Optimization of Selection of Tests in Diagnosing the Patient by General Practitioner

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Abstract. In General Practitioner's work the fundamental problem is the accuracy of the diagnosis under time constraints and health care cost limitations. The General Practitioner (GP) after an interview and a physical examination makes a preliminary diagnosis. The goal of the paper is to find the set of tests with such total diagnostic potential in verification of this diagnosis that is not smaller than a threshold value and with minimal total cost of tests. In proposed solution method, the set of preliminary diagnoses after the interview and the physical examination is given. For each preliminary diagnosis, for each test, diagnostic potential of the test in verification of the diagnosis is determined using Analytic Hierarchy Process based method with medical expert participation. Then binary linear programming problem with constraint imposed on total diagnostic potential of tests but with criterion function of minimal total test cost is solved for each diagnosis. For the case study when the patient with lumbal pain is coming to the GP, for each of six preliminary diagnoses, for each test, the diagnostic potentials of tests have been estimated. Then for each diagnosis, the solution of the binary linear programming problem has been found. A limitation of the case study is the estimation of diagnostic potential of tests by one expert only.

Keywords: Diagnostic Potential, Analytic Hierarchy Process, Binary Linear Programming Problem.

1 Introduction

In General Practitioner's work the fundamental problem is the accuracy of the diagnosis under time constraints and cost limitations on health care. The General Practitioner (GP) after interview with the patient or his/her guardian and physical examination makes a preliminary diagnosis. In order to check this diagnosis, GP can order the tests (examinations) from the admitted set of tests. Under-testing may result in delayed or missed diagnosis, while over-testing can cause a cascade of unnecessary activities, and costs [3], [8]. Medical diagnosis with medical tests cost is studied in the papers based on the following approaches: naïve Bayes classification [2], decision trees [6], genetic algorithms and fuzzy logic [4], rough sets [5], Analytic Hierarchy Process [1].

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The goal of the paper is to help General Practitioner (GP) to find the sets of tests with sufficiently great diagnostic potential but with minimal total cost of tests.

In the case study when the patient with lumbal pain is coming to the GP, for each of six assumed preliminary diagnoses, the BLPP is solved. In this case study, the AHP pair-wise comparisons of diagnostic potentials of tests has been based on expert's opinion. In general in these comparisons, statistical data about tests can be taken into account too.

The structure of the paper is as follows. The research problem and its solution method are outlined in Section 2. In next section the case study about patient with lumbal pain is presented. Finally there are summary and conclusions.

2 Research problem and research method

The following problem is studied in the paper. The patient with health problem is coming to GP. The GP should decide what to do: start the treatment by himself or direct the patient to a hospital or direct the patient to a specialist. The proposed solution method is as follows. The set of preliminary diagnoses after the interview and physical examination is given. For each diagnosis, for each test: urine, blood, ultrasound, X-ray, the diagnostic potential of the test in verification of the diagnosis is determined. In order to define the diagnostic potential of these tests when verifying a diagnosis, Analytic Hierarchy Process (AHP) [7], [9] based method with medical expert participation is applied. These tests are diagnosis dependent, e.g, the extent of blood test depends on diagnosis. For given diagnosis the costs of tests are calculated. The selected set of tests need to have the total diagnostic potential which is greater or equal to the given threshold. The criterion is the minimal cost of selected tests. Hence, for each diagnosis, the binary linear programming problem (BLPP) is defined. The analysis is executed for different values of threshold of total diagnostic potential of tests.

Now the proposed method will be presented in details.

Let

$D = \{D_1, \ldots, D_k, \ldots, D_s\}$ – the set of preliminary (hypothetical) diagnoses for the interview and the physical examination,

$T = \{T_1, \ldots, T_l, \ldots, T_t\}$ – the set of tests (examinations) that the GP can order.

Diagnostic potential of test T_l, where $l \in \{1, \ldots, t\}$, *for diagnosis* D_k, where $k \in \{1, \ldots, s\}$ is the number $0 \leq p_{T_l} \leq 1$ such that $\sum_{l=1}^{t} p_{T_l} = 1$. The diagnostic potential expresses a capacity of this test in verification of the diagnosis. Greater the capacity is greater the number is.

Selection algorithm of tests for diagnosis D_k

1. For each test from the set T define its extent (set of elementary tests) and estimate the diagnostic potential of the test in verification of the diagnosis D_k using AHP.
2. For each test calculate the total cost of all elements in the extent using a table.
3. Find such subset S of T that total diagnostic potential in verification of diagnosis D_k of elements of S is not smaller than required threshold diagnostic potential but the total cost is minimal using BLPP.

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AHP is linear algebra based method that is used in expressing the expert opinion in multi-step multi-criteria decision process. In this paper diagnostic potential estimation process is one-step. In this paper in AHP based approach the comparison between pairs of test diagnostic potentials in diagnosis D_k verification is performed. This comparison is done by domain (medical) expert. Scale of relative importance when test T_i is not weaker than test T_j is given in Table 1.

Table 1. Scale of relative importance when test T_i is not weaker than test T_j

Value a_{ij}	**Estimation of diagnostic potential (importance) of test T_i in regard to T_j**
9	T_i is extremely preferred (absolutely more important) in regard to T_j
7	T_i is very strongly preferred (definitely more important)
5	T_i is strongly preferred (clearly more important)
3	T_i is moderately preferred (slightly more important)
1	T_i is equivalent (equally important) with T_j

When intermediate values between those in Table 1. are required, then the values from the set $\{2,4,6,8\}$ can be assumed. For a_{ji} (estimation of diagnostic potential of test T_j in regard to T_i), the following condition $a_{ji} = {}^{1}/_{a_{ij}}$ need to be satisfied. Hence, if T_i is extremely preferred (absolutely more important) in regard to T_j, then T_j is absolutely less important than T_i, i.e. $a_{ji} = {}^{1}/_{9}$. Sample values of relative comparisons are contained in Table 2. Elements $a_{ii} = 1$, because they concern the comparison of importance of T_i with itself. Having all pairwise comparisons of all test diagnostic potentials, the diagnostic potential of all tests is calculated according to AHP approach. It will be illustrated in case study (Section 3.). Then *Consistency ratio* [7], [9] is calculated. It should be smaller than 0,1. Sometime, value 0,15 is accepted as the upper bound. If the requirement imposed on Consistency ratio is not satisfied then the above pair-wise comparisons need to be done once more.

In order to present BLPP for diagnosis D_k, the following notation will be introduced.
p_l – diagnostic potential of test T_l in verification of diagnosis D_k obtained in point 1. of the above algorithm,
x_l – binary decision variable; $x_l = 1$ if test T_l should be executed for diagnosis D_k, $x_l = 0$ otherwise,
$P_{min} \in <0,1]$ - required minimal total diagnostic potential (sum of potentials) of selected tests; the same value have been accepted for all diagnoses,
c_l – test T_l cost.

BLPP for selection of tests for verification of diagnosis D_k
Constraint: $\sum_{l=1}^{t} p_l \cdot x_l \geq P_{min}$
Criterion: $min \sum_{l=1}^{t} c_l \cdot x_l$

The constraint imposes that decision variables satisfy the threshold of total diagnostic potential of selected tests requirements, while the criterion requires the minimal total

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cost value of these tests. In order to solve the BLPP, SIMPLEX method will be applied and supported with software addition SOLVER for Excel.

In [1], diagnosis ability, cost of testing and other criteria are submitted to subjective AHP evaluation. In our approach the test cost is not subjectively weighted in comparing with the diagnostic potential. It is difficult to compare the test cost with diagnostic potential of the test.

3 Case study

The case of the patient with lumbal pain will be examined. Let the sample set of diagnoses D contain the following elements: Spine disease (SD), Urolithiasis (U), Aortic dissecting aneurysm (ADA), Oncological disease (OD), Pancreatic disease (PD), Acute pyelonephritis (AP). The abbreviations in the parentheses will be used further in the paper. The set of tests that the GP can order contain the elements: Urine (UR), Blood (B), Ultrasound examination (US), X-ray examination (X).

3.1 Analytic Hierarchy Process (AHP) in estimation of diagnostic potential of tests in verification of the diagnoses

Let us take urolithiasis as an example of the estimation of diagnostic potential using AHP. In Table 2. in rows and columns that are labelled by symbols p_{UR}, p_B, p_{US}, p_X of diagnostic potential of tests UR, B, US, X there are pair-wise comparison values a_{ij} of the relative importance of test T_i in regard to T_j when urolithiasis (U) is the preliminary diagnosis. The disease symbol U is put in the element of first row and first column. Then the sums of elements in columns are calculated. These sums are in the last row of this table.

Table 2. Values of pair-wise comparisons of diagnostic potentials p_{UR}, p_B, p_{US}, p_X of tests UR, B, US, X with sums of entries in columns for diagnosis U.

U	p_{UR}	p_B	p_{US}	p_X
p_{UR}	1	2	0,142857	1
p_B	0,5	1	0,111111	1
p_{US}	7	9	1	7
p_X	1	1	0,142857	1
Sum of entries in column	9,5	13	1,396825	10

In order to calculate normalized values of pair-wise comparisons of diagnostic potentials in Table 3., elements a_{ij} from Table 2. are divided by sums of entries in columns. Then mean values of entries in the rows are calculated (last column in Table 3.). These means are diagnostic potentials of tests for diagnosis U.

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Table 3. Normalized values of pair-wise comparisons of diagnostic potential of tests UR, B, US, X with means of entries in the rows that are diagnostic potentials of tests for diagnosis U

U	p_{UR}	p_B	p_{US}	p_X	Mean value of the row (diagnostic potential for U)
p_{UR}	0,105263	0,153846	0,102273	0,1	0,11534551
p_B	0,052632	0,076923	0,079545	0,1	0,077275028
p_{US}	0,736842	0,692308	0,715909	0,7	0,711264722
p_X	0,105263	0,076923	0,102273	0,1	0,096114741

The diagnostic potentials of tests UR, B, US, X in verification of the other diagnoses SD, ADA, OD, PD, AP are given in Table 4.

Table 4. Diagnostic potentials of tests UR, B, US, X in verification of the other diagnoses SD, ADA, OD, PD, AP

	Diagnostic potential for SD	Diagnostic potential for ADA	Diagnostic potential for OD	Diagnostic potential for PD	Diagnostic potential for AP
p_{UR}	0,06575919	0,065678508	0,067565247	0,059815141	0,506360018
p_B	0,296573726	0,162151802	0,251287775	0,443705986	0,263267192
p_{US}	0,057469716	0,706491183	0,613581731	0,443705986	0,195249668
p_X	0,580197368	0,065678508	0,067565247	0,052772887	0,035123122

For each diagnosis for the set of diagnostic potentials of the tests, the Consistency ratios have been calculated. In all above cases these ratios are smaller than 0,1.

3.2 Test costs for diagnoses

Table 5. Test costs for all preliminary diagnosis

Diagnosis	Urine test cost	Blood test cost	Ultrasound examination cost	X-ray examination cost
SD	10	112,00	100,00	50,00
U	30	126,00	100,00	35,00
ADA	10	35,00	100,00	35,00
OD	10	69,00	100,00	35,00
PD	10	71,00	100,00	35,00
AP	30	40,00	100,00	35,00

Now for each diagnosis the test costs will be given. Urine, blood, ultrasound, X-ray tests are complex tests. They consist of elementary tests, e.g. urine test for diagnosis U consists of general urine test and urine culture. Costs of tests will be given in polish zloty (PLN). Costs of elementary tests will be assumed according to service price list of University Clinical Hospital in Wrocław [10], provided there is such information in this list. Otherwise the prices of the following elementary tests are taken from the sources: blood count with smear [12], phosphorus [13], Vitamin D concentration

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25OHD3 [11], calcium [14]. Because of many medical terms the calculations of costs of tests will be omitted, and final results only are given in Table 5.

3.3 Binary linear programming problems for finding the sets of tests for diagnoses

The BLPP for diagnosis U for total diagnostic potential threshold equal to 0,8 is defined as follows.
Constraint:

$$0{,}11534551 \cdot x_1 + 0{,}077275028 \cdot x_2 + 0{,}711264722 \cdot x_3 + 0{,}096114741 \cdot x_4 \geq 0{,}8$$

Criterion:

$$\min(30 \cdot x_1 + 126 \cdot x_2 + 100 \cdot x_3 + 35 \cdot x_4)$$

where
x_1, x_2, x_3, x_4 , respectively, are binary decision variables for urine, blood, ultrasound examination, X-ray examination tests, respectively.

For all six diagnosis, the solutions of BLPP, values of total diagnostic potential, and total tests costs for the constraint: total diagnostic potential threshold equal to 0,8 are given in Table 6.

Table 6. For all diagnoses, the solutions of BLPP, values of total diagnostic potential, and total tests costs for the total diagnostic potential threshold equal to 0,8

	x_1	x_2	x_3	x_4	Total diagnostic potential	Total tests cost
SD	0	1	0	1	0,876771	162
U	1	0	1	0	0,82661	130
ADA	0	1	1	0	0,868643	135
OD	0	1	1	0	0,86487	169
PD	0	1	1	0	0,887412	171
AP	1	1	0	1	0,80475	105

The same results as in Table 6., however, for total diagnostic potential threshold equal to 0,9 are given in Table 7.

Table 7. For all diagnoses, the solutions of BLPP, values of total diagnostic potential, and total tests costs for the total diagnostic potential threshold equal to 0,9

	x_1	x_2	x_3	x_4	Total diagnostic potential	Total test cost
SD	1	1	0	1	0,94253	172
U	1	0	1	1	0,922725	165
ADA	1	1	1	0	0,934321	145
OD	1	1	1	0	0,932435	179
PD	1	1	1	0	0,947227	181
AP	1	1	1	0	0,964877	170

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For the total diagnostic potential threshold equal to 1,0, all four tests are required.

4 Summary and conclusions

The selection method of medical tests that General Practitioner can order for specific health problem, interview, and physical examination has been presented. The method of estimating the diagnostic potentials of tests is Analytic Hierarchy Process based. Having these potentials and test costs, binary linear programming problem is solved in order to find the set of tests with sufficiently great total diagnostic potential but with minimal cost. The method is different when comparing with typical usage of AHP where diagnostic potentials of tests and test costs are subjectively weighted. The approach has been applied for case study of patient with lumbal pain and six diagnostic hypotheses verified by medical tests. A limitation of the case study is the estimation of diagnostic potential of tests by one expert only. In the similar way, the method can be applied to a wider range of diagnostics of diseases.

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