The evolution of political views within the model with two binary opinions. ⋆

Magdalena Gołębiowska[0000−0001−7507−6737] and Katarzyna Sznajd-Weron[0000−0002−1851−8508]

Department of Theoretical Physics,Wrocław University of Science and Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

Abstract. We study a model aimed to describe political views within two-dimensional approach, known as the Nolan chart or the political compass, which distinguish between opinions related to economic and personal freedom. We conduct Monte Carlo simulations and show that in the lack of noise, i.e. at social temperature $T = 0$, the consensus is impossible if there is a coupling between opinions related to economic and personal freedom. Moreover, for $T > 0$ we show how the strength of the coupling between these opinions can hamper or facilitate the consensus.

Keywords: Complex Social Systems · Agent-Based Simulation · Opinion dynamics · Political Compass.

1 Introduction

The temptation to model human behavior appeared numerous times not only in the history of science [5] but also in the fiction science. Probably the most obvious example of such a temptation is psycho-history, introduced by Asimov in his famous Foundation cycle.

One of the approaches to understand the human behavior is agent-based modeling (ABM) [6, 12], which allows to describe the macroscopic behaviors based on the microscopic rules that define how individuals interact with each other. Among many different subjects studied within agent-based (i.e. microscopic) approach, one of the most popular and interdisciplinary one is the opinion dynamics [6, 12, 11, 15, 16, 14, 18].

When it comes to modeling opinions related to political views, binary variables seem to be particularly natural choice being a discretization of the traditional left–right/progressive-conservative division [10, 17, 13]. However, numerous empirical studies show that such a one-dimensional description may not be sufficient enough [1, 8].

Placing political views along two axes, representing economic and personal freedom, is known presently as the Nolan chart or the political compass. To our best knowledge, such a two-dimensional description of political views was used

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for the first time within ABM in [21]. Originally it was introduced as a non-equilibrium model described by the set of dynamical rules. However, this year, the model was reformulated in the spirit of statistical physics equilibrium model and studied analytically within the renormalization group technique [20].

Despite the obvious advantages of such an analytical method, there are several disadvantages when looking from the social point of view. Firstly, it allows to study only infinite systems, which do not exist in reality. Secondly, it does not allow to follow the temporal evolution of the system, which is particularly interesting from the social point of view. Therefore, we decided to analyze the model within computer simulations and focus mainly on the temporal behavior of the small systems.

2 Model

We consider a one dimensional lattice of size N with periodic boundary conditions. Each site $i = 1, \dots, N$ of the lattice is occupied by exactly one agent, who is described by two binary dynamical variables (S_i, σ_i), where $S_i = \pm 1$ denotes the view related to the economic freedom and $\sigma_i = \pm 1$ the view related to the private freedom [20, 21]. According to the political compass we define: (S)ocialists $S_i = -1, \sigma_i = +1$, they want the strong government in the economic area but they value the freedom in the personal life, (L)ibertarians $S_i = +1, \sigma_i = +1$, they value the freedom in both areas, (A)uthoritarians $S_i = -1, \sigma_i = -1$ they are against any freedom and (C)onservatives $S_i = +1, \sigma_i = -1$ they want the economic freedom but strict rules in the private area.

The model, as usually in the equilibrium statistical physics, is defined by the Hamiltonian [20]:

$$H = - J_1 \sum_{i=1}^{N} S_i S_{i+1} - J_2 \sum_{i=1}^{N} S_i S_{i+2} - K_1 \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - K_2 \sum_{i=1}^{N} \sigma_i \sigma_{i+2} - M_0 \sum_{i=1}^{N} \sigma_i S_i, \quad (1)$$

where J_1, J_2, K_1, K_2 are the coupling constants between agents, and M_0 describes the interaction between views related to the private and to the economic freedom.

In this paper we investigate the system within Monte Carlo simulations and we use the standard Metropolis algorithm. It means that the transition rate from one state $r \equiv (S_1, \dots, S_i, \dots, S_N)$ to the new one $r' \equiv (S'_1, \dots, S'_i, \dots, S'_N)$:

$$P(r \to r') = min\left(1, e^{-(H(r') - H(r))/T}\right), \quad (2)$$

where T is so called *social temperature* introduced within micro-sociology by Bahr and Passerini [4].

We use a random sequential updating, which mimics the continuous time. It means that within a single update we choose randomly only one agent and we

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try to update its state. In case of a single binary opinion it is straightforward – we try to change the state of an agent to the opposite one $S_i \to -S_i$. However, in our model each agent is described by two opinions and thus it is less obvious how we should update the system.

We can use many different updating schemes (US), analogously as it was done for the Ashkin-Teller two-spin model [7]: (US1) choose randomly agent i, update its opinion S_i and then σ_i, (US2) choose randomly agent i, update its opinion σ_i and then S_i, (US3) first update opinions S_i in the random order for all i and then do the same for σ_i, (US4) first update opinions σ_i in the random order for all i and then S_i. We conducted simulations within all above schemes and we obtained the same results for all of them. Therefore, here we present the algorithm and results only for US1.

As usually, we count the time in Monte Carlo steps (MCS) and one MCS consists of N elementary updates given by the following algorithm:

1. Choose randomly agent i from all N agents, $i \sim U\{1, N\}$
2. Update opinion related to the economic freedom:
   (a) Calculate the change $\Delta H = H(r') - H(r)$ caused by the potential change $S_i \to -S_i$
   (b) If $\Delta H \leq 0$ then update the state $S_i \to -S_i$ else
   (c) Choose a random number $r \sim U(0, 1)$. If $r < e^{-\Delta H/T}$ then update the state $S_i \to -S_i$.
3. Update opinion related to the private freedom:
   (a) Calculate the change $\Delta H = H(r') - H(r)$ caused by the potential change $\sigma_i \to -\sigma_i$
   (b) If $\Delta H \leq 0$ then update the state $\sigma_i \to -\sigma_i$ else
   (c) Choose a random number $r \sim U(0, 1)$. If $r < e^{-\Delta H/T}$ then update the state $\sigma_i \to -\sigma_i$.

3 Results

The model consists of 6 parameters – 5 coupling constants and the social temperature T. For $M_0 = 0$ there are no interactions between chains $(\sigma_1, \ldots, \sigma_N)$ and $(S_1, \ldots, S_N)$, and thus the model reduces to two independent next-nearest neighbor Ising (ANNNI) models [9]. Because of that we focus mainly on the role of M_0. We focus on the very particular case, $J_1 = K_2 = 0$ and $J_2 = K_1 > 0$, which means that in the private area we try to follow the nearest neighbors (like in the basic Ising model), whereas in the economic one the next-nearest neighbors (like in the basic Sznajd model) [22]. Such a choice was inspired by the original paper on the political compass within ABS [21].

Let us start with the case of $M_0 = 0$ as the reference one. In such a case for $T = 0$ the system evolves towards an absorbing state and its configuration can be easily predicted for all combinations of parameters J_1, J_2, K_1, K_2. For $J_2 = K_1 = 0$ and $J_1, K_2 > 0$, even if initially $\forall_i S_i(0) = \sigma_i(0)$ like in Fig. 1, during the evolution all four political views appear. However, eventually the system reaches one of possible states: (1) consensus, i.e. all agents have the same

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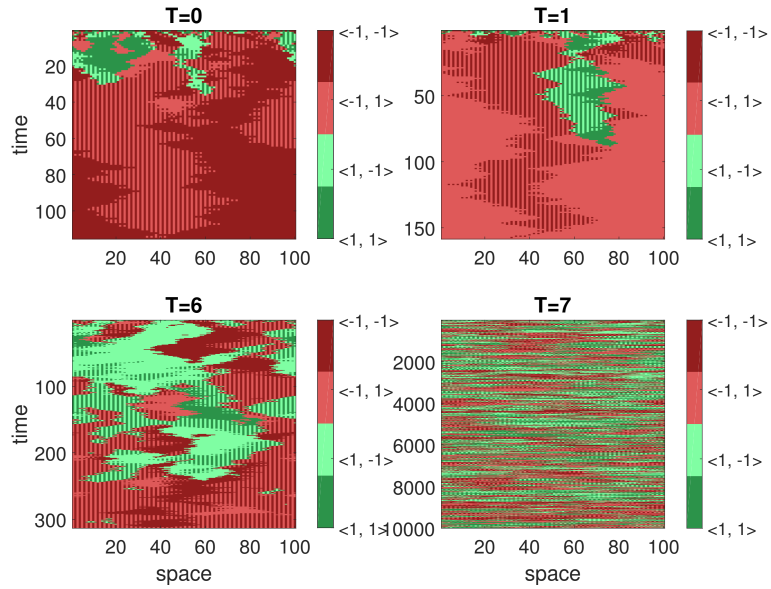

Fig. 1. The time evolution of the system of size $N = 100$ for the coupling constant $M_0 = 0$ and four values of the social temperature T, as indicated in the title of each subplot. Vertical axis represents the time and horizontal one the space, which means that each horizontal line is a visualization of the system's configuration at a given time. Individual states $< S_i, \sigma_i >$ are indicated by the color-bar.

political attitude (S,L,A or C) or (2) the war between two states (two-party system).

All these absorbing states can be identified by measuring four quantities: two magnetizations

$$m_S = \frac{1}{N}\sum_{i=1}^{N} S_i, \qquad m_\sigma = \frac{1}{N}\sum_{i=1}^{N} \sigma_i, \tag{3}$$

and two densities of active bonds

$$\rho_S = \frac{1}{2N}\sum_{i=1}^{N}(1 - S_i S_{i+1}), \qquad \rho_\sigma = \frac{1}{2N}\sum_{i=1}^{N}(1 - \sigma_i \sigma_{i+1}). \tag{4}$$

The consensus within a chain is reached if $|m_\alpha| = 1$, where $\alpha = \{S, \sigma\}$, whereas the war corresponds to $\rho_\alpha = 1$.

For $0 < T < T^*$ all these states can still be reached but they are not absorbing anymore, because due to the noise the system evolves for ever. It is not seen on presented figures, because we stop the simulation, once the consensus or the war, is reached, i.e., ($|m_S| = 1$ or $\rho_S = 1$) and ($|m_\sigma| = 1$ or $\rho_\sigma = 1$). For $T > T^*$ the we do not observe growth of any consensus or war domains, as shown in the bottom right panel of Fig. 1.

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What changes if we allow for the coupling between the economic and private area, e.g., $M_0 > 0$? For $M_0 > min(2|J_2|, 2|K_1|)$ the system almost immediately blocks and no time evolution is seen. Independently on the initial state, in the blocked state $S_i = \sigma_i$ for all agents. It means that Libertarians and Authoritarians coexist but consensus is never reached. Analogous discussion can be provided for $M_0 < 0$, but in such a case Socialists coexist with Conservatives.

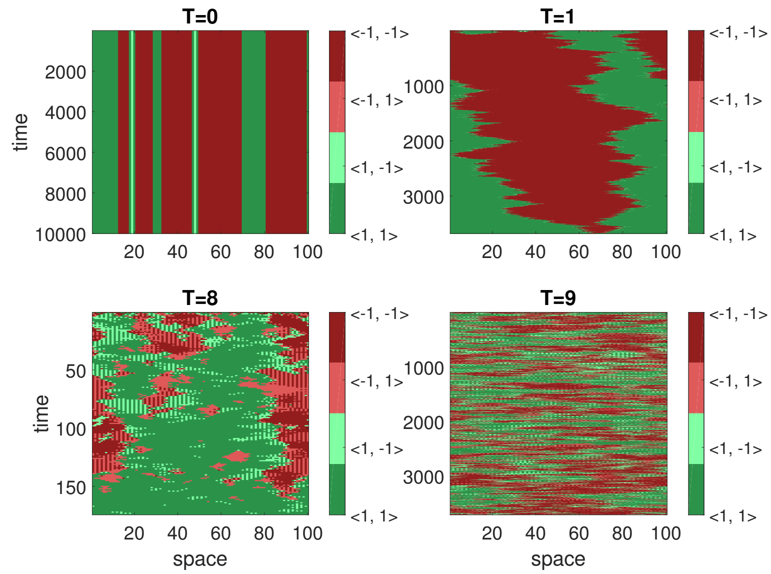

Fig. 2. The time evolution of the system of size $N = 100$ for the coupling constant $M_0 = 1, J_2 = K_1 = 10$ and four values of the social temperature T, as indicated in the title of each subplot. Vertical axis represents the time and horizontal one the space, which means that each horizontal line is a visualization of the system's configuration. Individual states $< S_i, \sigma_i >$ are indicated by the color-bar.

The most interesting behavior is observed for intermediate values $M \in (0, min(2|J_2|, 2|K_1|))$. Sample time evolution is shown in Fig. 2. In this case for $T = 0$ the system blocks after several time steps and coexistence of A and L (for $M_0 > 0$) or S and C (for $M_0 < 0$) is observed. On the other hand, for relatively small social temperature $0 < T < T_0^*$ it evolves very slowly towards consensus and all agents reach the state $S_i = \sigma_i$ (for $M_0 > 0$) or $S_i = -\sigma_i$ (for $M_0 < 0$). For $T_0^* < T < T^*$, all four political attitudes can appear and both types of clusters (consensus and war) are observed. However, only the consensus can spread over the whole system and it spreads relatively fast, what is clearly seen in Fig. 3. We do not give here any particular values of T_0^*, T^* because they depend on the particular values of the interaction constants, as seen in the right panel of Fig. 3.

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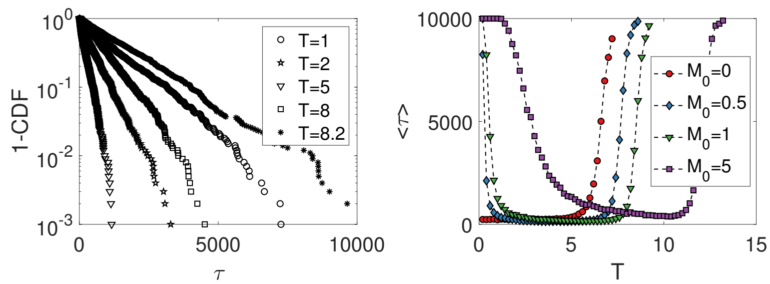

Fig. 3. Waiting time τ to reach consensus for the system of size $N = 100$ obtained within 10^4 samples for interaction coefficients $J_1 = 10, J_2 = 0, K_1 = 0, K_2 = 10$. **Left panel:** The tail of the empirical cumulative distribution function of the waiting times for $M_0 = 1$ and several values of social temperature T. **Right panel:** The average waiting time to reach consensus as a function of of social temperature T for several values of M_0.

4 Discussion

In Fall 2011 Reason-Rupe Poll reported that 24 % of Americans are Economically Conservative and Socially Liberal, 28 % Liberal, 28 % Conservative, and 20 % Communitarian (check on $https://reason.com/poll/$). This is one of numerous studies, which confirms that one-dimensional description of political views is not sufficient [8]. Here, we presented briefly results of the simple equilibrium model with two binary opinions.

We would like to stress that there is a significant difference between a multi-state but one-dimensional description, such as in the Potts model [23], and the multi-dimensional description, such as in the Axelrod model [3] or in the one discussed here. The natural question is why not to use the Axelrod model to represent political views, when it allows to describe many dimensions and to measure each of them by a multi-state variable? The answer, and simultaneously the justification of our approach, is that within the Axelrod model all dimensions are equivalent and there are no direct interactions between them.

We are aware of many limitations of our study, which should be treated as a zero-level approach to the real political system. First of all, a one-dimensional lattice with periodic boundary conditions is an appropriate structure for a round-table discussion but not for the large real-life social networks. In the future, one should rather consider heterogeneous graphs with a small-world property. Moreover, it has been suggested, based on the empirical research, that even a two-dimensional space may be not sufficient to describe political landscapes [2]. However, even a two-dimensional description is a step forward in relation to a single left-right representation of political attitudes, that has been mostly used so far [19, 10, 13].

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