

# An Attempt to Replace System Dynamics with Discrete Rate Modeling in Demographic Simulations

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**Abstract.** The usefulness of simulation in demographic research has been repeatedly confirmed in the literature. The most common simulation approach to model population trends is system dynamic (SD). Difficulties in a reliable mapping of population changes with SD approach have been however reported by some authors. Another simulation approach, i.e. discrete rate modeling (DRM), had not yet been used in population dynamics modelling, despite examples of this approach being used in the modelling of processes with similar internal dynamics. The purpose of our research is to verify if DRM can compete with the SD approach in terms of accuracy in simulating population changes and the complexity of the model. The theoretical part of the work describes the principles of the DRM approach and provides an overview of the applications of the DRM approach versus other simulation methods. The experimental part permits the conclusion that DRM approach does not match the SD in terms of comprehensive accuracy in mapping the behavior of cohorts of the complex populations. We have been however able to identify criteria for population segmentation that may lead to better results of DRM simulation against SD.

**Keywords:** Computer simulation · Population modeling · Decision support  
· Cross-sectional analysis · Operations research · Healthcare demand modeling

## 1 Introduction

Reliable demographic projections are often a key and essential part of research on a wide range of socio-economic issues, such as forecasting health needs [1], designs of pension schemes [15], shaping of urban plans [14], natural interest rate forecasting [23] and others. Population analysis can be conducted, among others, using simulation methods, from which system dynamics approach (SD) is particularly often selected. This choice is justified by the specific characteristics of the SD, namely: causal loops, a macro-perspective focused on the entire population rather than on individual members of the population, and the form of mathematical relationships that facilitate the mapping of birth, death and migration processes. The usefulness of SD in demographic research has been repeatedly confirmed in the literature, starting with Forrester's work in the late 1960s and early 1970s [8]. However, it is reasonable to note there are some difficulties in a reliable mapping of population changes with this approach [16]. Eberlein and

Thompson [6] discussed the so-called cohort blending problem of distortions in age structure: people entering a cohort at the same time "grow up" into the next cohort at different times. To overcome this problem, authors proposed the solution which assumes that the range of age-gender cohorts coincides with the simulation step. The disadvantage of this solution is the need to over-detail the model and to provide the model with data relating to one-year age groups. In our previous research [18] we proposed to modify the approach described in [6]. We suggested the use of so-called "cohort modeling", based simultaneously on complex (multi-annual) and elementary (annual) cohorts, in which the initial state and the intensity of the input and output stream can be taken individually. In the specific case, the parameters of the elementary cohorts that make up the compound cohort may be identical.

An analysis of the literature showed that another simulation approach, i.e. discrete rate modeling (DRM), had not yet been used in population dynamics modelling, despite examples of this approach being used in the modelling of processes with similar internal dynamics of change, cf. [5, 24]. The DRM literature emphasizes the advantages of this approach, such as the quick reaction to changes in state variables, an avoidance of rounding and a speed of calculation. The purpose of our research is to verify if DRM can compete with the SD approach in terms of accuracy in simulating population changes and the complexity of the model measured by the number of blocks used. To ensure an unbiased comparison between the two approaches we extended the SD model, developed during our previous studies, with an additional module of an elementary cohort according to DRM approach. Therefore, we do not compare the time needed to get results using individual approaches.

## 2 Rationale for the Study

Our previous research focused on the application of a simulation approach to study healthcare services [17-20]. We used the SD approach in demographic modeling and studied the impact of demographic changes on the volume of demand for health services. As part of this research, we developed a model [18] that maps 36 aggregate gender-age cohorts. Each cohort includes 5-year-old group of people, except for the oldest female and male cohort. The oldest cohort, i.e. 85+ is made up of 20-year-old group. Each aggregated cohort contains a number of (5 or 20) annual elementary cohorts. Thanks to the "cohort modeling" approach, we eliminated the so-called cohort blending problem described in [6]. The developed model has passed verification, but its disadvantage is the high complexity forced by the need to simultaneously control elementary and aggregate cohorts. Simplifying a model can be done in many ways, such as introducing blocks with a more versatile application, applying blocks tailored to a specific problem, or using a different simulation method. Our proposal is to apply the DRM approach in place of the SD approach inside of the hierarchical blocks. We will conduct all the research in the Extendsim environment.

Initial attempts to study the usefulness of the DRM have already been made by one of the authors. A comparison of DEVS (discrete event simulation) and DRM approaches was presented in the work [29]. It has been proved that DRM far exceeds

DEVS performance, especially when dealing with high flow rate of objects. Moreover, duration of the simulation run in DRM approach is independent of source streams intensity and initial resource values.

### 3 The Concept of DRM Approach

#### 3.1 Original Purpose of DRM

DRM is the approach for modelling the flow of uniform materials between different points. The flow is described by the actual flow rate (allowable and current one) and the initial and current state of the tanks with which the particular stream interacts. DRM can also be used to model systems whose dynamics are continuous or hybrid, i.e. continuous-discrete. The DRM approach can also be combined into a single, but compound, model with a discrete approach. For instance, discrete part may relate to road transport modelling and the DRM part may be related to the loading/unloading of bulk, liquid or even gaseous materials, i.e. when mapping processes of various and mixed materials, such as the production of foods [2, 24].

#### 3.2 Principles of Calculations of DRM Simulation

The key concepts in the DRM approach are *tank* and *stream* (valve). The flow rate that fills or empties the tank is described by two values: the maximum (acceptable) flow rate in a given stream, and the actual (efficient) flow. The actual flow intensity depends on external conditions. For the incoming stream it depends on the availability of the material in the source tank and the capacity of the target tank. For the outgoing stream, the actual flow intensity depends on the availability of the flowing material in the source tank and the available capacity of the target tank. The above rule is called flow maximization [5] rule. The DRM approach implemented in Extendsim already provides possibilities for modeling discrete events related to the initiation or termination of a continuous process. This is the case when, for example, the vehicle gradually fills up the entire cargo space and it can drive away with the load. It is also possible to mix materials from different sources, when many source streams merge into a single output stream. In the output stream, it can be detected whether the mixture actually flows efficiently in it, or whether the intensity of one of the flows is zero. In conclusion, changes in the flow rate and the condition of the tank are mutually determined.

The uniqueness of the DRM [13] is also the ability to use SD concepts in DEVS environment. This is especially useful for modeling fast-changeable or large-scale processes that highlight flows, flow rate, events, constraints, storage capacity and routing. The bigger accuracy of simulations in the DRM approach is due to avoiding rounding errors that occur when time clocks are incompatible between continuous models and the discrete nature of events. Similarity to SD is due to the capturing of the reality by flow categories. The difference is that in SD we have fixed intervals between calculations and in DRM calculations occur when events occur. Similarity to DEVS is that model state changes occur only at discrete points over time. However, in DRM, calculations are performed only when the intensity of one of the flows changes. According

to [13] it is possible to predict the occurrence of an event related to, for example, a system's state condition and reduce the number of calculations and the total error.

## 4 Literature Background

### 4.1 DRM Concept in Research

One of the early versions of the DRM library was called SD Industry. It was used by Siprelle and Phelps [24]. Authors pointed out that the DRM approach can be used in the food and pharmaceutical industries, when mass, continuous or semi-continuous materials are flowing at a fast rate. The alternative (DEVS) approach is not well suited for mapping these processes, as it focuses on individual parts or pieces of the flowing substance. Damiron and Nastasi [5] proved that in a situation where the main role is played by flows of things or fluids it is not appropriate to apply the DEVS or SD, and DRM is recommended. Additional advantages of the DRM approach include the ability to control the flow direction by prioritizing it and avoiding potential synchronization problems related to the creation of hybrid models that include discrete elements.

The literature contains studies comparing three basic simulation approaches, i.e. DEVS, SD and DRM. Muravjovs et al. [21] claimed that DRM resembles SD in terms of intensity and the usage of quantity/time units but the incorporation of randomness into the model is similar to DEVS. They also showed that results of simulation in the SD approach have a sharp step change, while the simulation results in the DEVS approach are gradual with minor changes of a step-by-step nature. The DRM approach produces more accurate results, represented by a fixed slope chart, without changes of a step-by-step nature. Similarly, Terlunen et al. [27] indicate that the DRM approach combines the key ideas and advantages of both DES and SD. They attempted to use DRM in supply chain modeling and identified two main problems. One problem is the need to perform conceptual abstractions to link supply chain elements and model elements. The second problem arises, because in the DRM approach an element of model can represent the flow of only one material type while real systems produce and transmit many different products and materials.

### 4.2 Demographic Simulations

We reviewed the literature on the use of simulation models in analysis of demographic processes. Our goal was to identify the target area of the demographic model and the type of simulation approach. In all models considered below, the following elements occur: resources, incoming streams, outgoing streams, initial states, and parameters. These are fundamental elements of any SD approach however implemented through different terms and technologies. Each of the following research problems does not solely concern the demographic model. The results of demographic simulations are used in target models, representing phenomena in areas such as finance, healthcare, construction and others.

Van Sonsbeek [25] presented a simulation model to analyze the impact of an ageing population on the burden of the pension system. He pointed out some shortcomings of

the presented model, such as: too few age-gender cohorts, failure to consider different models of households and types of family life, inclusion in the model only state pensions. McGrattan et al. [15] studied risks to the pension system, healthcare and long-term care expenditure arising from the decline in the population and the lowered rate of working per pensioner in Japan. Because expenditures per person increase with age the authors stressed the need to take into account the heterogeneity of different cohorts. Giesecke and Meagher [9] studied the professional activity of individual social groups in Australia. They used a simulation model to predict the impact of population ageing on skills shortages and consumption levels. Han [10] used simulation model to predict the natural rate of interest that supports the economy at full employment while keeping an inflation at constant level, considering the ageing of Japan's population in current and future years. The natural interest rate in the light of demographic changes in the euro area was also addressed in [23]. In this model the overlapping generations were considered and the most important reason for the changes was the number of people at the working-age evaluated according to the age-dependent productivity over the number of people in the entire population. Colacelli and Corugedo [4] used demographic forecasts to study several scenarios for economic reform, such as monetary policy choice and debt ratio. Lauf et al. [14] built a city population model to study housing demand patterns. Feeding the model with data from demographic forecasts is also crucial in predicting future healthcare needs. Kingston et al [12] predict that in the coming decades there will be a decrease in the need for assistance among younger older adults. Simulation was used in a number of studies to forecast the demand for long-term care [1], [7], [28]. Best et al. [3] based on death certificates, developed a model to analyze all-cause premature mortality and the commonest causes of premature death among different population groups. However, as indicated by the cancer incidence data provided by [26], morbidity rates for individual age cohorts may vary over time. Satyabudhi and Onggo [22] used a discrete parallel simulation approach in demographic research. They pointed out that it is more important to track the structure of the population than the size of the entire population.

In our opinion, in most of the examples discussed, the DRM approach can be used while retaining all the functionalities of models. We will use a multi-cohort model of regional population which was developed in accordance with the SD approach during our previous study [18]. We will suggest to replace the SD approach with a DRM approach. The use of the DRM approach should hypothetically allow for a more cross-cutting analysis of cohorts related to the size of residence, family situation, level of education or state of health. In models dealing with the impact of demographic change on the need for health services, the DRM approach should allow for the inclusion of healthcare facilities locations, thresholds for care qualifications, health status and etc.

## 5 Methods

### 5.1 Population Model

The reference population model describes two genders separately. Each age group is therefore represented by two cohorts: male and female. The model is based on blocks

representing 1-year populations, equipped with an interface for growing from the pre-age group (if any) and up to the following age group (if any), taking into account births in age group 0, deaths in all groups, and migrations. The model provides an option of using two schemes to determine the initial values of the 1-year population status within aggregate cohorts. These are: assignment of uniform initial states and descending assignment according to triangular distribution. The selection of the scheme is carried out after the execution of test simulations. A total of 210 one-year-old cohorts, 34 five-year-old cohorts and two 20-year-old cohorts were used in the model.

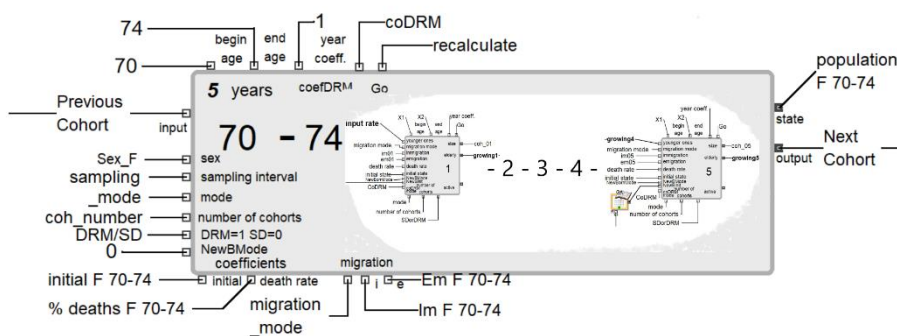
The model requires the following data :

- initial states for all 5-year cohorts and 20-year cohort,
- annual migration volumes (in absolute terms) for all higher-level cohorts.
- the birth rate and the death rate, which is reported in proportion to the population status at the beginning of the year for all higher-level cohorts.

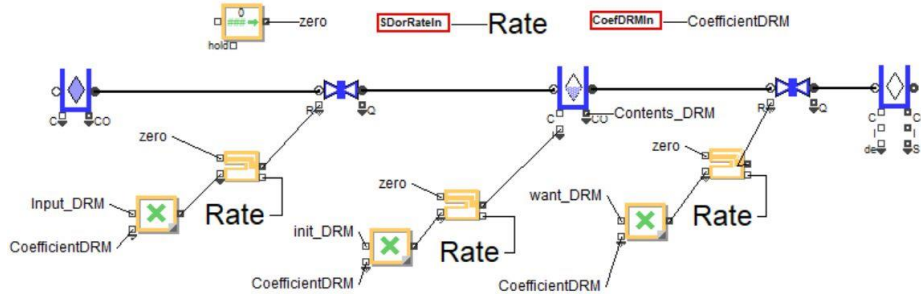
## 5.2 The Plan of Experiments

The research problem that we want to consider is as follows: will the replacement of the SD approach by DRM be efficient for a complex model which requires a wide range of input parameters and contains hundreds of parameters?

Each simulation experiment requires a range of tests to be performed to check for specific combinations of numeric values representing run parameters of the model. For example, the modeler needs to specify a time step for continuous simulation [11] to reconcile the batch order of objects in a DEVS simulation, as well as to agree on the order of calculations when performing parallel calculations in both approaches. Therefore it was decided that hierarchical blocks representing age-gender cohorts will be checked for different combinations of numerical values: initial state, birth rate and deaths rate. Fig. 1 presents the SD and/or DRM population model at higher levels of hierarchy and Fig. 2 presents DRM population model (at low level).



**Fig. 1.** The structure of one hierarchical (five-year) block representing a women's cohort aged 70-74. The block consists of five blocks (numbered 1 to 5) representing persons of a specific age range. It has 2 output connectors (cohort's population and rate of aging) and one main input connectors (rate of aging from younger cohort). ExtendSim environment was used



**Fig. 2.** Internal structure of a block representing persons of a specific age; fragment of DRM model. Extendsim environment was used

We want to verify if the use of the DRM approach will make it possible to obtain results comparable to the SD approach, taking into account the following criteria: absolute or relative deviations of simulation results from actual values and deviations of simulation results in the SD and DRM approach. Due to the models complexity, their modularity and the fact that observations take a matrix form (36 cohorts over many years), in part of our research we adopted extended criteria to consider the variation in deviations in each module. If the hypothesis cannot be considered true, we are interested in whether the DRM approach can be used to improve the SD approach, and when the DRM will be a better-rated approach than SD.

## 6 Results

### 6.1 Experiment 1: Comparing SD and DRM Results with Real Data

The first experiment was performed with a population model consisting of dozens of birth-population-death structures. This model is based on authors' previous study [18]. The model includes 36 main cohorts of the Wrocław, Poland region.

We take into account the size of the population by cohort: F0-4, F5-10, ..., F85+ and M0-4, M5-10, ..., M85+ from 2006 to 2019. We use the available actual data to verify the results of SD and DRM models. Calculations start in 2006. End-of-year simulation results are used to check the accuracy of calculations. The compatibility of the two models, i.e. the SD and DRM model, is compared using the X indicator (Formula 1).

$$X = \frac{\text{real population} - \text{simulated population}}{\text{real population}} \quad (1)$$

In the most favorable variant, the X will be equal to zero. A positive value of X occurs when the simulation result is less than the actual value. A negative value occurs when the simulation result is higher than the actual value. Because actual and simulated populations can only have positive values, the above claims are true. We start the experiment by performing simulations according to the SD (Table 1). Because there is no random factor in the model, a single run is sufficient to obtain simulation results.

**Table 1.** X-indicator values for the SD model (REAL-SD)/REAL (all values in percentages), separately for male and female population. MAD: Mean Absolute Deviation.

| Year | M0-4        | M5-9  | M10-14 | ...  | M80-84 | M85+       | F0-4          | F5-9  | F10-14 | ... | F80-84 | F85+  |
|------|-------------|-------|--------|------|--------|------------|---------------|-------|--------|-----|--------|-------|
| 2007 | 1.70        | -2.21 | -6.39  | ...  | -1.84  | -8.13      | 2.55          | -2.50 | -0.81  | ... | 1.79   | -6.04 |
| ...  | ...         | ...   | ...    | ...  | ...    | ...        | ...           | ...   | ...    | ... | ...    | ...   |
| 2019 | 3.86        | 3.36  | 10.98  | ...  | 6.01   | -22.54     | 3.41          | 3.11  | 12.15  | ... | 5.72   | -5.53 |
| MAD  | 4.63        | 5.61  | 3.58   | ...  | 3.79   | 10.41      | 4.81          | 5.83  | 3.81   | ... | 3.87   | 2.97  |
|      | Mean (Male) |       |        |      |        | 3.4        | Mean (Female) |       |        |     |        | 2.96  |
| Year | 2007        | 2010  | 2013   | 2016 | 2019   | Mean (All) |               |       |        |     |        | 3.18  |
| MAD  | 1.95        | 3.44  | 3.54   | 2.79 | 3.94   |            |               |       |        |     |        |       |

The largest deviations occur for the M85+ cohort. This cohort is characterized by a small population size and diverse death rates. The number of deaths increases rapidly as the age increases and the size of annual cohorts decreases, but these changes are not described by linear function. Moreover, there is a lack of the age structure data for the oldest cohorts. The relatively small mean deviation for the F85+ cohort can be explained by higher number of women than men of this age and perhaps an incidental but proper adjustment of the inner age structure of this cohort to the assumed variability in the model. Another explanation is that a uniform decrease in the size of one-year cohorts in both age-gender groups of the oldest population was assumed. Due to the lower life expectancy for men, probably after the age of 85 the number of individuals in this cohort decreases sharply, while in the F85+ cohort the number of women decreases gradually and is more evenly distributed over time.

We did a similar analysis for the DRM (Table 2) and we do not observe problems related to the M85+ cohort.

**Table 2.** The X values for the DRM model (REAL-DRM)/REAL (all values in percentages), separately for male and female population. MAD: Mean Absolute Deviation.

| Year | M0-4 | M5-9  | ... | M75-79 | M80-84 | M85+  | F0-4 | F5-9  | F10-14 | ..  | F40-44 | ..  | F85+  | MAD  |
|------|------|-------|-----|--------|--------|-------|------|-------|--------|-----|--------|-----|-------|------|
| 2007 | 3.48 | -3.09 | ... | 3.05   | 2.89   | 10.71 | 4.24 | -3.34 | -4.07  | ... | -2.27  | ... | 12.38 | 4.30 |
| 2008 | 7.4  | -4.72 | ... | 2.99   | 0.25   | 4.77  | 8.03 | -4.53 | -4.82  | ... | -3.95  | ... | 6.14  | 4.40 |
| 2009 | 8.03 | -4.53 | ... | 2.49   | -0.19  | -0.48 | 8.29 | -4.01 | -6.23  | ... | -3.83  | ... | 4.12  | 4.30 |
| ...  | ...  | ...   | ... | ...    | ...    | ...   | ...  | ...   | ...    | ... | ...    | ... | ...   | ...  |
| 2017 | 2.12 | 6.13  | ... | 0.59   | 1.43   | 4.42  | 2.40 | 6.64  | 8.52   | ... | 0.52   | ... | 5.86  | 3.52 |
| 2018 | 2.60 | 3.18  | ... | -2.52  | 5.81   | -3.50 | 2.73 | 2.82  | 12.61  | ... | 0.58   | ... | 3.44  | 3.96 |
| 2019 | 1.27 | 2.47  | ... | -4.27  | 3.40   | -1.17 | 0.80 | 2.23  | 13.45  | ... | 0.07   | ... | 3.64  | 3.93 |
| MAD  | 4.34 | 6.26  | ... | 2.44   | 2.47   | 3.78  | 4.45 | 6.42  | 5.67   | ... | 2.12   | ... | 6.31  |      |

For almost all years the SD simulation results were above the actual values. In the DRM, simulation results are almost uniformly distributed relative to the course of actual values. In the DRM we can observe a higher mean deviation for the F85+ cohort than in SD, but it does not exceed 7%. We note that the mean deviation increases non-



monotonically as the simulation time in the SD increases. In the DRM, the mean deviation is stable, however there are cohorts for which the deviation decreases non-monotonically over time - e.g. the F85+ and F10-14 cohorts.

In conclusion, the SD approach showed an advantage over DRM in terms of mean absolute deviation for male, female and both genders. However, there are cohorts for which the mean absolute deviation is lower in the DRM approach, e.g. M75-79, M80-84, M85+, F0-4, F40-44.

**6.2 Experiment 2: Comparing SD and DRM According to the Lowest Absolute Deviation Criterion**

Table 3 shows the results of the comparison between the SD and DRM models on the basis of absolute deviation criterion. In a small majority of cohorts (254 cells), the SD is a winning approach. The DRM approach resulted in better results in 214 cells. Note that for male cohorts, both approaches were found to be better exactly 117 times. For female cohorts, significantly better results are recorded for the SD approach. For each of the cohorts in the simulation, each approach won at least once.

**Table 3.** Comparison of simulation results according to DRM and SD approaches; The DRM or SD symbol indicates smaller relative differences between the simulation result for this approach and the actual data. For example, for the M0-4 cohort in 2019, the simulation result according to the DRM approach was more similar to historical data than the result for the SD approach.

| Year   | M0-4 | M5-9 | ... | M60-64 | ... | M85+    | F0-4   | F5-9 | ... | F60-64 | ... | F85+   | DRM | SD  |
|--------|------|------|-----|--------|-----|---------|--------|------|-----|--------|-----|--------|-----|-----|
| 2007   | SD   | SD   | ... | SD     | ... | SD      | SD     | SD   | ... | SD     | ... | SD     | 9   | 27  |
| 2008   | SD   | SD   | ... | SD     | ... | DRM     | SD     | SD   | ... | SD     | ... | SD     | 13  | 23  |
| ...    | ...  | ...  | ... | ...    | ... | ...     | ...    | ...  | ... | ...    | ... | ...    | ... | ... |
| 2018   | DRM  | DRM  | ... | DRM    | ... | DRM     | DRM    | DRM  | ... | DRM    | ... | DRM    | 18  | 18  |
| 2019   | DRM  | DRM  | ... | DRM    | ... | DRM     | DRM    | DRM  | ... | DRM    | ... | DRM    | 19  | 17  |
| DRM    | 8    | 4    | ... | 3      | ... | 12      | 8      | 4    | ... | 3      | ... | 2      |     |     |
| SD     | 5    | 9    | ... | 10     | ... | 1       | 5      | 9    | ... | 10     | ... | 11     |     |     |
| DRM/SD | MALE |      |     |        |     | 117/117 | FEMALE |      |     |        |     | 97/137 |     |     |

**6.3 Experiment 3: a Hybrid Approach**

We combined the results (Table 4) of both approaches as follows: first we calculate average of the results for each cohort and then we use the weighted average formula (2):

$$Com = \alpha \times SD + (1 - \alpha) \times DRM \tag{2}$$

where *Com* means a common approach;  $\alpha$  is a number between 0 and 1; *SD* is the average value for each cohort from SD model and *DRM* is the average value for each cohort from DRM model. We would like to find out which approach gives the closest real-world results in 13 simulated years for 36 cohorts: SD, DRM, or combined SD and DRM results (*Com*). It turned out that in most cases the SD approach wins (206 out of

468), the second is the DRM approach (172 out of 468), while the combination of the two approaches won 90 times. However, the Common approach appeared to be a winner for at least one year for all cohorts.

**Table 4.** The comparison of DRM, SD and hybrid approach (Com). DRM, SD, or Com designation means the smallest relative differences between the simulation result of the given approach and actual data. Numbers mean how many times specified approach won. We chose  $\alpha = 0.74$  because it provides the highest winning frequency by the common approach across the cohorts.

| Year     | M0-4 | M5-9 ... | M60-64 ... | M85+ | F0-4 | F5-9 ... | F60-64 ... | F85+ | DRM | SD  | Com |
|----------|------|----------|------------|------|------|----------|------------|------|-----|-----|-----|
| 2007     | SD   | SD ...   | SD ...     | Com  | SD   | SD ...   | SD ...     | Com  | 7   | 23  | 6   |
| 2008     | SD   | SD ...   | SD ...     | Com  | SD   | SD ...   | SD ...     | Com  | 10  | 19  | 7   |
| ...      | ...  | ...      | ...        | ...  | ...  | ...      | ...        | ...  | ... | ... | ... |
| 2017     | DRM  | DRM...   | DRM ...    | DRM  | DRM  | DRM...   | DRM ...    | Com  | 13  | 11  | 12  |
| 2018     | DRM  | DRM...   | DRM ...    | DRM  | DRM  | DRM...   | DRM ...    | Com  | 14  | 15  | 7   |
| 2019     | DRM  | DRM...   | DRM ...    | DRM  | DRM  | DRM...   | DRM ...    | Com  | 15  | 13  | 8   |
| DRM-M(F) | 8    | 4 ...    | 3 ...      | 10   | 8    | 4 ...    | 3 ...      | 0    |     |     |     |
| SD-M(F)  | 5    | 9 ...    | 10 ...     | 0    | 5    | 9 ...    | 10 ...     | 7    |     |     |     |
| Com-M(F) | 0    | 0 ...    | 0 ...      | 3    | 0    | 0 ...    | 0 ...      | 6    |     |     |     |

MALE; DRM: 97/SD: 94/Com: 43

FEMALE; DRM: 75/SD: 112/Com: 47

#### 6.4 Experiment 4: Analysis of Results by Gender

In experiments 1 to 3, we took into account each age-gender cohort separately. However, the different cohorts are not equinumerous. For example, the M85+ cohort had 3,361 individuals in 2006 and the F25-29 cohort had 53,727. Therefore we summed up the populations separately for each gender for each year of simulation and compared the deviations for aggregated cohorts (Table 5). It turns out that DRM almost always gives the best results and the combination of methods is never worse than SD.

The significant jump in the value of deviations in 2010 (Fig. 3) is generated by the significant increase in the population of M and F in reality. This deviation decreases over the years. Looking globally, these deviations are small and for the DRM the deviation never exceeds 1 percent.

**Table 5.** Population (in thousands) by gender; actual data, relative deviation values (in percentages) for SD, DRM, and COMMON approaches.

| Year | SD  |     | DRM |     | Com |     | REAL |     | DEVIATION |      |         |      |          |      |
|------|-----|-----|-----|-----|-----|-----|------|-----|-----------|------|---------|------|----------|------|
|      | M   | F   | M   | F   | M   | F   | M    | F   | SD. M/F   |      | DRM.M/F |      | Com. M/F |      |
| 2007 | 559 | 610 | 559 | 611 | 559 | 611 | 559  | 611 | 0,04      | 0,13 | -0,08   | 0,05 | -0,02    | 0,09 |
| 2008 | 559 | 612 | 560 | 612 | 559 | 612 | 560  | 613 | 0,14      | 0,25 | -0,01   | 0,13 | 0,07     | 0,19 |
| ...  | ... | ... | ... | ... | ... | ... | ...  | ... | ...       | ...  | ...     | ...  | ...      | ...  |
| 2018 | 584 | 640 | 585 | 641 | 584 | 640 | 592  | 646 | 1,39      | 0,93 | 1,14    | 0,75 | 1,26     | 0,84 |
| 2019 | 588 | 643 | 589 | 645 | 589 | 644 | 596  | 650 | 1,37      | 0,99 | 1,14    | 0,8  | 1,26     | 0,89 |

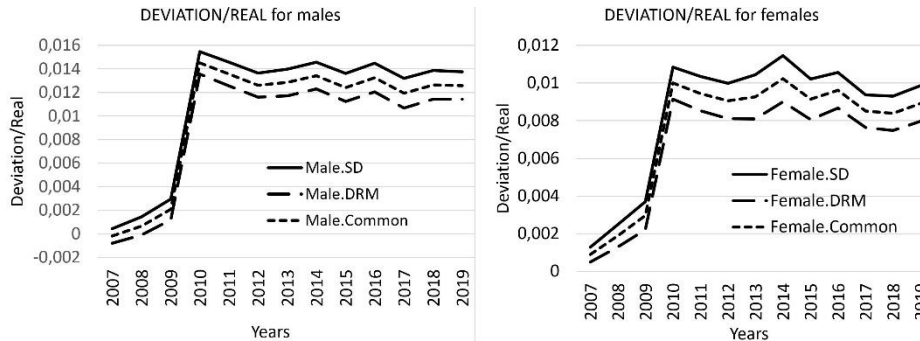


Fig. 3. Deviation graphs for SD, DRM, and COMMON relative to gender

### 6.5 Experiment 5: the Analysis of Results for Aggregated Age Cohorts

Another analysis is made for a population divided into two groups: younger (0-59) and elders (60+), Fig. 4. It is obvious that 85+ cohorts are the least numerous of all cohorts and that their compliance with actual data is the most difficult to ensure. We see that for younger by far the best is the DRM approach. However, when using other approaches, deviations are on the acceptable level equal to 1.6 %. For the elders, the SD approach is the best choice and deviations do not exceed 5 percent. Omitting population 85+ results in a decreasing difference between the two approaches. If the 85+ population is taken into account, the advantage of the SD approach is stable.

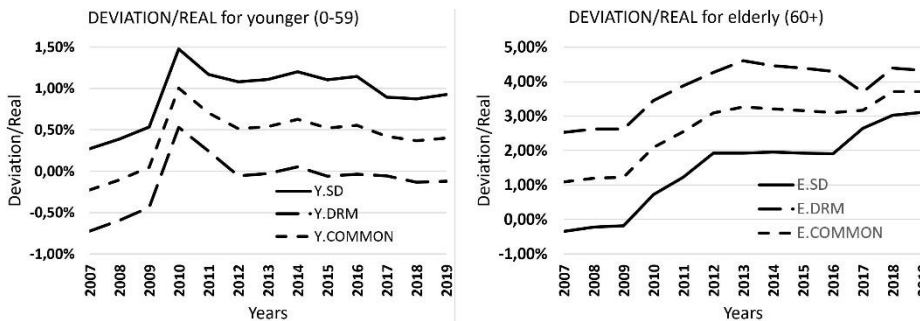


Fig. 4. Deviation graphs for SD, DRM, COMMON: population is divided relative to age

### 6.6 Comparison of the Complexity of the Models

We define the complexity of two versions of the model by the additional number of blocks necessary for the proper operation of the elementary cohort. In the case of the DRM approach, we used 12 blocks. These are (Fig. 2): Tank (3), Valve (3), Multiply (3), Select Value In (3), Constant (1). In the case of the SD approach, we used 5 blocks: Holding Tank (2), Select Value In (2) and Constant (1). The presented data shows that we create a model with a simpler structure using the SD approach.

## 7 Conclusions

Continuous simulation modelling (SD) is widely used in demographic research. Population projections are, in turn, a key component of many socio-economic models created with discrete approaches. This leads to continuous-discrete hybrid models in which various technical and methodological problems arise. In industrial processes of material flows from one location to another, DRM method is used. This simulation approach combines certain aspects of a continuous and a discrete approach and outperforms DEVS for systems with high flow rates of objects in industrial processes.

The overall goal of our study was to investigate whether and under what conditions the SD approach could be replaced with DRM in demographic simulations, through the technique of modifying the elementary cohorts representing fragments of populations of the similar age and the same gender. The premises were as follows: the intent to test DRM hybridity in the sense of integration with modules implemented in accordance with the DEVS, literature reports on the usefulness of the DRM in non-socio-economic fields, the willingness to check the adequacy of demographic simulations using the DRM approach compared to results obtained from SD approach. Our goal was also to compare the complexity of models built using DRM and SD approaches.

The theoretical part of the work describes the principles of the DRM and provides an overview of the applications of the DRM versus two other simulation methods, i.e. SD and DEVS. The issue of simulation in demographic modeling is also discussed. For the research part, we developed a complex model consisting of dozens of feedback loops and 210 one-year-old cohorts, 34 five-year-old cohorts and two 20-year-old, and performed simulation experiments. We compared the results obtained from our simulation model with real data based on the example of the population of the Wrocław region in 2006-2019. We tested the SD, DRM and the combination of SD and DRM by performing the comparative analysis between all three approaches. We determined how many times (taking into account cohorts and years) specified approach won.

Taking into account the deviations between simulation results and actual data for individual cohorts, we come to the conclusion that DRM does not match the SD accuracy. The combination of SD and DRM has however in some cases shown an advantage over both components applied separately. If we consider the fact that the different cohorts differ significantly in number of individuals, we may conclude that DRM enables to achieve the accuracy level compared to SD. DRM approach does not match the SD in terms of comprehensive accuracy in mapping the behavior of cohorts of the complex populations. However, we have been able to identify criteria for population segmentation that may lead to better results of DRM simulation than SD. DRM results are more accurate compared to real data for younger populations (0-59 years) and for two genders. In some cases, it is also worth considering to use a hybrid method of weighted average of DRM and SD results.

The satisfactory results of our research entitle us to conclude that the research on using DRM in demographic simulations may help to overcome some of the limitations of the SD approach. One of them is the need to treat various factors affecting demographic dynamics as homogeneous. Since DRM enables mixing of input streams it is possible to model heterogeneous populations or subpopulations.

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