A Study on a Marine Reservoir and a Fluvial Reservoir History Matching Based on Ensemble Kalman Filter

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Abstract. In reservoir management, utilizing all the observed data to update the reservoir models is the key to make accurate forecast on the parameters changing and future production. Ensemble Kalman Filter (EnKF) provides a practical way to continuously update the petroleum reservoir models, but its application reliability in different reservoirs types and the proper design of the ensemble size are still remain unknown. In this paper, we mathematically demonstrated Ensemble Kalman Filter method; discussed its advantages over standard Kalman Filter and Extended Kalman Filter (EKF) in reservoir history matching, and the limitations of EnKF. We also carried out two numerical experiments on a marine reservoir and a fluvial reservoir by EnKF history matching method to update the static geological models by fitting bottom-hole pressure and well water cut, and found the optimal way of designing the ensemble size. A comparison of those the two numerical experiments is also presented. Lastly, we suggested some adjustments of the EnKF for its application in fluvial reservoirs.

Keywords: History matching, EnKF, Marine reservoir, Fluvial reservoir

1 Introduction

History matching is the act of adjusting a model of a reservoir until it closely reproduces the past behavior of a petroleum reservoir, which is a crucial component in reservoir management. Once a geological model has been history matched, it can be used to simulate future reservoir behavior with a higher degree of confidence, particularly if the adjustments are constrained by known geological properties in the reservoir. With the advancement of the computer science, automatic history matching uses computer algorithms to solve the optimization problem based on the reasonable objective function. By applying proper history matching approaches, geological settings of the reservoir can be preserved.

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Ensemble Kalman Filter (EnKF) works with an ensemble of the reservoir models, with its inherent forecasting and updating process (correct the forecasted data with the new measurements), the EnKF gives a suite of realizations of reservoir models which are consistent with the prior geological settings and the dynamic data. Recently, the EnKF has gain increasing attention for real time reservoir management and history matching, using data from the permanent down whole sensors. However, there are still great challenges of using the EnKF. For example, with limited computational power, a small ensemble size is viable in the practical history matching, but a small ensemble size also leads to an erroneous result. EnKF also may fail to detect facies boundaries.

2 Theoretical Formulations

2.1 Kalman Filter

We first review the formulations of Kalman filter, which is a set of mathematical equations that provides an efficient recursive estimation of the states of a process, in a way that with minimized mean and standard variances.

For a linear stochastic model

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (1)$$

with a measurement

$$z_k = Hx_k + v_k \quad (2)$$

where variable x_k is the state variable at time k, matrix A is the state transition matrix, B is the control operator, z_k is the measurement vector at time k and H is the measurement operator. The random variables w_{k-1} and v_k represent the process and measurement noise respectively, and they are assumed to be independent, white, with a standard Gaussian distribution with the noise covariance Q and R respectively.

Define $\hat{x}_k^- \in R^n$(note the “super minus”) to be our a prioristate estimate at step k given knowledge of the process prior to step k, and to be our a posteriori state estimate at step k given measurement. We can then define a priori and posteriori estimate errors as

$$e_k^- = x_k - \hat{x}_k^- \quad (3)$$

$$e_k = x_k - \hat{x}_k \quad (4)$$

The priori estimate error covariance and the a posteriori estimate error covariance are

$$P_k^- = E[e_k^- e_k^{-T}] \quad (5)$$

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$$P_k = E[e_k e_k^T] \tag{6}$$

The equation that computes an a posteriori state estimate $\hat{x}_k^-$ as a linear combination of an a priori estimate $\hat{x}_k^-$ and a weighted difference between an actual measurement z_k and a measurement prediction $H\hat{x}_k^-$ as shown below:

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right) \tag{7}$$

where the Kalman gain K is

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{8}$$

Kalman filter is very suitable for the linear problems, and it is a quite effective in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. However, Kalman filter only works for linear models and it has two main drawbacks: large dimensionality and error covariance propagation.

2.2 Ensemble Kalman Filter

Some adjustments to Kalman filter have been raised in order to apply Kalman filter in non-linear problems, and eliminate the pertaining drawbacks of the standard Kalman filter. A Kalman filter that linearizes about the current mean and covariance is referred to as an extended Kalman filter (EKF), but EKF only works to weakly non-linear problems and the computational power is even more than standard Kalman filter. For the complex non-linear problems with a large number of variables, like history matching in reservoir management, ensemble Kalman filter, which is first introduced by Evensen, provides a plausible solution.

The EnKF is a Monte Carlo method based on Markov chain approach. First, it samples many realizations from the prior probability density function. Second, for each realization, it use the model forecast function (in history matching, the forecast function is the reservoir simulator) to estimate the dynamic data at the next time step. Third, it uses those predicted realizations to calculate the approximation of the predicted covariance.

Similar to Kalman filter, the EnKF also consists of two sets of equations: the state forecast equation, and the update equation. For nonlinear models, the time update equations are no longer linear.

$$\hat{x}_k^- = f(\hat{x}_{k-1}) + w_{k-1} \tag{14}$$

Where $f(x)$ is the forecast function, the other notations keep the same as they are explained in Kalman filter. The measurement is assumed to be a linear relationship by adding a Gaussian white noise.

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$$z_k = Hx_k + v_k \tag{15}$$

Then the update equation can be expressed as

$$\hat{x}_k = \hat{x}_k^- + K(z_k - K\hat{x}_k^-) \tag{16}$$

Here, the Kalman gain is defined the same as it is in Kalman filter. But to calculate the Kalman gain, in Monte Carlo approach, it is different from its linear case. For N realizations, the unbiased covariance is

$$P_k^- = \frac{1}{N-1}(\hat{x}_k^- - \overline{\hat{x}_k^-})(\hat{x}_k^- - \overline{\hat{x}_k^-})^T \tag{17}$$

Where "—" over denotes the statistical mean. The covariance is in a large dimension for a typical highly underdetermined history matching problem, and it takes much computational power and storage to get it. However, it is not necessary to explicitly having it. Because we substitute the estimated covariance into equation (16), and therefore K can be expressed as

$$K = \frac{1}{N-1}(\hat{x}_k^- - \overline{\hat{x}_k^-})(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T \bullet [H^T \frac{1}{N-1}(\hat{x}_k^- - \overline{\hat{x}_k^-})[(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T] + R]^{-1} \tag{18}$$

For a highly underdetermined problem, to reduce the dimension, when calculation K, combine the term $(\hat{x}_k^- - \overline{\hat{x}_k^-})^T$ and H^T together as $[(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T]$, so K can also be expressed as

$$K = \frac{1}{N-1}(\hat{x}_k^- - \overline{\hat{x}_k^-})[(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T] \bullet [H^T \frac{1}{N-1}(\hat{x}_k^- - \overline{\hat{x}_k^-})[(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T] + R]^{-1} \tag{19}$$

In this way, without calculating the estimated covariance P_k^- explicitly, the computational power is reduced significantly. There is an inversion in calculating the Kalman gain, and the term $(\hat{x}_k^- - \overline{\hat{x}_k^-})^T H^T$ might be singular, therefore, by applying singular value decomposition, the pseudo-inversion might be required. The sampling error is inevitable for large problems, because in order to avoid prohibitive forward simulations, ensemble size N is usually much less than state or sometimes even observation size.

3 Numerical Experiments Setup

To study the EnKF history matching method on its application limitations and to find the optimal design of the ensemble size. We used the EnKF to update two reservoir types: one is a marine reservoir, which has a smooth distribution of permeability; the other is a fluvial reservoir, which has distinct facies boundaries. In experiments, we used the grid block-oriented parameterization and the reservoirs are represented by two 50 by 50 Cartesian grid blocks models. **Fig. 1.** shows the permeability model of the reference marine reservoir and the reference fluvial reservoir respectively.

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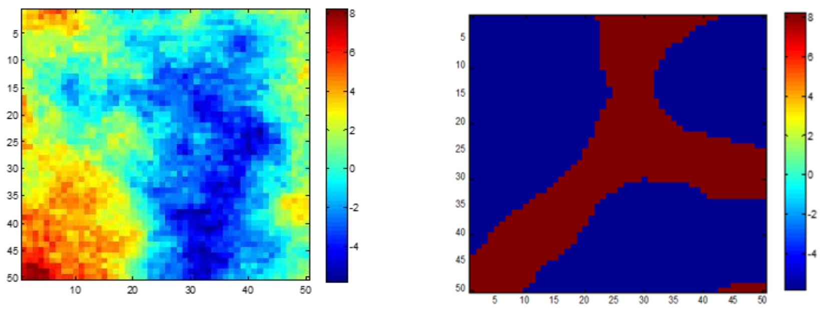
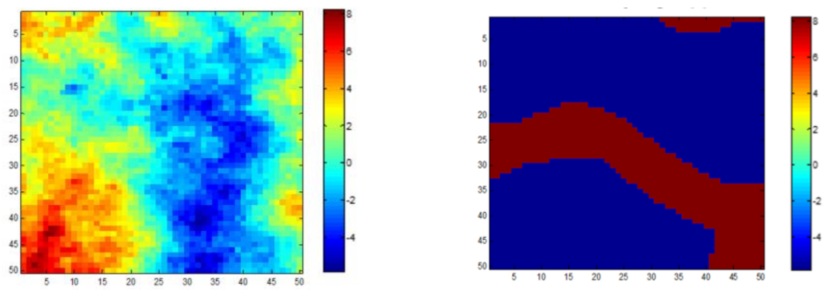


a. Marine reservoir **b.** Fluvial reservoir

Fig. 1. The reference permeability of the marine reservoir and the fluvial reservoir.

3.1 Build prior models for the Reservoirs

Both prior realizations are generated from the hard data by proper geostatistical methods. We used Sequential Gaussian simulation, and we used a training image and by sequential indicator simulation and to establish the priors of the fluvial reservoir. **Fig. 2.** represents the one of the prior ensemble of the two reservoirs respectively.



a. Marine reservoir **b.** Fluvial reservoir

Fig. 2. The reference permeability of the marine reservoir and the fluvial reservoir.

3.2 Build prior dynamic data

Typically, the dynamic data of a petroleum reservoir includes oil production, bottom-hole pressure, well water cut, oil saturation, water saturation. In this study, we used the bottom hole pressure (BHP) and water cut (WWCT) as the dynamic data. The well configuration is an inverted 9-point pattern, which consists an injector in the center and 8 producers at the edge of the reservoir. For both reservoirs, load the true permeability data for the simulator. Add the proper white noise to the outputs form the simulator.

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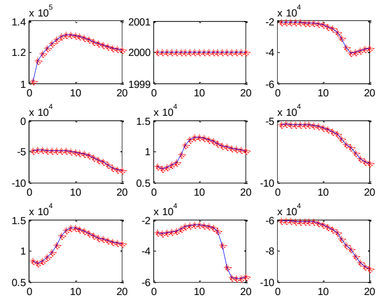
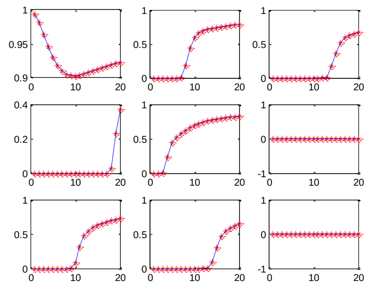
The standard deviation is 0.48 and 0.07 for BHP and WWCT noise. In this way, the observation data is generated for this project. See **Fig.3.** and **Fig. 4.** accordingly.

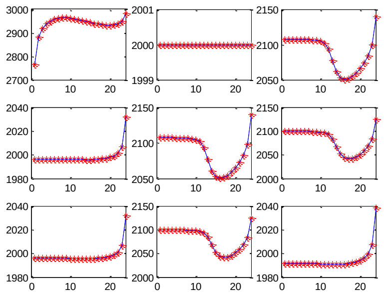


a. Bottom hole pressure data

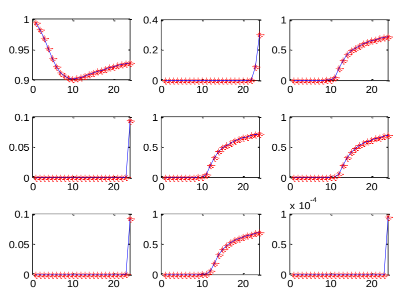


b. Well water cut data

Fig. 3. The observation data of the marine reservoir.



a. Bottom hole pressure data



b. Well water cut data

Fig. 4. The observation data of the fluvial reservoir.

4 Numerical Experiments Results

4.1 Result of EnKF history matching for the marine reservoir

After using EnKF for the history matching of a marine reservoir, with the ensemble size of 10 the results are shown in **Fig.5.** the computational time is 65 seconds; with the ensemble size of 30, the results are shown in **Fig.6.**, and the computational time is 106 seconds.

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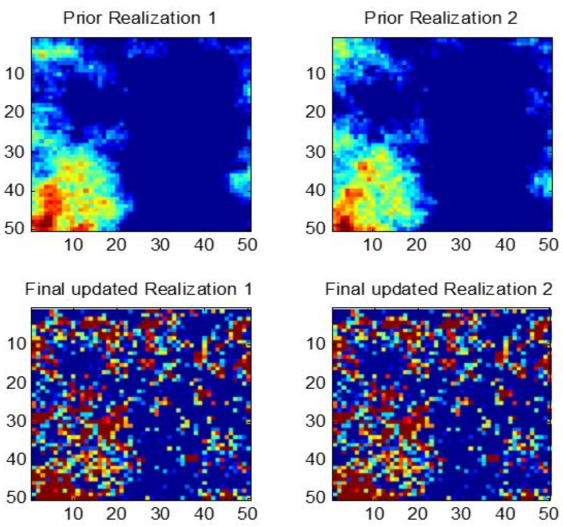

Fig. 5. The permeability model result of the EnKF history matching with the ensemble of 10.

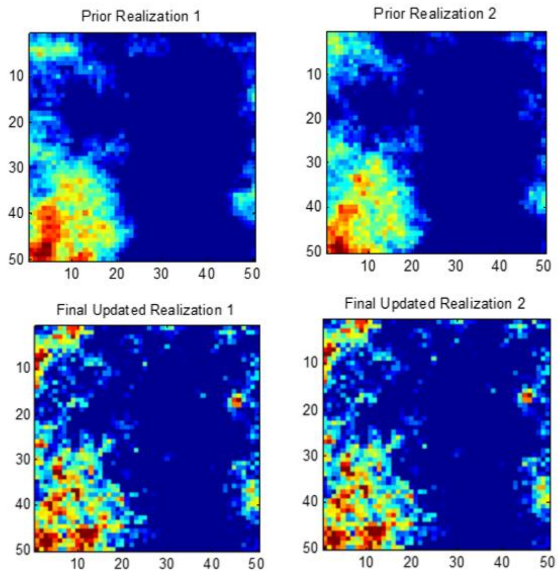

Fig. 6. The permeability model result of the EnKF history matching with the ensemble of 30.

With a larger ensemble size, the results are very likely to be improved. However, it also took longer computational time. We designed different ensemble sizes, and plotted the result in **Fig.7.** shows the relationship between ensemble size and computational time. For a practical problem, to balance the time and accuracy, an ensemble size of 100 is a popular choice, however, it is still not large enough to eliminate the errors. **Fig. 8.** shows the permeability model updates with the ensemble size of 100. Generally, the

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EnKF is more efficient than the traditional gradient based method.

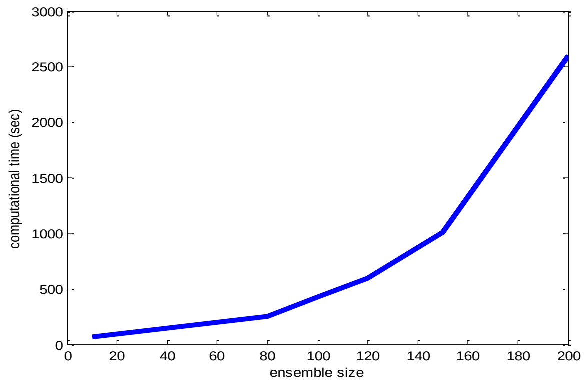

Fig. 7. The plot of ensemble size of which computational time of history matching.

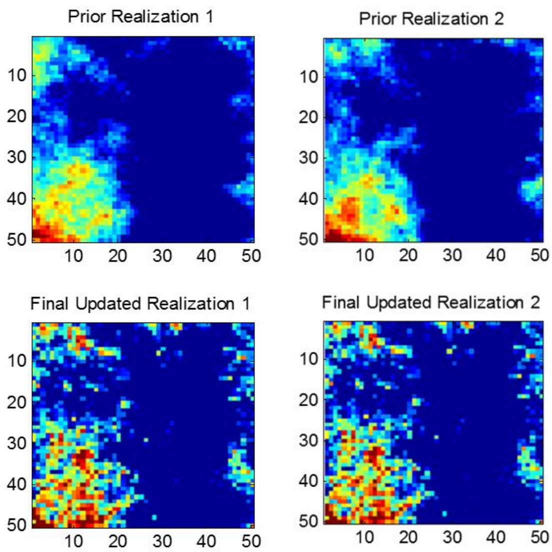

Fig. 8. The permeability model result of the EnKF history matching with the ensemble of 100.

Fig. 9. shows the history matched BHP plot and WWCT plot respectively. The red line is the read observation data, and the green lines are the simulated dynamic data with the prior ensemble.

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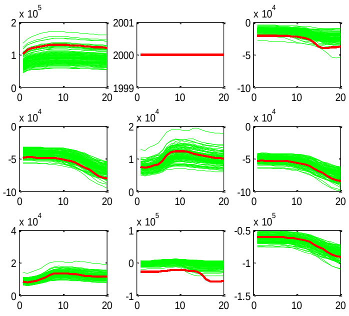
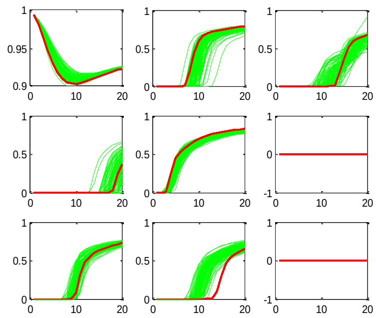


b. Bottom hole pressure data



b. Well water cut data

Fig. 9. The EnKF history matching result for the dynamic data

We can see the reliability of EnKF is highly dependent on the size of the ensemble. If it is so small that the ensemble is not statistically representative the system is said to be under sampled. Under sampling causes three major problems: inbreeding, filter divergence and spurious correlation.

Inbreeding refers to a problem that the analysis error covariance are inherently underestimated after each of the observation assimilations. The Kalman gain uses a ratio of the error covariance of the forecast state and the error covariance of the observations to calculate how much emphasis or weight should be placed on the background state and how much weighting should be given to the observations. Therefore without a proper weighting for the two terms, then the adjustment of the forecast state will be incorrect.

4.2 Result of EnKF history matching for the fluvial reservoir

Similarly, applying EnKF into history matching for a fluvial reservoir, with a binary permeability system, the results are shown in **Fig. 10.**

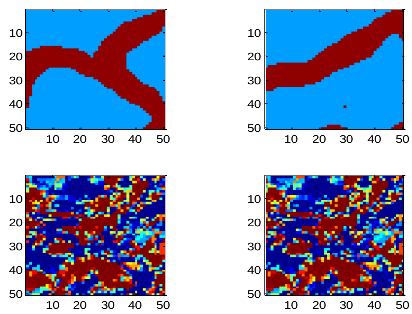


Fig. 10. The permeability model result of the EnKF history matching for the fluvial reservoir with the ensemble of 100.

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It is clearly indicated that even the prior has a binary permeability, only applying EnKF in history, the results will a system be the continuous permeability distribution. If we put an arbitrary threshold for the shale face and forced it into a binary system, the results still fallacious, since we lose the conductivity feature and lose continuity, as the **Fig. 11.** shows.

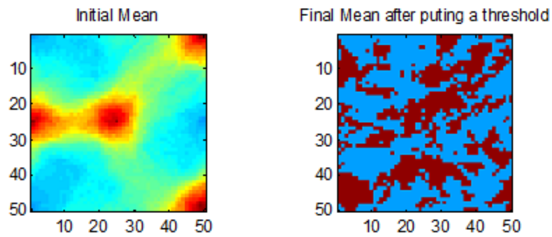

Fig. 11. The mean of the initial and updated fluvial reservoir realizations.

5 Conclusions and Recommendations

The EnKF provides a framework for real-time updating and prediction in reservoir simulation models. Every time new observations are available and are assimilated there is an improvement of the model parameters and of the associated dynamic data.

The EnKF works very well for marine reservoir which has a continuous and smooth distribution of permeability, when the prior has an efficient ensemble size, and the prior models can capture the conductivity features of the real reservoir.

To overcome the problem of under sampling, it is recommended to apply the following methods:

1) Implement covariance inflation, which can correct an underestimation in the forecast error covariance matrix. The aim is to increase the forecast error covariance by inflating, for each ensemble member, the deviation of the background error from the ensemble mean by factor. In the experiments of Whitaker and Hamill the optimal values of the inflation factors were 7% for the EnKF.

2) Covariance localization is a process of cutting off longer range correlations in the error covariance at a specified distance. It is a method of improving the estimate of the forecast error covariance. It is ordinarily achieved by applying a Schur product to the forecast error covariance matrix.

Standard EnKF can be applied in history matching for reservoirs with a continuous and smooth permeability distribution. However, for fluvial reservoirs, despite for its large computational requirement, Gradient based methods, with the first norm in its objective function, works well, and easy to implement.

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