Currently, the Polish government prepared a draft bill on possibility to employ doctors from outside of the European Union on more advantageous conditions than at present. Other types of delays occurring in the operating theatre are also caused by lack of medical personnel, including anaesthetists and operating

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room nurses. The reasons for occurrence of these delays, and thus significant costs incurred by the hospital related to overtime worked by employees, also include mistakes in the organization of work in the operating theatre.

As a result of the lateness analysis described in the article, in the studied hospital several corrective activities were introduced, the main purpose of which is to minimize latenesses. Due to them, the daily surgery plan is not implemented. The most important improvements concerned work of the cleaning staff and of anaesthetists. For the first employee group, to reduce work delays it was enough to reorganize the work system. It was indicated in the procedures that rooms in which surgeries were to be performed on a given day should be cleaned first. Anaesthetists who assist with the first surgery on a given day have been released from the obligation to attend the morning briefing. Implementation of the recommendations improved implementation of the plan of surgeries, reducing the number of surgeries not performed by 16 percent. It should be noted that the introduced improvements did not require additional financial outlays, but only reorganization of work and improvement of existing procedures.

Let's compare the method presented in this article with the classical PERT method [21]. Let's assume as the activities time evaluation (i,j): the optimistic duration t_{ij}^{opt}, the most possible duration t_{ij} and the pessimistic one t_{ij}^{pes} for stable (normal, most typical, scenarios) conditions (circumstances) for the realization of the project. If the delay of the activity (i,j) is possible, we assume $t_{ij}^{pes} + \tau_{ij}^{pes}$ as the pessimistic time. Critical time distributions of two surgeries and their characteristics for these cases are given in Table 4. In PERT method, lateness time interval estimates are significantly wider than time interval estimates calculated according to the method used in this article. It is due to the fact that, among others, in PERT method, it is assumed that project's critical time has normal distribution with variance equal to the sum of variances of activities times on critical path. Whereas in the case of project time analysis presented in this article, critical time variance is lower than the sum of critical activities times, (see [22]). In an actual case, the decision maker should establish which of the methods (presented in this article, PERT) is more adequate to conditions of the surgery in the operating theatre.

Table 4. Critical time, probability of finish on time and confidence interval for lateness (0.95).

Type of lateness	Critical time	Probability of finish on time	Confidence interval for lateness
Possible each lateness	$N(398.6, 77.4)$	0.95	(-274.4, 23.8)
No lateness	$N(329.67, 76.1)$	0.77	(-207.9, 95.3)

Let's analyze the study case presented in Santibáñez et. al. [23]. The authors analyze the time spent by the patient in British Columbia Cancer Agency's ambulatory care. The simulation technique is used to determine the total time spent by the patient in the clinic. They assume that patient wait times at the various stages of the process (waits for exam room, consults and discharge) have normal distributions. Therefore, total patient wait time in process for an ACU appointment has normal distribution with variance equal to the sum of variances of the wait times in particular points of the process

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in the ACU. Expected time of total patient wait time (min.) in Clinic of Medical Oncology equals 20.1, and a confidence interval (at confidence level 0.95) equals (19.7, 20.5), whereas, expected actual patient wait time equals 16.8, and confidence interval (at confidence level 0.95) equals (13.7, 20.0). Therefore, also in this case, interval estimate of patient wait time is significantly wider than the actual wait time. It is due to the fact that, among others, normal distribution of patient wait time at the various stages of the process was used.

6 Conclusions

As a result of our study we came up with a proposal of time analysis of a project implemented in conditions of risk associated with delayed start of activities. We assumed that activity times are type-2 fuzzy numbers. When experts estimate shapes of membership functions of times of activities, they take into account both situations when particular activities of the project start on time and situations when they start with a delay. Therefore, when estimating shapes of membership functions, experts should take into account availability of the personnel which is, among others, the condition for timely realization of particular stages of the project. There was also suggested a method of a sensitivity analysis of these delays to meeting the project completion deadline. At the end we presented a case study in which we analysed a typical daily plan of surgeries in the operating ward of a hospital in Poland. In this case, when estimating the shape of membership functions, experts should take into account availability of the personnel, the qualifications of the medical staff, as well as medical factors. As a result of the surgical process time analysis and the sensitivity analysis, we identified a certain type of delay that significantly increases the time of the surgery process. In this particular case, the largest delay is caused by waiting for the operator due to the fact that e.g. the operator is in the ward, fills in documentation. If in the surgery process there are no non-procedural activities (no delay in starting any procedural activity), then the expected time reserve is equal to 7 minutes.

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