

# Scalable Computing System with Two-Level Reconfiguration of Multi-Channel Inter-Node communication

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**Abstract.** The paper presents the architecture and organization of a reconfigurable inter-node communication system based on hierarchical embedding and logical multi-buses. The communication environment is a physical network with a bus topology or its derivatives (e.g. folded buses, mesh and toroidal bus network). In the system, multi-channel communication is forced through the use of tunable signal receivers/transmitters, with the buses or their derivatives being completely passive. In the physical environment, logical components (nodes, channels, paths) are distinguished on the basis of which logical connection networks separated from each other are created. The embedding used for this purpose is fundamentally different from the previous interpretations of this term. Improvement of communication and computational efficiency is achieved by changing the physical network architecture (e.g. the use of folded bus topologies, 2D and 3D networks), as well as the logical level by grouping system elements (processing nodes and bus channels) or their division. As a result, it is possible to ensure uniformity of communication and computational loads of system components. To enable formal design of the communication system, a method of hierarchy description and selection of its organization was proposed. In addition, methods of mathematical notation of bus topologies and the scope of their applications were analyzed. The work ends with a description of simulations and empirical research on the effectiveness of the proposed solutions. There is high flexibility of use and relatively low implementation price.

**Keywords:** Hierarchical embedding · Multi-channel architecture · Scalable systems

## 1 Introduction

Until recently, it was thought that personal computers would meet the needs for most users in the field of science and industry. There is a possibility of remote use of supercomputers, however, at the time of the almost universal use of artificial

intelligence and the mass of cyber attacks, such assumptions became incorrect. It is a result of the fact, that performing calculations using AI methods may turn out to be unsafe from the perspective of the possible occurrence of cyber attacks. This situation can significantly limited possibility the correctness of the results or even disturb the proper functioning of the system based on them. The security aspect is extremely important for users, so sometimes during remote access there are concerns about the integrity or confidentiality of the transmitted data. The construction of cheap, easy-to-use and scalable computer units with increased resilience to possible damage from cyber attacks has become deliberate.

There may be several solutions to the above problem. The first of them is usage of commonly available computing units based on Arduino and Raspberry. The creation of an appropriate infrastructure consisting of a dozen or so devices of this type can significantly improve the computing power needed to perform more complex tasks. The idea behind the authors of the text is to create a supercomputer structure made of the aforementioned devices together with the definition of the method of communication.

The second solution to the problem involves the use of Internet of Things devices. The authors assume that the communication structure created by them will allow for the connection of various types of devices used every day, which are elements of the Internet of Things. Each of them has its own computing powers, which, when combined with each other, make it possible to obtain additional resources needed to perform complex calculations.

The authors decided to look for solutions to the above problem in the area of application of passive optical communication to connect computational units based on Arduino or Raspberry platforms. Previous studies in the design, construction and operation of multi-machine units assumed, as in most cluster systems, the integration of computational nodes by switching electrical signal in the network or transport layer [23, 24]. This solution provided scalability of computational power, however, balancing communication channels was difficult to achieve.

Another solution described in the literature was the use of optical technologies similar to those used, among others in WDM switches, however, many times cheaper [30]. A special optical system sent a light wave to the input of the appropriate receiver. Also in this case, the solution was effective in increasing the computational power of parallel computer.

The proposed solution is based on passive optical technologies. The basis of the whole system is a multi-channel optical bus network, based on which various derivative bus networks are built using hierarchical embedding. The network configuration is performed from the level of processing nodes, which greatly simplifies its operation. The use of a multi-channel optical connection network allows to efficiently connect or disconnect computational nodes from the subsystem. Depending on the formally determined need, the subsystem autonomously changes its architecture, organizing its resources to ensure maximum efficiency, reliability or minimum response time to service requests.

The result of the research is to prepare the technical basis for the functioning of the infrastructure consisting of Internet of Things devices or Arduino and Raspberry units. Regardless of the chosen method of managing computing power, proper communication should be ensured. The proposed idea assumes improvement in the following areas: **a.** availability for everyone (due to the low cost of production), **b.** scalability - the ability to connect additional devices in the event of a need for additional computing power, **c.** safety - in the event of a threat, you can turn off specific devices that have been infected without having to disconnect the entire system.

## 2 Multi-channel, embedding and hierarchy in the construction of connection networks

The basis for new communication solutions is hierarchy understood in a slightly different way. So far, analyzing data transmission systems, the focus has been on two-level architectures based on the projection of logical topology on physical network resources [7, 19]. Nowadays, when using hierarchy to build scaled computational environments based on IoT, this approach trivializes the problem and has several significant disadvantages. The most important of them are:

1. Focusing on logical paths as the basic elements of connection network, which simplifies the problem solved too much;
2. Ignoring the hierarchy of functional relationships of communication architecture components;
3. Limitation to two amounts of analyzed levels, preventing the synthesis and analysis of real systems;
4. Lack of parameters determining the level of physical and logical correlation of connection characteristics;
5. Application of classic graph or matrix representations to the morphological description of the network, limiting thus the possibilities of its study.

These disadvantages can be minimized by using graph theory methods [6, 8, 17, 29] and multi-level hierarchical systems [12, 27, 28]. Therefore, connection architecture should be reduced to a multi-level hierarchical structure. Then, combinatorial optimization methods implemented to solve the tasks of multi-level hierarchical systems theory can be used for its synthesis and analysis. Hierarchization will be performed using embedding implemented through multi-channeling. This approach assumes that the primary concept of further consideration is multi-channeling offered, among others by communication technologies used to build IoT network. In order to limit the area of interest, it was assumed that the channel hierarchy has a multi-echelon organization [9, 10, 26].

Analysis of the use of IoT technology in hierarchical networks requires an original definition of the embedding concept. In classic works [1, 2, 21] embedding consists in placing guest graph in the host graph nodes, for which the mapping function is used. The result of embedding is a new network whose parameters are inherited from the source graphs. Until now, Cartesian product was most often

used as a mapping function, ensuring additive and multiplicative inheritance of parameters. This embedding is of theoretical nature and has been called topology embedding. Previous research focused on using the Cartesian product to build scalable topological organizations, sorting networks and fault-tolerant connection networks. Standard topologies, such as ring, tree, hypercube, toroid, etc. were used as bases. From the point of view of considered issues, topology embedding is of secondary importance.

Embedding is proposed to be expanded with new types: embedding of communication components, embedding of physical networks, and embedding of logical connection networks. The concept of communication component is defined as real or virtual transmission environment, connecting a pair of nodes (logical or physical) at the same level of the technology hierarchy. Examples of components are logical channels and paths, as well as physical components analogous to them. It is assumed that multi-channeling is available in all technologies used and is the basis for this type of embedding. As a result, there may be many subchannels of lower ( $k + 1$ ) level in the  $k$ -th layer's channel. A set of channels of  $k$ -th level can be embedded in the logical channel of ( $k - 1$ ) layer or directly in the physical channel of level 1 (so-called vertical embedding). The logical path of the  $k$ -th level can be a set of its logical channels, based on various channels of the ( $k - 1$ ) layer (so-called horizontal embedding). Embedding components is described by a hierarchy that reflects the interrelationship between inter-node communication means, called the hierarchy of communication components. An example of a hierarchy of logical (virtual) components is shown in Figure 1a. The logical channel is created by connecting two logical nodes (or more for group transmission) using a logical connection. In turn, a logical path is a sequential combination of a set of logical channels, and a logical topology is a set of paths.

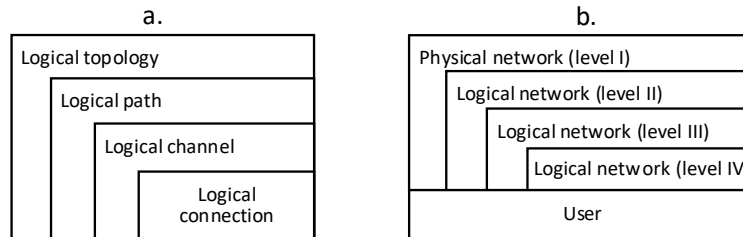


Fig. 1: Hierarchical organization of communication

A hierarchy of physical components could be presented in a similar way. Its levels are respectively: connections, channels, paths and physical topologies. A distinctive feature of the presentation from previous works is the narrowing of the set of logical topologies used only to the bus and its derivatives. Logical networks are built on their basis, the range of which is very wide.

Embedding an hierarchization can also be used to build a hierarchy of communication networks, which in general, consists of a core, a set of access networks, and end-user serving nodes. In this case, we are talking about embedding physical connection networks. This type of embedding is similar to the traditional one, described among others in [1, 2, 25]. The lower level connection network is embedded in a higher layer network node or in a set of them. From the graph theory point of view, access networks are represented by a forest of disconnected graphs. Because embedding may not apply to all nodes and can only include a limited set of them, the hierarchy of physical connection networks is heterogeneous. An example of this hierarchy is shown in Figure 1b. Although in real solutions the end user is primarily connected to level IV access networks, there is nothing to prevent him from joining to other layers, including directly to the core of the network.

Another new type of embedding is connection networks embedding. Unlike the classical approach, the lower-level connection network is inscribed in all or a significant part of the upper-layer network, and not only in its communication node or channel. This is accomplished by embedding logical paths of  $k$ -th layer in the network of  $(k - 1)$  level, while maintaining the equivalence relationship of nodes of different hierarchy levels. The result of such embedding will not be new networks, but only implementation of lower-level networks on network resources located higher in the hierarchy of connection networks. The graph theory properties of the embedded network are determined solely by itself, and the transport characteristics passed through levels higher than the one on which the network was created. This embedding can be considered as a functional development of embedding communication components. In the literature, network embedding has a two-level character (the logical network is embedded on physical network resources) [1, 2, 22]. The conducted research did not limit the number of embedding levels constituting the hierarchy.

The last type of embedding described in this paper is the embedding of technology, involving the introduction in the environment of one communication technology, another, using the resources of the first. This solution is already known and described in the literature [3, 11]. An example of technology embedding is widely used Ethernet network allocation on MPLS network resources functioning in the WDM physical network environment. Also in this case, on the basis of embedding, a hierarchy of technology is created, whose layers are associated with a specific level of communication, and not with a specific type.

According to the classification presented in Figure 2, the basic element forming any hierarchy is the communication channel, understood as the environment of information transmission at the physical level. Based on this, using hierarchy of components, basic communication paths are created from the transmission point of view. These, in turn, constitute the basis of hierarchically connected interconnection networks that are used as the environment for functioning of communication technologies. Summarizing, the global hierarchical structure is built by embedding the hierarchy of components in the network hierarchy, and the result obtained in the hierarchy of communication technologies.

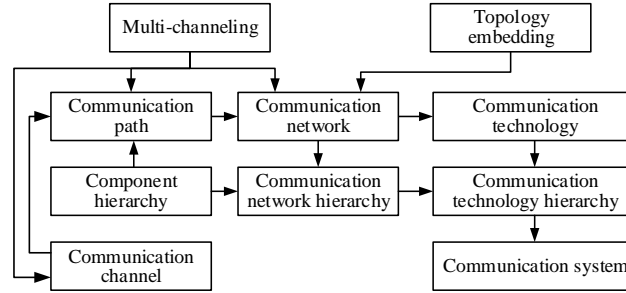


Fig. 2: Relationship between hierarchical communication components

The synthesis and analysis of systems described as multi-level hierarchical structures is one of the known directions in the study of large-scale systems [12, 27]. For every hierarchy considered (components, networks, technologies), it is recommended to formally determine the number of levels, the selection of elements for every individual layer and the ways of their connection, so that the resulting structure is characterized by minimal construction and operation costs and maximum efficiency. Until now, based on a compact description, a set of acceptable structures and criteria for their assessment was defined. In addition, the task of hierarchy synthesis is performed only at the qualitative level, and quantitative models are either not considered at all, or have only a special character [4, 5]. To formally solve the above task, it was proposed to use methods of construction an optimal hierarchy, previously used mainly in management, control and bioinformatics [18, 20].

### 3 Bus networks and their mathematical representation

In the further part of the paper it is assumed that the physical network of the communication system is based solely on the bus, so let's consider its definitions. A bus network is a combination of two types of equal objects  $N$  nodes and  $B$  buses. Each node can be incidental with any number of buses. The incidental means that each node can be connected with any number of buses. A network of  $n$  nodes and  $m$  buses is usually marked as  $[n, m]$  and describes the incidence matrix  $A = \{i, h\}$  with size  $n \times m$ . Element  $a_{ij} \in A$  is equal to 1 if with node of number  $i = 1, \dots, n$  there is incidental bus with number of  $j = 1, \dots, m$ , otherwise  $a_{ij} = 0$ . From the definition of the incident matrix it follows that bus networks do not allow loops for both buses and nodes. Therefore, if there are multiple connections in the designed system (i.e. the selected node will be integrated with the selected bus with several connections), the traditional way of describing the bus will not be able to be used.

If the number of incident buses with  $i$ -th node (node level) is  $s_i^w = 1, \dots, n$ , and the number of incident nodes with  $j$ -th bus (bus level) as  $s_j^m$ , then for any

bus network  $[n, m]$  there is a relationship between the summary degree of nodes and buses:

$$\sum_{i=1}^n s_i^w = \sum_{j=0}^m s_j^m = s \quad (1)$$

Expression (1) is the basis of the bus network synthesis method developed by the authors with single connections presented, among others in [13]. Unlike networks with direct connections, for the bus network  $[n, m]$  with the incidence matrix  $A$ , there is always a network with the transposed incidence matrix  $A^T$ , where  $[n, m]^T = [m, n]$ .

Expression (1) also shows that bus networks and hypergraphs are structurally equivalent. To analyze the buses represented as hypergraphs, graph theory tools can be used to present the hypergraphs as bipartite graphs.

**Definition 1.** *Undirected graph  $G = (V, E)$  is a bipartite graph, if the set of its vertices can be divided into two parts  $X_0 \cup X_1 = V$  in such a way that: no vertex from the part  $X_0$  is connected with the vertices of part  $X_0$ ; no vertex from the part  $X_1$  is connected to the vertices of part  $X_1$ . In this case, the subsets  $X_0, X_1$  are called parts of bipartite graph  $G$ .*

If additional connections appear in the analyzed system, for example additional cables integrating buses with processing nodes, PBL neighborhood graphs can be used to describe the bus processing system. Let's assume that the number of nodes in parts  $X_0, X_1$  of the bipartite graph is equal to  $n$  and  $m$  respectively. Its edges are local connections  $l_{p,q}$ , where:  $p, q$  - number of processing nodes and buses, respectively, whose task is to connect the computational node with a bus. This description is in fact a description of the PBL neighborhood graph.

**Definition 2.** *The PBL graph  $G = (V, B, L)$  containing  $|V_G| = n$  nodes,  $|B_G| = m$  buses and  $L_G$  link set is the bipartite graph  $G_{PBL}$  which can be described by following pair:  $(V_{G_{PBL}}, B_{G_{PBL}})$  and  $V_{G_{PBL}} = VV_{G_{PBL}} \cup VB_{G_{PBL}}$ , where  $VV_{G_{PBL}} = V_G$  and  $VB_{G_{PBL}} = B_G$ . Connections in graph  $G$  between buses and nodes are represented by  $B_{G_{PBL}}$ . The nodes  $v_{G_{PBL},i}$  and  $b_{G_{PBL},j}$  are connected by the edge  $l_{G_{PBL},k}$  if and only if in the source bus system, the bus  $b_i$  is connected to the node  $v_j$  with link  $l_k$ .*

The representation of the bus system according to Definition 2 is shown in Figure 3a. In the IIoT and in traditional information systems, buses will connect two types of nodes: service providers  $K$  (measuring sensors, thin clients, etc.) and service recipients  $S$  (computational servers, data). Then, tripartite graphs should be used to represent the bus system. A graphical representation of the connection network with a single channel bus is shown in Figure 3b.

In order to analyze the characteristics of the bus connection network, algebraic topology notation based on algebra of connected finite non-directed graphs has been proposed. Graph algebra is defined as follows:

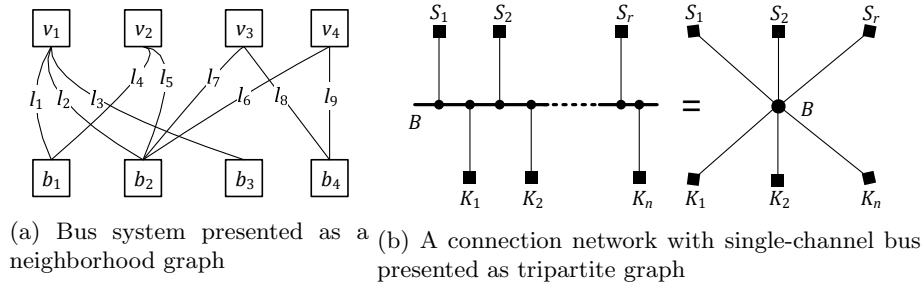


Fig. 3: Possible bus system representations as different graphs

**Definition 3.** The pair  $A = (D, \Omega)$  will be called the universal graph algebra over the universe  $U$ , if  $D$  is a set of graph with vertices from the  $U$  set, and the  $\Omega$  signature enables the zero operation  $\Lambda$ , binary operations for adding a vertex, adding edges, remove the vertex and remove edges.

First, let's define the theorem, which will be then used to write the topology of selected networks.

**Theorem 1.** The minimal elements of the algebra  $AG = (A, \Omega)$  will be trees of the form  $G_u^0$ , where:  $G_u^0 = (V = \{u\}, E = \emptyset)$  - an empty tree composed of the vertex  $u$ .

*Proof.* In the graph algebra, trees play a special role, because of them any connected graph is built. However, if the end vertex removal operation is applied to the tree, it is reduced to the tree of  $T_u^0$  form. That is why empty trees can act as minimal elements in algebra. Empty trees, consisting of only one vertex, cannot be further reduced.

Using the theorem 1, following trees will be analyzed:

1.  $T_{u,v} = (V = \{u, v\}, E = \{(u, v)\})$ ;
2.  $T_{u,v}^w = (V = \{u, v, w\}, E = \{(u, w), (w, v)\})$ .

Using algebra, the above trees can be represented by the minimum elements:

1.  $T_{u,v} = (V = \{u, v\}, E = \{(u, v)\}) = w_{ik}(G_u^0 \cup G_v^0, u, v) = T_{u,v} = G_u^0 * G_v^0 = G_u^0 * f G_v^0(f(u) = v)$ ;
2.  $T_{u,v}^w = (V = \{u, w, v\}, E = \{(u, w), (w, v)\}) = T_{u,w} \cup T_{w,v} = w_{ik}(T_u^0 \cup T_w^0, u, w) \cup w_{ik}(T_w^0 \cup T_v^0, w, v) = (T_u^0 * T_w^0) \cup (T_w^0 * T_v^0)$ .

where:  $w_{ik}$  - the operation of adding an edge into connected graph;  $*$  - the operation of combining two connected graphs.

In addition, the result of combining two graphs will be a connected graph, if even one or two of them does not meet connectivity condition. In particular, for a connected graph  $G$ , graphs corresponding to the expressions  $(T_u^0 \cup T_v^0) * G$



and  $(T_u^0 \cup T_v^0) * (T_u^0 \cup T_v^0) * (T_w^0 \cup T_s^0)$ , where:  $u, v, w, s$  in pairs different vertices, will be connected graphs. The algebraic expression describing the network in Figure 3b in graph algebra notation has the following form:  $(T_{S_1}^0 \cup \dots \cup T_{S_r}^0 \cup T_{K_1}^0 \cup \dots \cup T_{K_n}^0) * T_B^0$ .

Technical solutions developed on the basis of research are based solely on the multi-channel bus. Suppose the complete multi-bus (i.e. each service provider and recipient is connected to each of the buses) consists of  $m$  channels,  $S_r$  recipients and  $K_n$  service providers. Then its physical form and its notation in the form of a tripartite graph have the form presented in Figure 4. It can be assumed from a technical point of view, that the  $B_1, \dots, B_m$  buses are logical channels functioning in the one common physical channel.

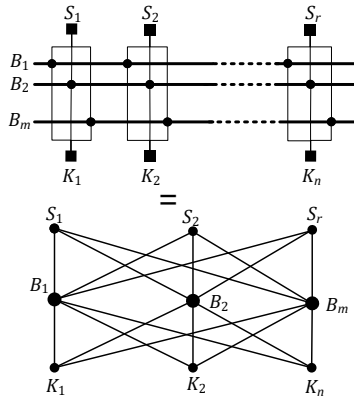


Fig. 4: A network with multi-channel bus presented as a tripartite graph

The algebraic expression describing the above network has the following form:  
 $(T_{S_1}^0 \cup \dots \cup T_{S_r}^0) * (T_{B_1}^0 \cup \dots \cup T_{B_m}^0) \cup (T_{B_1}^0 \cup \dots \cup T_{B_m}^0) * (T_{K_1}^0 \cup \dots \cup T_{K_n}^0) = (T_{S_1}^0 \cup \dots \cup T_{S_r}^0 \cup T_{K_1}^0 \cup \dots \cup T_{K_n}^0) * (T_{B_1}^0 \cup \dots \cup T_{B_m}^0)$

Graph algebra also allows to describe solutions in which the preference relationship appears. The formalization of the above process has been extensively described in other works of the authors [14, 15]. Informally, however, connections between service providers  $K$  and service recipients  $S$  can be described by the following rules:

1. The recipient prefers the selected service provider. Preferences are not permanent and can be changed without restrictions during work;
2. The connection of service recipients with service providers is performed by means of logical bus channels.

## 4 Modification of the multi-bus networks

The modifications to the bus networks described are intended to improve their functional parameters, in particular to compensate for bus communication loads and computational processing nodes. They all require a specific physical network organization.

In traditional bus networks, the communication network is single-channel and each of the system components is connected to it once. The bus usually carries out broadcasts on one common channel. In the case of traditional multi-channel buses, permanent user assignment to a specific logical channel is most common. In the offered solution, the logical communication channel is selected by the transceiver, which can be tuned. Thanks to this, the communication system can highlight the channel through which the node will communicate with the outside world. In addition, it is possible to equip computational nodes with a variable number of transceivers. It allows one or many times to join any logical bus or their set.

Because physical bus lengths are small, device prices are relatively low. Built groups (node clusters) have their own communication environment and can be isolated from external interference. For example, a separate group can be created by users using services insensitive to communication delays, another generating low, traffic, yet another having bursty character of the generated traffic. Logical buses are also grouped (clustered). In this way, not only computational power, but also the bandwidth of logical communication channels is scaled. Mixed grouping, in which computational nodes and logical communication buses are used, is the most effective. The multiplicity of node interfaces allows the use of folded buses (single, double or triple) ensuring better usage of the bus. Another method consists in the hierarchization of logical channels where channels are combined into groups that support different sets of servers. It is also possible to divide overloaded buses into smaller parts.

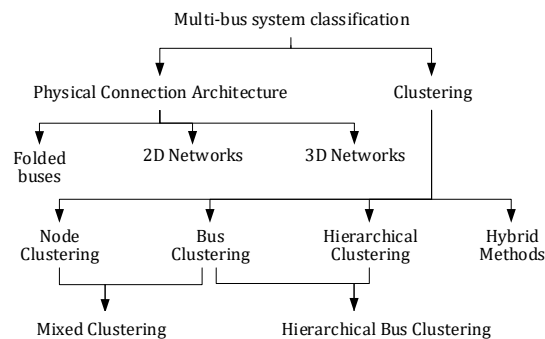


Fig. 5: Classification of the multi-bus system organization

Thanks to the variety of methods proposed, the connection architecture can be adapted to the traffic patterns present in the network (characteristic values of the communication load of the channels) and communication requirements of the clients. Because in optical systems, the change in wavelength used by the transceiver takes milliseconds, adapting the connection architecture to the current requirements of users can be dynamic and can be implemented in real time. The classification of methods for improving the efficiency of bus connection system architecture is shown in Figure 5.

## 5 Simulations and empirical studies

In order to validate the concept of building new communication environment, simulation and empirical studies were performed. Various organizations of the communication system and the resulting computational system were examined, the generalized architecture was presented in [16]. Simulation studies concerned the optical multi-bus communication environment, and empirical were calculated for the electrical one.

A model simulation basic operational parameters has been prepared for each of the architectures. The models are based on the probability and queue theory. The organization of analyzed solutions is defined below. In complete bus systems, each service provider and recipient is connected to each of the logical buses once. In systems with single connections, they are made only to one of the buses. Usually this applies only to service providers. In a multi-channel hierarchical bus system, logical buses are divided into parts and then combined into groups with identical division into parts. Division parameters are defined by:  $u$  - number of hierarchy levels;  $k_u^i$  - the number of bus components of the  $i$ -th hierarchy level;  $\kappa_j$  - a way to connect service providers to  $j$ -th level buses. In hierarchical buses with a limited partition coefficient at the recipient's output, the number of their connections is limited in advance. In a homogeneous request model,  $\omega$  means the likelihood of making requests. Additionally, the following symbols have been introduced for the hierarchical request model:  $\omega_p$  - probability of creating a request to the preferred service provider;  $\omega_k$  - probability of requesting the same group of service providers;  $\omega_O$  - probability of request to other groups;  $p$  - priority of the request. Obtained results are presented in Figure 6 and Figure 7.

In the Figure 6 we can see a features of multi-bus systems. Figure 6a presents hardware complexity of multi-bus system with  $B_m = 16$ . We have a five different bus. The bus (e) is a Hierarchical bus limited partition at output  $u = 4$ ,  $k_u^2 = k_u^3 = k_u^4 = 2$ ,  $\kappa_2 = \kappa_3 = \kappa_4 = 2$ . Figure 6b presents bandwidth dependence on number of logical elements for a complete bus system with  $K_s = K_k = 16$ ,  $\omega_p = 0.6$ ,  $\omega_k = 0.3$ ,  $\omega_O = 0.1$ .

In the Figure 7 we can also see a features of multi-bus systems, but this indicate a bandwidth of transmission. Figure 7a presents dependence of bandwidth on the number of processing elements for the performance of computational system with a hierarchical and homogeneous request model,  $B_m = 16$ ,  $u = 3$ ,  $k_u^2 = k_u^3 = 2$ ,  $\kappa_1 = \kappa_2 = 2$ ,  $\kappa_3 = 4$ ,  $\omega_1 = 0.5$ ,  $\omega_2 = \omega_3 = 0.25$ . Figure 7b presents

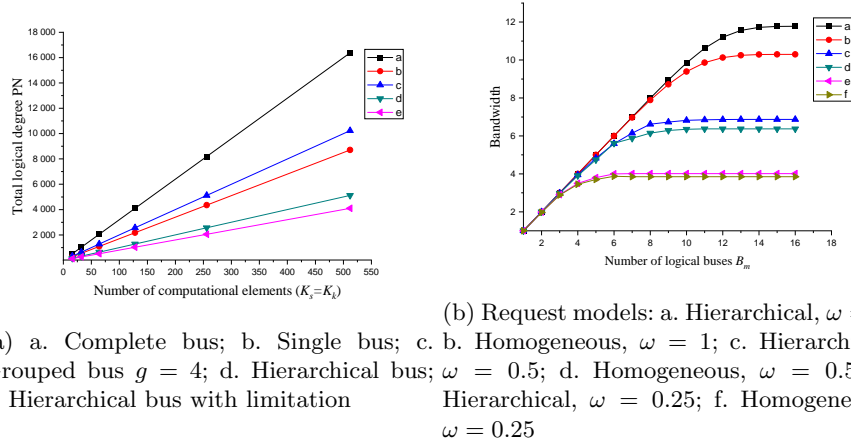


Fig. 6: Features of multi-bus system

dependence of the probability of handling the request on the number of logical buses. At the beginning, when the number of computational elements are low, the bandwidth for (a),(c) and (b),(d) buses (pairwise) is identical. The difference can be seen after the buses become saturated - they are loaded with traffic. The hierarchical version offers then much better performance than homogeneous one.

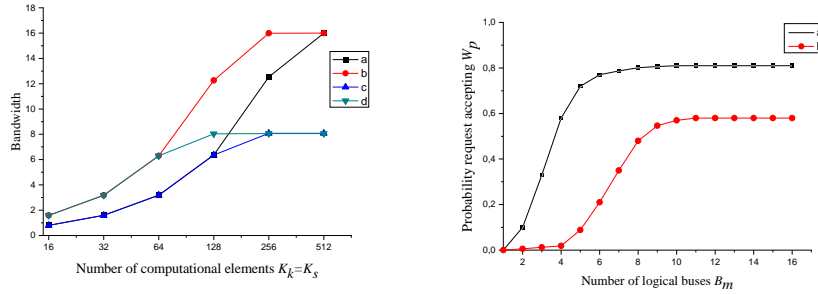


Fig. 7: Features of multi-bus systems

## 6 Summary

The research showed the desirability of construction and operating multi-bus optical communication environments both in the case of computational power scalability as well as its communication efficiency. Particularly favorable results were obtained when balancing the communication loads of the system. Balancing computational loads depends on many other factors and the results obtained are not as good, although still satisfactory.

While the reconfiguration process during simulation tests and empirical observations went smoothly, communication within a single bus channel seems unsatisfactory. This applies to the number of interfaces connected to the bus channel - the channel functioned efficiently with a smaller number of interfaces connected than it resulted from earlier calculations based on the conditional probability. The findings made indicate the need to modify the access protocol.

Further improvement of the system properties (in particular, communication efficiency) should be seen in the use of physical network with grid or toroid bus topology, as well as similar 3D topologies. This will require the development of original routing algorithms.

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