New rank-reversal free approach to handle interval data in MCDA problems

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Abstract. In many real-life decision-making problems, decisions have
to be based on partially incomplete of uncertain data. Since classical
MCDA methods were created to be used with numerical data, they are
often unable to process incomplete or uncertain data. There are several
ways to handle uncertainty and incompleteness in the data, i.e. interval
numbers, fuzzy numbers, and their generalizations. New methods are de-
veloped, and classical methods are modified to work with incomplete and
uncertain data. In this paper, we propose an extension of the SPOTIS
method, which is a new rank-reversal free MCDA method. Our exten-
sion allows for applying this method to decision problems with missing or
uncertain data. The proposed approach is compared in two study cases
with other MCDA methods: COMET and TOPSIS. Obtained rankings
would be analyzed using rank correlation coefficients.

Keywords: MCDA · COMET · SPOTIS · TOPSIS · Uncertainty · In-
terval numbers

1 Introduction

There are many complex problems which require handling a relatively signif-
icant number of opposing criteria to evaluate decision alternatives. Classical
multi-criteria decision problem consists of three elements: a set of criteria, a
set of the alternatives and criteria weights. For that kind of problems, Multi-
Criteria Decision-Analysis (MCDA) methods help support decision-maker in the
decision process. Applying the MCDA methods to the decision problem allows
determining the most reliable solution for this particular decision problem [8].

The complete dataset about alternatives should be collected to use the MCDA
method to solve a particular decision problem. However, in many real-life cases,
we faced with uncertain or incomplete data. This problem could appear in differ-
ent cases, for example, when we collect data from various sources, or when some
values in the data just not provided [22, 26]. There are several methods, which
decision-makers could apply to handle uncertain data, e.g. interval numbers [21],

fuzzy numbers [5] and their generalizations [6, 23]. Besides, if a single criterion attribute is missing for a particular alternative, we have to consider all possible values from the domain [24].

The other problem to cope with in the decision-making process is a rank reversal paradox [1, 25]. It is a phenomenon of reversing alternative's order in ranking when the set of alternatives is changed, e.g., alternative A_1 which was better than A_2 in the initial ranking could be worse than A_2 in the ranking calculated after adding a new alternative. The most MCDA methods, such as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), VlseKriterijumska OptimizacijaI Kompromisno Resenje (VIKOR) are susceptible to this paradox [3, 7, 11]. Classical MCDA methods are modified in order to eliminate rank reversal paradox in them. However, there are also new methods created which were designed to eliminate rank reversal paradox in them, e.g., Ranking of Alternatives through Functional mapping of criterion sub-intervals into a Single Interval (RAFSI)[27], Characteristic Object METhod (COMET)[14] and Stable Preference Ordering Towards Ideal Solution method (SPOTIS)[4].

SPOTIS is a new MCDA method which aims to eliminate rank reversal paradox by design [4]. It is a simple method that uses reference objects to evaluate final preferences, similarly to COMET and TOPSIS methods. Unlike classic MCDA methods, such as TOPSIS, VIKOR and PROMETHEE, SPOTIS method requires criteria bounds to be defined. Using criteria bounds as reference objects allows distributing alternatives linearly between ideal positive and negative solutions. Thus SPOTIS method stays completely rank reversal free [4].

In this paper, we propose extending the SPOTIS method, which allows applying this method in the decision problems with incomplete or uncertain data using interval values. The proposed approach is based on using monotonic criteria, i.e., each criterion is profit or cost type. Moreover, we compare the proposed approach with two other MCDA methods that also use the reference objects' concept, i.e., COMET and TOPSIS. In order to compare these three methods, we present two numerical study cases. The final preferences are determined based on interval number comparison according to priority degree. Finally, the rankings are compared using ranking similarity coefficients and literature reference results.

The rest of the paper is organized as follows: In section 2, basic preliminary concepts on selected MCDA methods are presented. Section 3 introduces the proposed approach. In Section 4, we present and discuss two study cases that show the efficiency of the proposed approach. In Section 5, we present the summary and conclusions.

2   Preliminaries

2.1   TOPSIS

The Technique of Order Preference Similarity (TOPSIS) method measures the distance of alternatives from the reference elements, respectively, positive and

negative ideal solution (PIS and NIS). This method was widely presented in [2, 12]. The TOPSIS method is a simple MCDA technique used in many practical problems. Thanks to its simplicity of use, it is widely used in solving multi-criteria problems. Below we present its algorithm [2]. We assume, that we have decision matrix with m alternatives and n criteria is represented as $X = (x_{ij})_{m \times n}$.

Step 1. Calculate the normalized decision matrix. The normalized values r_{ij} calculated according to equation (1) for profit criteria and (2) for cost criteria. We use this normalization method because [13, 20] shows that it performs better than the classical vector normalization. However, we can also use any other normalization method.

$$r_{ij} = \frac{x_{ij} - min_j(x_{ij})}{max_j(x_{ij}) - min_j(x_{ij})} \tag{1}$$

$$r_{ij} = \frac{max_j(x_{ij}) - x_{ij}}{max_j(x_{ij}) - min_j(x_{ij})} \tag{2}$$

Step 2. Calculate the weighted normalized decision matrix v_{ij} according to equation (3).

$$v_{ij} = w_i \cdot r_{ij} \tag{3}$$

Step 3. Calculate Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) vectors. PIS is defined as maximum values for each criteria (4) and NIS as minimum values (5). We do not need to split criteria into profit and cost here, because in step 1 we use normalization which turns cost criteria into profit criteria.

$$v_j^+ = \{v_1^+, v_2^+, \cdots, v_n^+\} = \{max_j(v_{ij})\} \tag{4}$$

$$v_j^- = \{v_1^-, v_2^-, \cdots, v_n^-\} = \{min_j(v_{ij})\} \tag{5}$$

Step 4. Calculate distance from PIS and NIS for each alternative. As shows equations (6) and (7).

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \tag{6}$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \tag{7}$$

Step 5. Calculate each alternative's score according to equation (8). This value is always between 0 and 1, and the alternatives which got values closer to 1 are better.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{8}$$

2.2   The COMET method

The Characteristic Objects METhod (COMET) is based on fuzzy logic and triangular fuzzy sets. The COMET method's accuracy was verified in previous works [15–17]. The formal notation of the COMET must be recalled based on [14]. Figure 1 presents the flowchart of the COMET method as summarizing.
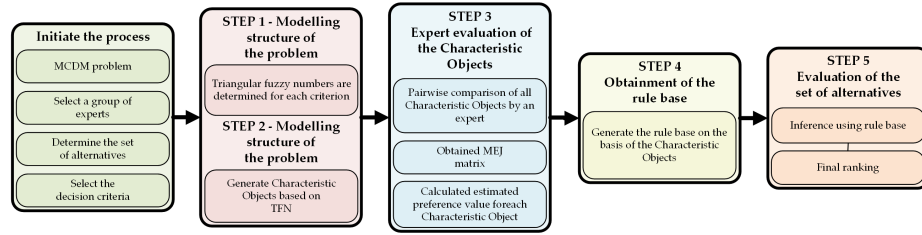

Fig. 1. The procedure of the COMET method

Step 0. Initiate the process – it is a preparatory stage, which aims to identify the problem to be further analysed clearly. In the beginning, it is necessary to define the purpose of the research and determine the specificity of the MCDA problem. We should then select an expert or a group of experts whose task will be to select decision alternatives and criteria for their evaluation. After selecting a group of alternatives, a set of criteria that should be taken into account in the further analysis should also be selected.

Step 1. Definition of the space of the problem – the dimensionality of the problem is determined by the expert, which selecting r criteria, $C_1, C_2, \ldots, C_r$. For each criterion C_i, e.g., $\{\tilde{C}_{i1}, \tilde{C}_{i2}, \ldots, \tilde{C}_{ic_i}\}$ (9) a set of fuzzy numbers is carefully selected:

$$\begin{aligned} C_1 &= \left\{ \tilde{C}_{11}, \tilde{C}_{12}, \ldots, \tilde{C}_{1c_1} \right\} \\ C_2 &= \left\{ \tilde{C}_{21}, \tilde{C}_{22}, \ldots, \tilde{C}_{2c_2} \right\} \\ &\quad \cdots \\ C_r &= \left\{ \tilde{C}_{r1}, \tilde{C}_{r2}, \ldots, \tilde{C}_{rc_r} \right\} \end{aligned} \tag{9}$$

where $c_1, c_2, \ldots, c_r$ are the cardinality for all criteria.

Step 2. Generation of the characteristic objects – the characteristic objects (CO) are obtained with the usage of the Cartesian product of the fuzzy numbers' cores of all the criteria (10):

$$CO = \langle C\left(C_1\right) \times C\left(C_2\right) \times \cdots \times C\left(C_r\right) \rangle \tag{10}$$

As a result, an ordered set of all CO is obtained (11):

$$
\begin{aligned}
CO_1 &= \langle C(\tilde{C}_{11}), C(\tilde{C}_{21}), \ldots, C(\tilde{C}_{r1}) \rangle \\
CO_2 &= \langle C(\tilde{C}_{11}), C(\tilde{C}_{21}), \ldots, C(\tilde{C}_{r1}) \rangle \\
&\quad \ldots \\
CO_t &= \langle C(\tilde{C}_{1c_1}), C(\tilde{C}_{2c_2}), \ldots, C(\tilde{C}_{rc_r}) \rangle
\end{aligned}
\tag{11}
$$

where t is the count of COs and is equal to (12):

$$
t = \prod_{i=1}^{r} c_i
\tag{12}
$$

Step 3. Evaluation of the characteristic objects – the Matrix of Expert Judgment (MEJ) is determined by the expert, which comparing the COs pairwise. The MEJ matrix is presented as follows (13):

$$
MEJ = \begin{pmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1t} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2t} \\
\cdots & \cdots & \cdots & \cdots \\
\alpha_{t1} & \alpha_{t2} & \cdots & \alpha_{tt}
\end{pmatrix}
\tag{13}
$$

where α_{ij} is the result of comparing CO_i and CO_j by the expert. The function f_{exp} express the individual judgement function of the expert. It is a representation of the knowledge of the selected expert, whose preferences can be presented as (14):

$$
\alpha_{ij} = \begin{cases}
0.0, & f_{\exp}(CO_i) < f_{\exp}(CO_j) \\
0.5, & f_{\exp}(CO_i) = f_{\exp}(CO_j) \\
1.0, & f_{\exp}(CO_i) > f_{exp}(CO_j)
\end{cases}
\tag{14}
$$

The number of query is equal $p = \frac{t(t-1)}{2}$ because for each element α_{ij} we can observe that $\alpha_{ji} = 1 - \alpha_{ij}$. After the MEJ matrix is constructed, a vertical vector of the Summed Judgments (SJ) is obtained by using moudus ponens tautology as follows (15):

$$
SJ_i = \sum_{j=1}^{t} \alpha_{ij}
\tag{15}
$$

Finally, the values of preference are estimated for each characteristic object, and a vertical vector P is obtained. The $i - th$ row includes the estimated value of preference for CO_i.

Step 4. The rule base — each characteristic object and its value of preference is converted to a fuzzy rule as (16):

$$
IF \ \ C\left(\tilde{C}_{1i}\right) \ \ AND \ \ C\left(\tilde{C}_{2i}\right) \ \ AND \ \ \ldots \ THEN \ P_i
\tag{16}
$$

In this way, a complete fuzzy rule base is obtained, which will then be used to infer alternatives' evaluation.

Step 5. Inference and the final ranking – each alternative is represented as a set of values, e.g. $A_i = \{\alpha_{i1}, \alpha_{i2}, \alpha_{ri}\}$. This set is addressed to the criteria $C_1, C_2, \ldots, C_r$. Mamdani's fuzzy inference technique is used to calculate the preference of the $i - th$ decision variant. The constant rule base guarantees that the determined results are unequivocal, and it makes the COMET completely rank reversal free.

2.3   SPOTIS

Stable Preference Ordering Towards Ideal Solution (SPOTIS) is a new method for multi-criteria decision support. [4]. The authors of this method aim to create a new method free of the Rank Reversal problem (the phenomenon of reversing the ranking when changing the number of alternatives in the input data). This method uses the concept of reference objects. Unlike other MCDA methods such as TOPSIS and VIKOR, which creates reference objects based on decision matrix, SPOTIS requires defining the data boundaries. Using data borders to define Ideal Positive and Ideal Negative Solution allows for a linear distribution of alternatives between IPR and INR and avoids ranking reversals.

To apply this method, data boundaries should be defined. For each criterion C_j the maximum S_j^{max} and minimum S_j^{min} bounds should be selected. Ideal Positive Solution S_j^* is defined as $S_j^* = S_j^{max}$ for profit criterion and as $S_j^* = S_j^{min}$ for cost criterion. Decision matrix is defined as $X = (x_{ij})_{m \times n}$, where x_{ij} is attribute value of the i-th alternative for j-th criterion.

Step 1. Calculation of the normalized distances to Ideal Positive Solution (17).

$$d_{ij}(A_i, S_j^*) = \frac{|S_{ij} - S_j^*|}{|S_j^{max} - S_j^{min}|} \tag{17}$$

Step 2. Calculation of weighted normalized distances $d(A_i, S^*) \in [0, 1]$, according to (18).

$$d(A_i, S^*) = \sum_{j=1}^{N} w_j d_{ij}(A_i, S_j^*) \tag{18}$$

Final ranking should be determined based on $d(A_i, S^*)$ values. Better alternatives have smaller values of $d(A_i, S^*)$.

This method has an alternative algorithm which is described in [4]. We describe and use this version because it is easier to understand, and both versions give identical results.

3   The proposed approach

An interval number is a set of real numbers with the property that any number that lies between two numbers included in the set is also included in the set. The interval of numbers between a^L and b^R , including a^L and b^R , is denoted $[a^L, b^R]$. The two numbers a^L and b^R are called the endpoints of the interval. Interval numbers are used when an attribute has an indefinite or uncertain value. This entails analysing all the values from a given interval. Only one of the interval's values is the unknown real value.

The SPOTIS method is designed to solve problems with crisp numbers. The values of the decision attributes will be converted to interval numbers, which will be noted as (19):

$$a_j = [\alpha_j^L, \alpha_j^R] \tag{19}$$

where j means number of crierion. Each real number can be written as a degenerate interval numbers, i.e., $\alpha_j^L = \alpha_j^R$. Let suppose there is no attribute value for an individual alternative in a given set of alternatives. In that case, the smallest and the biggest value in the criterion should be taken respectively. Each alternative will be written as an interval data set (20).

$$A = \{[\alpha_1^L, \alpha_1^R], [\alpha_2^L, \alpha_2^R], ..., [\alpha_n^L, \alpha_n^R]\} \tag{20}$$

Note here that when evaluating an alternative with at least one attribute given in terms of an interval number that is not degenerate, the assessment result will always be returned as an interval number in the proposed approach. Also, for monotonic decision criteria, the lowest and the highest evaluation value will always be on the interval boundaries. Therefore, to calculate the resulting evaluation interval, it suffices to determine the set of alternatives A', which will arise as the Cartesian product of all interval boundaries of the form (21):

$$A' = \{\{\alpha_1^L, \alpha_1^R\} \times \{\alpha_2^L, \alpha_2^R\} \times ... \times \{\alpha_n^L, \alpha_n^R\}\} \tag{21}$$

The set of alternatives A' contains exactly 2^n crisp alternatives which must be calculated by using SPOTIS algorithm. The final ranking's left boundary is the lowest preference value determined from the set A', and the right boundary is the highest value.

4   Study cases

In order to demonstrate the proposed approach, we have chosen two MCDA problems with interval data from recent studies, which are presenting in the subsections 4.1 and 4.2. Both topics deal with the assessment of electric vehicles, which is motivated by phasing out diesel and petrol engines in Europe. Many of these vehicles' parameters are of an interval nature, demonstrating the superiority of the proposed approach.

4.1   Assessment of electric bikes

The first problem is the choice of the best electric bicycles for city transport. There is currently an increasing tendency to look for more sustainable transport solutions, especially in highly congested urban areas. It seems that electric bicycles can be a good option, as they allow more benefits than combustion cars. Because of missing data in the manufacturer's specifications, we should apply interval MCDA methods to handle incompleteness in the data.

The alternatives used for this study case are adopted from [18]. Criteria used for the analysis are presented in the Table 1, where we can also find characteristic values, which are needed to be defined to use COMET and SPOTIS methods. Characteristic values are also needed to create intervals instead of missed data.

Table 1. The selected criteria C_1 - C_8 and their characteristic values [18].

C_i	Name	Unit	Low	Medium	High
C_1	Battery capacity	Ah	4	9	15
C_2	Charging time	hours	3	5	8
C_3	Number of gears	units	1	7	21
C_4	Engine power	W	250	350	500
C_5	Maximum speed	km/h	20	27	35
C_6	Range	km	20	60	100
C_7	Weight	kg	10	20	25
C_8	Price	USD	300	2500	6300

Table 2 presents chosen alternatives from the original study. Alternatives A_1 - A_8 contains several interval attribute values and alternatives A_9 - A_{13} contains only real values. This selection of alternatives shows how the proposed approach works when only part of the alternatives have interval data.

The structured COMET approach was used in the original paper to solve the considered MCDA problem. In this study, we also use COMET, but with the monolithic approach [22]. This is because we assume that the structure of the problem is unknown to the decision-maker. We have used stochastic optimization methods to obtain preference values for CO from preference values from the alternatives [10].

Table 3 presents raw preference values calculated for each alternative by using three MCDA methods. For SPOTIS method, smaller values means better alternative, for other methods bigger values means better alternative. However, preference data alone is not sufficient for determining the rankings, as these intervals overlap to some extent, which poses a problem in unambiguously assessing the final ranking.

We apply the approach described in [9] to obtain ranking values from interval data. Rankings from Table 3 are calculated in two steps: build comparison matrix for interval values using $P(a \geq b)$ and then rank sums for each row of this matrix. This ensures obtained the most likely ranking.

Table 2. The performance table of the alternatives A_1–A_{13}.

A_i	Name	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	Aceshin	8	[4,6]	21	250	30	40	22.2	730
A_2	ANCHEER Plus	8	5	21	250	25	[25,50]	23	615
A_3	Carrera Crossfuze	11	[6,7]	9	400	25	80	20.3	2300
A_4	ECOTRIC	12	[5,8]	7	500	32	55	24.9	999
A_5	Emu Crossbar	14.5	[6,8]	7	250	25	[55,100]	23	1560
A_6	Kemanner	8	[4,6]	21	250	25	[35,70]	20	[615,700]
A_7	Merax 26" Aluminum	8.8	[5,6]	7	350	32	[35,45]	22	690
A_8	Rattan	10.4	[4,5]	7	350	32	50	23.5	740
A_9	Desiknio Pinion Classic	7	3	6	250	24.8	80	15.7	6135
A_{10}	e-Joe Gadis	11	5	7	350	32	72	24.9	1699
A_{11}	California Bicycle S	8	4	1	250	32	56	22.6	2499
A_{12}	Coboc ONE Soho	9.6	3	1	250	24.8	88	13.1	5520
A_{13}	Gazelle CityZen C8 HM	11	3.5	8	350	32	94	23.1	2999

Table 3. Considered alternatives and their results A_1–A_{13}.

A_i	Preference				Ranking			
	Ref	SPOTIS	COMET	TOPSIS	Ref	SPOTIS	COMET	TOPSIS
A_1	[0.4414 0.4804]	[0.4756 0.5256]	[0.4439 0.5030]	[0.4727 0.5050]	9	5	9	6
A_2	[0.3752 0.4693]	[0.5309 0.5700]	[0.4015 0.4380]	[0.4207 0.4421]	10	9	10	12
A_3	[0.4802 0.4918]	[0.4875 0.5125]	[0.4853 0.5007]	[0.4565 0.4765]	8	7	8	7
A_4	[0.5308 0.5686]	[0.4056 0.4806]	[0.5615 0.5959]	[0.5228 0.5759]	5	2	5	2
A_5	[0.4219 0.6119]	[0.5111 0.6314]	[0.4284 0.5954]	[0.3970 0.4782]	6	11	6	10
A_6	[0.4116 0.5959]	[0.4496 0.5561]	[0.4356 0.5619]	[0.4288 0.4982]	7	6	7	8
A_7	[0.5264 0.5778]	[0.5020 0.5426]	[0.5301 0.6368]	[0.4738 0.5036]	4	8	4	5
A_8	[0.5800 0.6056]	[0.4646 0.4896]	[0.5991 0.6580]	[0.5168 0.5344]	2	3	2	4
A_9	0.3945	0.5950	0.3603	0.4071	12	12	12	13
A_{10}	0.5555	0.4800	0.6262	0.5248	3	4	1	3
A_{11}	0.3669	0.5991	0.3736	0.4344	13	13	11	11
A_{12}	0.4016	0.5497	0.2609	0.4499	11	10	13	9
A_{13}	0.6204	0.4140	0.6046	0.5768	1	1	3	1

The rankings obtained by the methods used are quite different. Alternative A_{13} has the first position in the reference ranking and the ranking obtained with interval SPOTIS and TOPSIS method. The COMET method placed this alternative in the third position in the ranking. Alternative A_{10} has first position in monolithic COMET ranking, but has lower values in other rankings. Alternative A_8 has the second position in the reference ranking, where only COMET return the same position. It should be noted that both SPOTIS and TOPSIS have different ranked this alternative, but in the case of TOPSIS, the position is more distant. In order to comprehensively assess the similarity of the obtained rankings, r_w and WS values will be determined [19].

Table 4 presents r_w ranking correlation coefficient values. These values point out that ranking obtained monolithic COMET method is strongly correlated with reference ranking. Other two rankings, obtained with interval SPOTIS and

interval TOPSIS methods have a high correlation between themselves, but the quite good correlation with reference ranking.

Table 4. Comparison of rankings using r_w coefficient.

r_w	Ref	SPOTIS	COMET	TOPSIS
Ref	1.0000	0.7975	0.9560	0.8615
SPOTIS	0.7975	1.0000	0.7316	0.9317
COMET	0.9560	0.7316	1.0000	0.8144
TOPSIS	0.8615	0.9317	0.8144	1.0000

Table 5 shows WS ranking similarity coefficient values. According to calculated values, ranking obtained with interval SPOTIS method is strongly correlated with the reference ranking. Ranking obtained using monolithic COMET approach also has a quite strong correlation with reference ranking. The last ranking obtained with interval TOPSIS method also has a good correlation with the reference.

Table 5. Comparison of rankings using WS coefficient.

WS	Ref	SPOTIS	COMET	TOPSIS
Ref	1.0000	0.9111	0.8915	0.9240
SPOTIS	0.8904	1.0000	0.7929	0.9696
COMET	0.8915	0.7735	1.0000	0.8157
TOPSIS	0.9044	0.9650	0.7957	1.0000

4.2   Assessment of electric vans

An ecological footprint in the urban environment is made by urban freight transport. This problem has become the key challenge for all groups involved in freight transport in urban areas. Therefore electric vans should be considered as an alternative for combustion vehicles. The second study case is on assessing electric vans and the data with reference ranking for this investigation is taken from [26]. In the original study, the authors use PROMETHEE II and Fuzzy TOPSIS methods to rank electric vans for city logistic. In this work, we would use chosen alternatives from the original paper in order to demonstrate how efficient the proposed methods are.

Table 6 presents criteria description and characteristic values calculated from complete decision matrix. Characteristic values are necessary to determine when using COMET and SPOTIS methods. Criteria C_6, C_7 and C_9 are cost type criteria and should be minimized. Other criteria are profit type.

Table 7 presents alternatives chosen from original work with criteria attributes values. Alternatives A_5, A_6 and A_9 have only real number attributes,

Table 6. The selected criteria C_1 - C_9 and their characteristic values.

C_i	Name	Unit	Low	Medium	High
C_1	Carrying capacity	kg	340.00	1770.00	3200.00
C_2	Max velocity	km/h	40.00	95.00	150.00
C_3	Travel range	km	100.00	250.00	400.0
C_4	Engine power	kW	9.00	104.50	200.00
C_5	Engine torque	Nm	80.00	490.00	900.00
C_6	Battery charging time 100%	h	2.00	7.00	12.00
C_7	Battery charging time 80%	min	10.00	95.00	180.00
C_8	Battery capacity	kWh	2.70	61.35	120.00
C_9	Price	thous. USD	12.90	81.45	150.00

and the other alternatives have missed data which are substituted with intervals based on characteristic values from Table 6.

Table 7. Considered alternatives and their results A_1-A_{10}.

A_i	Name	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	Berlingo Electric	695	110	170	49	200	7.5	30	22.5	[12.9, 150.0]
A_2	Boulder Delivery Truck	2700	104	160	80	[80.0, 900.0]	8	[10.0, 180.0]	80	100.0
A_3	Ecomile	935	80	120	28	[80.0, 900.0]	8	[10.0, 180.0]	14.4	51.5
A_4	Electric Delivery Van 1000	830	40	118	14	98	8	120	2.7	[12.9, 150.0]
A_5	EVI MD	3000	96	145	200	610	10	120	99	120.0
A_6	e-NV200+	705	120	170	80	270	4	30	24	25.0
A_7	Kangoo Maxi Z.E.	650	130	170	44	226	8	[10.0, 180.0]	22	22.0
A_8	Mercedes-Benz Sprinter E-CELL	1200	80	135	100	220	2	[10.0, 180.0]	35.2	[12.9, 150.0]
A_9	Minicab-MiEV Truck	350	100	110	30	196	4.5	15	10.5	12.9
A_{10}	Peugeot eBipper	350	100	100	30	[80.0, 900.0]	3	[10.0, 180.0]	20	60.0

In the Table 8 calculated preference values are presented. Reference ranking is a ranking obtained with Fuzzy TOPSIS from the original work [26]. Next columns show preferences obtained with the proposed interval SPOTIS method, monolithic COMET, and interval TOPSIS method.

For obtained the rankings we use the same methodology as for section 4.1, and rankings from Table 8 are calculated in two steps: build comparison matrix for interval values using $P(a \geq b)$ and then rank sums for each row of this matrix. In this problem, obtained rankings are quite similar, according to the Table 8. Alternative A_5 has the first position in all four rankings. Alternative A_6 has the third position in the reference ranking and the second position in other rankings. Rank positions for alternative A_2 are also similar: second position in the reference ranking and third position in other rankings.

Table 9 contains r_w ranking correlation coefficient values. Despite very similar first positions in rankings, correlations between reference ranking and other rankings are quite low. Other rankings have quite strong correlations between themselves.

Table 8. Vans preferences and rankings

A_i	Preference			Ranking			
	SPOTIS	COMET	TOPSIS	Ref	SPOTIS	COMET	TOPSIS
A_1	[0.5721 0.6833]	[0.2390 0.4164]	[0.4370 0.5103]	7	8	8	5
A_2	[0.3997 0.6220]	[0.3144 0.7213]	[0.4791 0.6431]	2	3	3	3
A_3	[0.5604 0.7827]	[0.1028 0.4328]	[0.2875 0.4709]	4	9	9	9
A_4	[0.7742 0.8853]	[0.0235 0.0974]	[0.1667 0.3137]	5	10	10	10
A_5	0.4635	0.5802	0.5737	1	1	1	1
A_6	0.5036	0.5476	0.5668	3	2	2	2
A_7	[0.5534 0.6645]	[0.2616 0.4571]	[0.4549 0.5227]	6	6	6	4
A_8	[0.4772 0.6994]	[0.2100 0.6026]	[0.3960 0.5588]	8	4	5	6
A_9	0.5977	0.3794	0.4497	10	5	4	7
A_{10}	[0.5152 0.7375]	[0.1589 0.5359]	[0.3509 0.4976]	9	7	7	8

Table 9. Comparison of rankings using r_w coefficient for vans.

r_w	Ref	SPOTIS	COMET	TOPSIS
Ref	1.0000	0.5592	0.5405	0.6672
SPOTIS	0.5592	1.0000	0.9857	0.8766
COMET	0.5405	0.9857	1.0000	0.8645
TOPSIS	0.6672	0.8766	0.8645	1.0000

Finally, Table 10 shows WS ranking similarity coefficient values calculated for obtained rankings. This values point that correlation is quite strong, because first positions in the ranking is more important when WS similarity coefficient is calculated. As we can see, the results obtained show that the interval SPOTIS method provides solutions comparable to other methods, while being very simple to apply.

Table 10. Comparison of rankings using WS coefficient for vans.

WS	Ref	SPOTIS	COMET	TOPSIS
Ref	1.0000	0.8630	0.8634	0.8570
SPOTIS	0.8731	1.0000	0.9833	0.9574
COMET	0.8647	0.9833	1.0000	0.9533
TOPSIS	0.9051	0.9510	0.9528	1.0000

5   Conclusions

In this paper, we present a way to extend SPOTIS method to work with interval numbers which allow handling uncertainty and incompleteness in decision problems. We also compare the proposed approach with two other interval methods, COMET and TOPSIS to show how it performs in real-life decision problems.

The main contribution is providing an extension of the SPOTIS method, which is performed comparably with other interval methods. The main advantage of the SPOTIS method is its simplicity. The SPOTIS method consists of two simple steps, and the only additional requirement is defining criteria bounds. The study cases confirm it performs as good as COMET and TOPSIS methods, but it is much simpler to use than COMET and is also rank-reversal free, unlike TOPSIS. In order to compare the performance of these methods, the priority degree approach was used to build rankings. Then, the rankings were compared using rank correlation coefficients.

The future works may include

– developing the more complex approach which would be possible to apply to any criteria types without limitations,
– research of possibility using other number generalizations instead of interval numbers,
– comparing the proposed approach with other MCDA methods.

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