Analysis of complex partial seizure using non-linear duffing Van der Pol oscillator model

Beata Szuflitowska[0000-0002-5825-7234] and Przemyslaw Orlowski[0000-0001-6407-9668]

West Pomeranian University of Technology,
70-313 Szczecin, Sikorskiego 37, Poland
Email bszuflitowska@zut.edu.pl

Abstract. Complex partial seizures belong to the most common type of epileptic seizures. The main purpose of the case study is the application of the Van der Pol model oscillator to study brain activity during temporal left lobe seizures. The oscillator is characterized by three pairs of parameters: linear and two nonlinear, cubic and Van der Pol damping. The optimization based on the normalized power spectra of model output and real EEG signal is performed using a genetic algorithm. The results suggest that the estimated parameter values change during the course of the seizure, according to changes in brain waves generation. In the article, based on values of sensitivity factor of parameters, and, sample entropy non-stationary of considered seizure phases are analyzed. The onset of the seizure and the tangled stage belongs to strongly non-stationary processes.

Keywords: Van der Pol oscillator, EEG, parameter estimation, biological signal.

1 Introduction

Electroencephalography (EEG) reflects the averaged electrical activity of neurons associated with different neural processing placed in different brain regions and structures [1-2]. The International Federation of Societies for Electroencephalography and Clinical Neurophysiology considers EEG as non-invasive, safe for human health and easily controlled by clinicians technique [2]. The EEG signals belong to non-stationary and quasi-rhythmic signals that produce contained oscillations [3-5]. The main purpose of EEG measurement is the diagnosis of epilepsy [6-7]. The epilepsy is theoretically characterized by abnormal synchronization between brain regions. In 2017, the International League Against Epilepsy (ILAE) released a new classification of seizure types, including focal motor and non-motor onset, generalized motor, and absences, unknown motor and non-motor onset and the last unclassified types [8]. Time series during some epileptic seizures or Parkinson's disease are much more ordered oscillatory than in healthy records [2, 5, 7, 9]. EEG signals have been studied in literature as a Bag-of-Words modelas a random and the back propagation (BP) neural networks, and coupled oscillators [10-12]. Interesting results related to modelling of a biological control system using a system composed of two coupled internal van der Pol oscillators [13]. A classical van der Pol attractors are applied to distinguish between chaotic and stochastic behaviors of stationary EEG recorded from five normal subjects. The modification of Van der Pol oscillators ,i.e. a generalized Van der Pol equation with fractional-order derivative and parametric excitation is derived from the Fitz–Hugh–Nagumo equations

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

or the Wilson and Cowan model was considered as might be an efficient tool to control the dynamics of the action potentials [14-15]. Even more popular in the analysis of EEG time series is duffing oscillator alone or in combination with Holmes and Lorenz oscillators [13]. Based on the results presented in the literature using these oscillators Ghorbanian at all proposed a coupled duffing Van der Pol Oscillator Model to distinguish two states healthy and Alzheimer's disease [2, 16]. While analyzing the literature, we noticed the possibility of using the couple duffing system in the analysis of epileptic seizures. The duffing equations lead to show different phase states between normal and epileptic signals [17]. Therefore we used the proposed by Ghorbanian deterministic duffing Van der Pol Oscillator Model to modelled pre-,ictal and post-ictal signals for the first time [18]. The paper is an attempt to analyze the possibility of using the model in the detection of stages of an epileptic seizure occurring in one patient, i.e. the onset of the seizure, during patient's moves, movement automatics, tangled stage and the end of the seizure. To the best of our knowledge, this model has not been used to analyse ictal carefully extracted phases yet. The estimated values of model parameters are determined for each considered phase. The parameters have been obtained using cost function L in the form mean square of normalized power spectrum of real and corresponding generated EEG signals. Additionally, the non-stationary character of the individual ictal stage is studied and compared using sensitive optimal values u, of model parameters, and sample entropy. The signal analysis and model oscillator are presented in section 2. Sections 3 and 4 contain results and discussion, respectively. In Section 5, the main conclusions have been collected.

2 Materials and Methods

2.1 EEG Signals

EEG signals presented in this paper were recorded from right-handed 55 aged female who takes Phenytoin, at Temple University Hospital and is seizure-free since 7 months. This patient is selected for analysis for few reasons. Primarily, the description of records have been performed very carefully by the doctor. Patient behaviour has been associated with changes in the brain wave patterns. At the beginning of the EEG, the patient was calm and relaxed. The seizure begins at the end of hyperventilation when the patient's resting comfortably [19]. Digital video EEG is performed in the lab using standard 10-20 system of electrode placement with 1 channel of EKG. We considered sequences 10 s (length of samples N=2500) registered by electrode T3. The sequences, according to clinical description include the onset of seizure without symptoms, behavior changes in the form shaking and moving leg, the movement automatisms, the confusion and the end of the epileptic seizure.

The medical equipment records the signal in a discretized form of time. To determine the number of the sequence we introduced the parameter d, where d=1,2,..,.12. Therefore, the signal will be marked hereinafter as $x^d(n)$. Before calculating the discrete Fourier transform of each sample of EEG sequence has been multiplied by the appropriate Blackman's window coefficient:

$$x_w^d(n) = x^d(n)\,w(n) \tag{1}$$

where:

$$w(n) = 0.42 - 0.5\cos(2\pi n/N - 1) + 0.08\cos(4\pi n/N - 1)$$

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

The discrete Fourier transform (DFT) takes the form:

$$X^d(k) = \sum_{n=0}^{N-1} x_w^d(n)\,\omega_N(n,k) \tag{2}$$

where:

$\omega_N = \exp(-j2\pi nk/N)$ is the N^{th} root of unity.

Next, the amplitude of DFT of the signal is normalized in the range of [0, 1] according to the following formula:

$$\hat{X}^d(k) = \frac{\left|X^d(k)\right|}{\max\left|X^d(k)\right|} \tag{3}$$

The power P_b^d of normalized DFT amplitude sequences in five major frequency bands are calculated according to the formula:

$$P_b^d = \frac{1}{|S_b|}\sum_{k\in S_b}\left(\hat{X}^d(k)\right)^2 \tag{4}$$

where: b=1,…,5 is the number of frequency band, S_b - set of discrete frequencies, corresponding to five major frequency bands [2]: delta (δ, 1-4 Hz, b=1), theta (θ, 4-8 Hz, b=2), alpha (α, 8-13 Hz, b=3), beta (β, 13-30 Hz, b=4) and gamma (γ, 30-60 Hz, b=5).

2.2 Duffing Van der Pol Oscillator

The coupled system of duffing Van der Pol oscillators analyzed in this section was proposed by Ghorbanian and all to distinguish healthy and Alzheimer's disease signals [1, 5]. A four state equations representing coupled duffing Van der Pol oscillators model can be written as:

$$\begin{aligned}
\dot{x}_1^m &= x_3^m \\
\dot{x}_2^m &= x_4^m \\
\dot{x}_3^m &= -\left(\varsigma_1+\varsigma_2\right)x_1^m + \varsigma_2 x_2^m - \varsigma_1\left(x_1^m\right)^3 - \rho_2\left(x_1^m - x_2^m\right)^3 + \varepsilon_1 x_3^m\left(1-x_1^m\right) \\
\dot{x}_4^m &= \varsigma_2 x_1^m - \varsigma_2\left(x_1^m - x_2^m\right)^3 + \varepsilon_2 x_4^m\left(1-\left(x_2^m\right)^2\right)
\end{aligned} \tag{5}$$

where superscript m indicates that the signal is generated by the model, ς is the linear stiffness coefficient, ρ is the nonlinear stiffness coefficient, which indicates the strength of the duffing nonlinearity resulting in multiple resonant frequencies, ε is the Van der Pol damping coefficient which determines the strength of van der Pol nonlinearity. Parameters ς_1, ρ_1, ε_1 and ς_2, ρ_2, ε_2 belong to the first and second oscillator, respectively. The output may be selected as any combination of the positions and velocities to mimic an EEG signal. In this study, the velocity of the second oscillator is selected as the model output. The initial conditions are equal to:

$$x_1^m(0) = 0,\ x_2^m(0) = 1,\ x_3^m(0) = 0,\ x_4^m(0) = 0$$

Runge-Kutta iterative method is selected from standard numerical integration methods to solve these dynamic equations. An important mathematical property of Runge-Kutta methods is their stability properties which make them suitable to solve a wide

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

class of problems. The amplitude of the generated signal is discretized and normalized according to the formula 2-3. The components of the power spectra of the signal generated by the oscillator model in five considered frequency bands were also needed to build the objective function. The genetic algorithm is adapted to perform the optimization procedure. The algorithm encodes the decision variables of a search problem into finite-length strings of alphabets of certain cardinality. In the optimization process, we set the population size as 2000. The cost function is chosen as a root mean square of the errors in the power spectra of signal EEG P_b^d and generated $\breve{P}\left(\varsigma_1,\varsigma_2,\rho_1,\rho_2,\varepsilon_1,\varepsilon_2\right)$ in each selected brain frequency band, as shown in [3]. The functions L can be formally written as (7):

$$L(\mathbf{\Omega},\ d)=\sum_{\nu=1}^{5}\left(P_b^d-\breve{P}\left(\varsigma_1,\varsigma_2,\rho_1,\rho_2,\varepsilon_1,\varepsilon_2\right)\right)^2 \qquad (6)$$
$$\mathbf{\Omega}=\left[\varsigma_1,\varsigma_2,\rho_1,\rho_2,\varepsilon_1,\varepsilon_2\right]$$

where: L is the cost function, $\mathbf{\Omega}$ is the vector of design model variables, ς_1, ς_2, ρ_1, ρ_2, ε_1, and ε_2 are the decision variables of the optimization. The initial guesses for the optimization search were randomly generated within the bounds defined as:

$$0 \le \varsigma_{1,,2} \le 200,\ 0 \le \rho_{1,,2} \le 100,\ 0 \le \varepsilon_{1,,2}$$

The optimization goal is error minimization:

$$\min_{\varsigma_1,\varsigma_2,\rho_1,\rho_2,\varepsilon_1,\varepsilon_2} L\left(\varsigma_1,\varsigma_2,\rho_1,\rho_2,\varepsilon_1,\varepsilon_2\right) \qquad (7)$$

The solution set of decision variables for each sequence d will be denoted as:

$$\varsigma_1^d,\varsigma_2^d,\rho_1^d,\rho_2^d,\varepsilon_1^d,\varepsilon_2^d$$

3 Results

The analysis based on the optimal values of parameters, relative sensitivity factor of them and the objective function is performed taking into account types of sequences. Table 1 shows the results obtained in highlighted phases of the epileptic seizure.

It can be clearly seen, that high estimated values of linear stiffness parameters ς_1 and ς_2 are obtained for sequences associated with changes in the patient's behavior: during leg movement, movement automatics, and entanglement. The initial seizure is accompanied by high values of non-linear stiffness coefficients ρ_1 and ρ_2. A high value of ρ_2 is also observed in the final stage of the seizure. The small values of cost function L indicate obtaining a similar power spectrum of the real and generated signal. The obtained values of sensitivity coefficients for the onset are very high. In order to assess the application of sensitivity factor of optimal parameter values in the determination of range non-stationary nature of the signal, the sample entropy is calculated for each considered sequence. The results are collected in Table 2. According to the value of the sensitivity factor, the initial stages of the seizure are characterized by the highest entropy values.

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

Table 1. The model's parameters, the sensitivity factor *u*, and the cost function *L*

Phase		Estimated Values			Sensitivity factor *u*		
		Seq.1	Seq.2	Seq.3	Seq.1	Seq.2	Seq.3
Onset	ς_1	10.79	11.79	10.97	4567.00	424.82	-215.60
Leg movement		131.88	88.99	54.00	4764.00	1218.00	-123.70
Movement automatics		164.00	120.34	11.32	0.25	44.85	9.85
Entanglement		98.81	53.56	69.69	164.40	330.19	-98.18
End		12.81	12.74	8.33	0.05	8.07	0.87
Cost function *L*		0.06	0.03	0.06			
Onset	ς_2	59.33	47.33	40.50	4567.00	424.82	-215.60
Leg movement		164.00	120.34	13.24	0.56	1.43	-1.08
Movement automatics		35.84	45.54	37.03	8.11	23.07	2.01
Entanglement		139.63	115.00	72.39	-72.84	3.40	139.63
End		18.34	17.63	22.22	0.07	1.10	0.28
Cost function *L*		0.18	0.22	0.06			
Onset	ρ_1	68.61	48.90	35.67	4715.00	22.69	-58.90
Leg movement		28.21	31.54	31.89	0.34	0.77	0.07
Movement automatics		54.20	47.23	45.02	8.27	70.91	2.44
Entanglement		33.97	43.00	59.59	172.82	-495.01	-98.18
End		6.72	6.45	7.87	0.09	6.44	1.27
Cost function *L*		0.06	0.19	0.06			
Onset	ρ_2	70.52	54.20	48.45	4875.00	481.49	-87.60
Leg movement		49.91	52.13	35.00	0.34	0.60	-0.79
Movement automatics		51.00	49.91	64.82	10.75	69.74	0.88
Entanglement		99.55	81.53	56.99	164.40	330.19	99.55
End		93.45	110.32	84.45	0.49	13.89	1.73
Cost function *L*		0.08	0.03	0.06			
Onset	ε_1	7.53	5.13	3.43	2416.00	51.10	52.55
Leg movement		5.59	6.34	17.16	0.44	17.16	0.50
Movement automatics		2.78	4.29	4.72	2.21	0.07	-1.17
Entanglement		19.06	13.66	16.02	0.60	3.75	19.06
End		16.75	15.60	23.03	0.60	3.34	0.95
Cost function *L*		0.17	0.21	0.07			
Onset	ε_2	15.35	23.34	24.67	-6287.0	173.81	149.97
Leg movement		14.35	13.45	20.00	0.07	20.00	-0.07
Movement automatics		12.34	25.10	14.86	44.60	62.34	-5.38
Entanglement		22.59	28.47	19.17	0.49	-265.84	25.59
End		21.34	20.97	22.63	0.49	13.89	1.73
Cost function *L*		0.08	0.03	0.06			

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

Table 2. Values of the sample entropy calculated for the each sequence.

Phase	Sample entropy		
	Seq.1	Seg.2	Seq.3
Onset	0.24	0.12	0.18
Leg movement	0.22	0.16	0.08
Movement automatics	0.004	0.0004	0.05
Entanglement	0.03	0.13	0.08
End	0.08	0.03	0.08

4 Discussion

The seizures described in the work are characterized by a sudden onset occurring within the neural network in the left hemisphere [19]. The results show that the onset epileptic seizure is a strongly non-stationary process. In the early stages of the seizure, according to the description, in the record high-amplitude left temporal spike and slow-wave complexes. The female began to move her left leg. The increase of amplitude that occurs here can be due to an increase in the force acting on the springs, which is represented by linear stiffness parameters ς_1 and ς_2. Next, the patient has motor automatisms. In the EEG recording can be seen slow frontal delta waves, which, combined with the characteristic of the waveform of the temporal lobe, associated with high, similar values of ς_2, ρ_1, and ρ_2. In this phase, occur the greatest changes in the value of the sensitivity factor of the model parameters (from 0.07 for ε_2 to 70.91 for ρ_1). During the seizure, the patient experiences further behavioral changes, including confusion. According to the doctor describing the study, the EEG record during cis difficult to interpret at this moment. High values of ς_1, ς_2, ρ_1, and ρ_2 changing the kinematics of the seizure, which allows the spread of discharges to the other hemisphere (two excitation networks). Under the influence of damping, the force decreases. At the end of the seizure, in the EEG recording bilateral slow brain wave synchronization with sharp waves is observed. The stationary character dominates until the seizure has completely ceased. The low values ς_1 and ρ_1 suggest, that the first oscillator no longer stimulates the system to further vibrate. The seizures initially persist due to higher values of the second oscillator.

5 Conclusions

Having some facts about temporal lobe epilepsy, including individual phases of seizure propagation, the deterministic coupled duffing Van der Pol oscillator model is proposed to model the brain activity of epileptic patients. From the results obtained from an individual patient, it is shown, that the proposed model explains the problem of the generation of different rhythms of the EEG ictal signal. The model allows determining the EEG changes that occur during the seizure. An increase in linear parameter values is seen during slow delta and sharp wave detection,and the generation of fast spikes and high-amplitude is associated with decrease values of liner parameters and increase values of nonlinear cubic parameters.

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33

References

1. Verma, A. Radtke, R. EEG of Partial Seizures. Journal of Clinical Neurophysiology 4(23), 333-339 (2006).
2. Ghorbanian, P. Non-Stationary Time Series Analysis and Stochastic Modeling of EEG and its Application to Alzheimer's Disease. Ph.D. dissertation, Villanova University, 2014.
3. Sornmo, L. Laguna, P. Bioelectrical signal processing in cardiac and neurological applications. Burlington. MA: Elsevier Academic Press (2005).
4. Fisher, R.S. Cross, J.H. French, J.A. et all, Operational classification of seizure types by the International League Against Epilepsy: Position Paper of the ILAE Commission for Classification and Terminology. Epilepsia, 4(58), 522-530 (2017).
5. Ghorbanian, P. Ramakrishnan, S. Ashrafiuon, H. Stochastic non-linear oscillator models of EEG: the Alzheimer's disease case. Frontiers in Computational Neuroscience, 9(48) (2015). DOI: 10.3389/fncom.2015.00048.
6. Acharya, U.R. Molinari, F. Vinitha, S. Chattopadhyay, S. Automated diagnosis of epileptic EEG using entropies. Biomedical Signal Processing and Control, 4(7), 401-408, (2012).
7. Soltesz, I. Staley, K. Computational Neuroscience in Epilepsy. Elsevier Science, (2011).
8. Fisher, R.S. Cross, J.H. D'Souza, C. Instruction manual for the ILAE 2017 operational classification of seizure types. Epilepsia, 4(58),531-542 (2017).
9. Botcharova, M. Modelling and analysis of amplitude, phase and synchrony in human brain activity patterns" Ph.D. dissertation, University College London (2014).
10. Hussein, R. Elgendi, M. Wang, J.Z. Ward, R.K. Robust detection of epileptic seizures based on L1-penalized robust regression of EEG signals. Expert Systems with Applications, 104, 153-167 (2018).
11. Ghorbanian, P. Ramakrishnan, S. Ashrafiuon, H. Nonlinear Dynamic Analysis of EEG Using a Stochastic Duffing-van der Pol Oscillator Model, proceedings of the ASME 2014 Dynamic Systems and Control Conference, DOI: 10.1115/DSCC2014-5854.
12. Liu, L. Recognition and Analysis of Motor Imagery EEG Signal Based on Improved BP Neural Network. *IEEE Access*, 7, 47794 – 47803, April 2019, DOI. 10.1109/ACCESS.2019.2910191
13. Ohsuga, M., Jamaguchi, J.Schimuzi, H. Entrainment of two coupled van der pol oscillators by an external oscillation, *Biological Cybernetics* volume 51, pages 325–333(1985).
14. Kawahara, T. Coupled Van der Pol oscillators-A model of excitatory ad inhibitory neural interactions Biological Cybernetics 39, 37–43 (1980).
15. Tabi, C., B. Dynamical analysis of the FitzHugh-Nagumo oscillatons through a modified Van der Pol equation with fractional-order derivative term. International Journal of Non-Linear Mechanics, 105, 173-178 (2018).
16. Meijer, H.G.E. Eissa, T.L. Kiewiet. B. et all, Modeling Focal Epileptic Activity in the Wilson–Cowan Model with Depolarization Block. J Math Neurosci, 5(7), 2015.
17. Rakshit, S., Bera, B., Majhi, S. et al. Basin stability measure of different steady states in coupled oscillators. Sci Rep 7, 45909 (2017). https://doi.org/10.1038/srep45909
18. Szuflitowska B., Orlowski P., Statistical and physiologically analysis of using a Duffing-van der Pol oscillator to modeled ictal signals, ICARCV Shenzhen, China, 2020, pp. 1137-1142, doi: 10.1109/ICARCV50220.2020.9305339.
19. Obeid, I. Picone, J. Harabagiu, S. Automatic Discovery and Processing of EEG Cohorts from Clinical Records. In Big Data to Knowledge All Hands Grantee Meeting (p. 1). Bethesda, Maryland, USA: National Institutes of Health, 2016. [Online]. www.isip.piconepress.com/publications/conference_presentations/2016/nih_bd2k/cohort/.

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_33