

An application of a pseudo-parabolic modeling to texture image recognition^{*}

Joao B. Florindo¹[0000-0002-0071-0227] and Eduardo Abreu¹[0000-0003-1979-3082]

Institute of Mathematics, Statistics and Scientific Computing - University of Campinas, Rua Sérgio Buarque de Holanda, 651, Cidade Universitária “Zeferino Vaz”
- Distr. Barão Geraldo, CEP 13083-859, Campinas, SP, Brasil
<http://www.ime.unicamp.br>
{florindo,eabreu}@unicamp.br

Abstract. In this work, we present a novel methodology for texture image recognition using a partial differential equation modeling. More specifically, we employ the pseudo-parabolic equation to provide a dynamics to the digital image representation and collect local descriptors from those images evolving in time. For the local descriptors we employ the magnitude and signal binary patterns and a simple histogram of these features was capable of achieving promising results in a classification task. We compare the accuracy over well established benchmark texture databases and the results demonstrate competitiveness, even with the most modern deep learning approaches. The achieved results open space for future investigation on this type of modeling for image analysis, especially when there is no large amount of data for training deep learning models and therefore model-based approaches arise as suitable alternatives.

Keywords: Pseudo-parabolic equation · Texture recognition · Image classification · Computational Methods for PDEs.

1 Introduction

Texture images (also known as visual textures) can be informally defined as those images in which the most relevant information is not encapsulated within one or a limited set of well-defined objects, but rather all pixels share the same importance in their description. This type of image has found numerous application in material sciences [19], medicine [11], facial recognition [15], remote sensing [33], cybersecurity [27], and agriculture [24], to name but a few fields with increasing research activity.

While deep learning approaches have achieved remarkable success in problems of object recognition and variations of convolutional neural networks have prevailed in the state-of-the-art for this task, texture recognition on the other hand still remains a challenging problem and the classical paradigm of local

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image encoders still is competitive with the most modern deep neural networks, presenting some advantages over the last ones, like the fact that they can work well even when there is little data available for training.

In this context, here we present a local texture descriptor based on the action of an operator derived from the Buckley-Leverett partial differential equation (PDE) (see [1, 2] and references cited therein). PDE models have been employed in computer vision at least since the 1980's, especially in image processing. The scale-space theory developed by Witkin [31] and Koenderink [17] are remarkable examples of such applications. The anisotropic diffusion equation of Perona and Malik [23] also represented great advancement in that research front, as it solved the problem of edge smoothing, common in classical diffusion models. Evolutions of this model were later presented and a survey on this topic was developed in [30].

Despite these applications of PDEs in image processing, substantially less research has been devoted to recognition. As illustrated in [29, 4], pseudo-parabolic PDEs are promising models for this purpose. An important characteristic of these models is that jump discontinuities in the initial condition are replicated in the solution [10]. This is an important feature in recognition as it allows some control over the smoothing effect and would preserve relevant edges, which are known to be very important in image description.

Based on this context, we propose the use of a pseudo-parabolic equation as an operator acting as a nonlinear filter over the texture image. That image is used as initial condition for the PDE problem and the solution obtained by a numerical scheme developed in [1] is used to compose the image representation. The solution at each time is encoded by a local descriptor. Extending the idea presented in [29], here we propose two local features: the sign and the magnitude binary patterns [13]. The final texture descriptors are provided by simple concatenation of histograms over each time.

The effectiveness of the proposed descriptors is validated on the classification of well established benchmark texture datasets, more exactly, KTH-TIPS-2b [14] and UIUC [18]. The accuracy is compared with the state-of-the-art in texture recognition, including deep learning solutions, and the results demonstrate the potential of our approach, being competitive with the most advanced solutions recently published on this topic.

This paper is structured as follows. In Section 2, we present the pseudo-parabolic partial differential equation we are considering to texture image recognition, along with the key aspects of the discretization method being used. In Section 3, we introduce the proposed methodology for the application of a pseudo-parabolic modeling to texture datasets recognition. We also highlight the effectiveness of the proposed descriptors and validate them on the classification of well established benchmark and state-of-the-art databases in texture recognition. In Section 4, we present some numerical experiments to show the efficiency and accuracy of the computed solutions verifying that the results demonstrate the potential of our approach, being competitive with the most advanced solutions recently available in the specialized literature on this subject matter. In

Section 5, we concrete to present some results and discussion for the proposed descriptors in the classification of KTH-TIPS-2b and UIUC. We discuss in particular the accuracy of the proposed descriptors compared with other texture descriptors in the literature, confirming its potential as a texture image model. Finally, in the last Section 6, we present our concluding remarks.

2 Partial differential equation and numerical modeling

We consider an advanced simulation approach for the pseudo-parabolic PDE

$$\frac{\partial u}{\partial t} = \nabla \cdot \mathbf{w}, \quad \text{where} \quad \mathbf{w} = g(x, y, t) \nabla \left(u + \tau \frac{\partial u}{\partial t} \right), \quad (1)$$

and let $\Omega \subset \mathbb{R}^2$ denote a rectangular domain and $u(\cdot, \cdot, t) : \Omega \rightarrow \mathbb{R}$ be a sequence of images that satisfies the pseudo-parabolic equation (1), in which the original image at $t = 0$ corresponds to the initial condition, along with zero flux condition across the domain boundary $\partial\Omega$, $\mathbf{w} \cdot \mathbf{n} = 0$, $(x, y) \in \partial\Omega$. We consider the discretization modeling of the PDE (1) in a uniform partition of Ω into rectangular subdomains $\Omega_{i,j}$, for $i = 1, \dots, m$ and $j = 1, \dots, l$, with dimensions $\Delta x \times \Delta y$. The center of each subdomain $\Omega_{i,j}$ is denoted by (x_i, y_j) . Given a final time of simulation T , consider a uniform partition of the interval $[0, T]$ into N subintervals, where the time step $\Delta t = T/N$ is subject to a stability condition (see [29, 28] for details). We denote the texture configuration frames in the time levels $t_n = n\Delta t$, for $n = 0, \dots, N$. Let $U_{i,j}^n$ and $W_{i,j}^{n+1}$ be a finite difference approximations for $u(x_i, y_j, t_n)$ and \mathbf{w} , respectively, and both related to the pseudo-parabolic PDE modeling of (1). In the following, we employ a stable cell-centered finite difference discretization in space after applying the backward Euler method in time to (1), yielding

$$\frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta t} = \frac{W_{i+\frac{1}{2},j}^{n+1} - W_{i-\frac{1}{2},j}^{n+1}}{\Delta x} + \frac{W_{i,j+\frac{1}{2}}^{n+1} - W_{i,j-\frac{1}{2}}^{n+1}}{\Delta y}. \quad (2)$$

Depending on the application as well as the calibration data and texture parameters upon model (1), we will have linear or nonlinear diffusion models for image processing (see, e.g., [23, 6, 30]). As a result of this process the discrete problem (2) would be linear-like $\mathbf{A}^n \mathbf{U}^{n+1} = \mathbf{b}^n$ or nonlinear-like $\mathbf{F}(\mathbf{U}^{n+1}) = 0$ and several interesting methods can be used (see, e.g., [2, 29, 28, 23, 6, 30]). We would like to point out at this moment that our contribution relies on the PDE modeling of (1) as well as on the calibration data and texture parameters associated to the pseudo-parabolic modeling in conjunction with a fine tuning of the local descriptors for texture image recognition for the pertinent application under consideration. In summary, we have a family of parameter choice strategies that combines pseudo-parabolic modeling with texture image recognition.

Here, we consider the diffusive flux as $g(x, y, t) \equiv 1$, which results (1) to be a linear pseudo-parabolic model. For a texture image classification based on a pseudo-parabolic diffusion model to be processed, we just consider that each

subdomain $\Omega_{i,j}$ corresponds to a pixel with $\Delta x = \Delta y = 1$. As we perform an implicit robust discretization in time (backward Euler), we simply choose the time step $\Delta t = \Delta x$ and the damping coefficient $\tau = 5$. More details can be found in [29]; see also [28, 1].

Therefore, this description summarizes the basic key ideas of our computational PDE modeling approach for texture image classification based on a pseudo-parabolic diffusion model (1).

3 Proposed methodology

Inspired by ideas presented in [29] and significantly extending comprehension on that study, here we propose the development of a family of images $\{u_k\}_{k=1}^K$. These images are obtained by introducing the original image u_0 as initial condition for the 2D pseudo-parabolic numerical scheme presented in Section 2. u_k is the numerical solution at each time $t = t_k$. Here, $K = 50$ showed to be a reasonable balance between computational performance and description quality.

Following that, we collected two types of local binary descriptors [13] from each u_k . More exactly, we used sign $LBP_S_{P,R}^{riu2}$ and magnitude $LBP_M_{P,R}^{riu2}$ descriptors. In short, the local binary sign pattern $LBP_S_{P,R}^{riu2}$ for each image pixel with gray level g_c and whose neighbor pixels at distance R have intensities g_p ($p = 1, \dots, P$) is given by

$$LBP_S_{P,R}^{riu2} = \begin{cases} \sum_{p=0}^{P-1} H(g_p - g_c) 2^p & \text{if } \mathcal{U}(LBP_{P,R}) \geq 2 \\ P + 1 & \text{otherwise,} \end{cases} \quad (3)$$

where H corresponds to the Heaviside step function ($H(x) = 1$ if $x \geq 0$ and $H(x) = 0$ if $x < 0$) and \mathcal{U} is the uniformity function, defined by

$$\mathcal{U}(LBP_{P,R}) = |H(g_{P-1} - g_c) - H(g_0 - g_c)| + \sum_{p=1}^{P-1} |H(g_p - g_c) - H(g_{p-1} - g_c)|. \quad (4)$$

Similarly, the magnitude local descriptor is defined by

$$LBP_M_{P,R}^{riu2} = \begin{cases} \sum_{p=0}^{P-1} t(|g_p - g_c|, C) 2^p & \text{if } \mathcal{U}(LBP_{P,R}) \geq 2 \\ P + 1 & \text{otherwise,} \end{cases} \quad (5)$$

where C is the mean value of $|g_p - g_c|$ over the whole image and t is a threshold function, such that $t(x, c) = 1$ if $x \geq c$ and $t(x, c) = 0$, otherwise.

Finally, we compute the histogram h of $LBP_S_{P,R}^{riu2}(u_k)$ and $LBP_M_{P,R}^{riu2}(u_k)$ for the following pairs of (P, R) values: $\{(1, 8), (2, 16), (3, 24), (4, 24)\}$. The proposed descriptors can be summarized by

$$\mathfrak{D}(u_0) = \bigcup_{\text{type}=\{S,M\}} \bigcup_{\substack{(P,R)=\{(8,1), \\ (16,2), (24,3), \\ (24,4)\}}} \bigcup_{k=0}^K h(LBP_type_{P,R}^{riu2}(u_k)). \quad (6)$$

Final descriptors correspond to the concatenated histograms in (6). Figure 1 shows the concatenated histograms for one time ($t = 1$) of the PDE operator. To reduce the dimensionality of the final descriptors, we also apply Karhunen-Loève transform [22] before their use as input to the classifier algorithm. The diagram depicted in Figure 2 illustrates the main steps involved in the proposed algorithm.

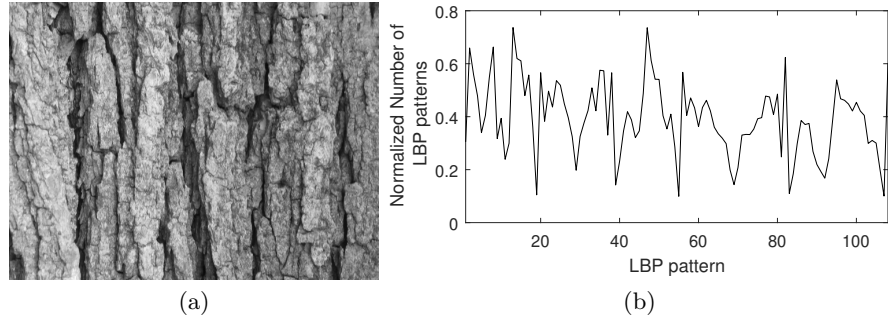


Fig. 1. Histograms used to compose the final descriptors obtained from the PDE operator at $t = 1$. (a) Original texture. (b) Normalized histograms.

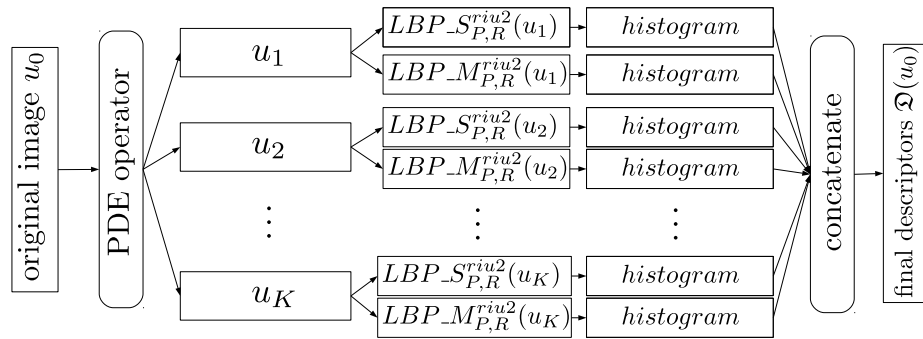


Fig. 2. Main steps of the proposed method.

4 Experiments

The performance of the proposed descriptors is assessed on the classification of two well-established benchmark datasets of texture images, namely, KTH-TIPS-2b [14] and UIUC [18].

KTH-TIPS-2b is a challenging database focused on the real material depicted in each image rather than on the texture instance as most classical databases. In this way, images collected under different configurations (illumination, scale and pose) should be part of the same class. The database comprises a total of 4752 color textures with resolution 200×200 (here they are converted to gray scales), equally divided into 11 classes. Each class is further divided into 4 samples (each sample corresponds to a specific configuration). We adopt the most usual (and most challenging) protocol of using one sample for training and the remaining three samples for testing.

UIUC is a gray-scale texture dataset composed by 1000 images with resolution 256×256 evenly divided into 25 classes. The images are photographed under uncontrolled natural conditions and contain variation in illumination, scale, perspective and albedo. For the training/testing split we also follow the most usual protocol, which consists in half of the images (20 per class) randomly selected for training and the remaining half for testing. This procedure is repeated 10 times to allow the computation of an average accuracy.

For the final step of the process, which is the machine learning classifier, we use Linear Discriminant Analysis [12], given its easy interpretation, absence of hyper-parameters to be tuned and known success in this type of application [29].

5 Results and Discussion

Figures 3 and 4 show the average confusion matrices and accuracies (percentage of images correctly classified) for the proposed descriptors in the classification of KTH-TIPS-2b and UIUC, respectively. The average is computed over all training/testing rounds, corresponding, respectively, to 4 rounds in KTH-TIPS-2b and 10 rounds in UIUC. This is an interesting and intuitive graphical representation of the most complicated classes and the most confusable pairs of classes. While in UIUC there is no pair of classes deserving particular attention (the maximum confusion is of one image), KTH-TIPS-2b exhibits a much more challenging scenario. The confusion among classes 3, 5, 8, and 11 is the most critical scenario for the proposed classification framework. It turns out that these classes correspond, respectively, to the materials “corduroy”, “cotton”, “linen”, and “wool”. Despite being different materials, they inevitably share similarities as at the end they are all types of fabrics. Furthermore, looking at the sample from these classes, we can also observe that the attribute “color”, that is not considered here, would be a useful class discriminant in that case. In general, the performance of our proposal in this dataset is quite promising and the confusion matrix and raw accuracy confirm our theoretical expectations.

Table 1 presents the accuracy compared with other methods in the literature, including several approaches that can be considered as part of the state-of-the-art in texture recognition. First of all, the advantage over the original Completed Local Binary Patterns (CLBP), whose part of the descriptors are used here as local encoder, is remarkable, being more than 10% in KTH-TIPS-2b. Other advanced encoders based on Scale Invariant Feature Transform (SIFT) are also outper-

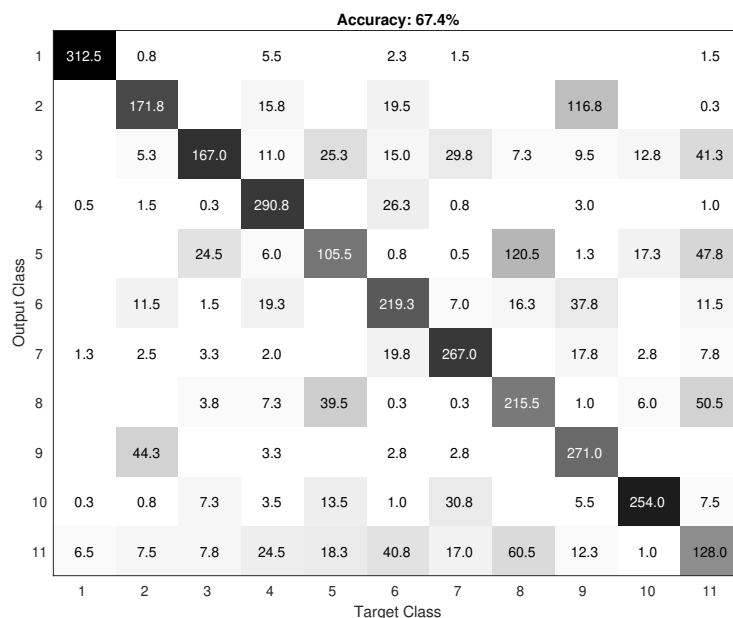


Fig. 3. Average confusion matrix and accuracy for KTH-TIPS-2b.

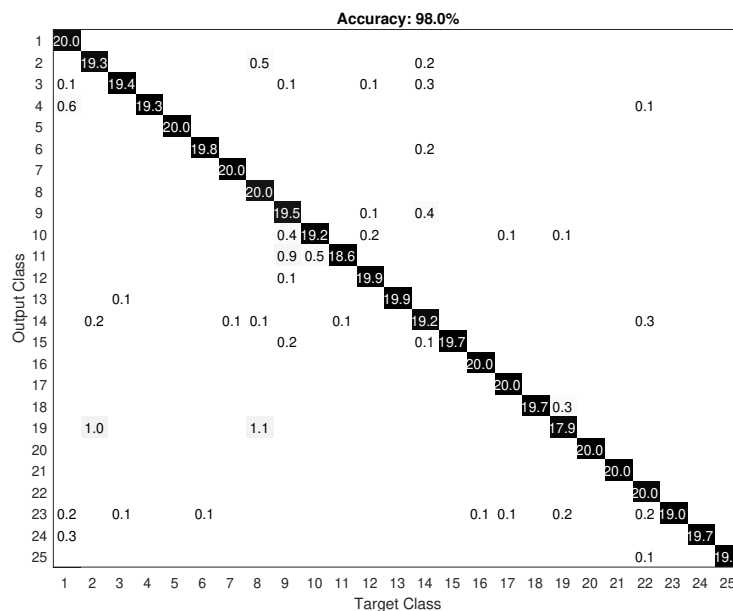


Fig. 4. Average confusion matrix and accuracy for UIUC.

formed in both datasets (by a large margin in the most challenging textures of KTH-TIPS-2b). SIFT descriptors are complex object descriptors and were considered the state-of-the-art in image recognition for several years. Compared with the most recent CNN-based approaches presented in [9], the results are also competitive. In UIUC, the proposed approach outperforms CNN methods like Deep Convolutional Activation Features (DeCAF) and FC-CNN AlexNet. Here it is worth to observe that FC-CNN AlexNet is not the classical “off-the-shelf” architecture, but an improved version of that algorithm especially adapted for texture images in [9]. These correspond to complex architectures with a high number of layers and large requirements of computational resources and whose results are pretty hard to be interpreted.

Table 1. Accuracy of the proposed descriptors compared with other texture descriptors in the literature. A superscript ¹ in KTH-TIPS-2b means training on three samples and testing on the remainder (no published results for the setup used here).

KTH-TIPS-2b		UIUC	
Method	Acc. (%)	Method	Acc. (%)
VZ-MR8 [25]	46.3	RandNet (NNC) [7]	56.6
LBP [21]	50.5	PCANet (NNC) [7]	57.7
VZ-Joint [26]	53.3	BSIF [16]	73.4
BSIF [16]	54.3	VZ-Joint [26]	78.4
LBP-FH [3]	54.6	LBP _{riu2} /VAR [21]	84.4
CLBP [13]	57.3	LBP [21]	88.4
SIFT+LLC [9]	57.6	ScatNet (NNC) [5]	88.6
ELBP [20]	58.1	MRS4 [26]	90.3
SIFT + KCB [8]	58.3	SIFT + KCB [8]	91.4
SIFT + BoVW [8]	58.4	MFS [32]	92.7
LBP _{riu2} /VAR [21]	58.5 ¹	VZ-MR8 [25]	92.8
PCANet (NNC) [7]	59.4 ¹	DeCAF [8]	94.2
RandNet (NNC) [7]	60.7 ¹	FC-CNN AlexNet [9]	91.1
SIFT + VLAD [8]	63.1	CLBP [13]	95.7
ScatNet (NNC) [5]	63.7 ¹	SIFT+BoVW [8]	96.1
FC-CNN AlexNet [9]	71.5	SIFT+LLC [9]	96.3
Proposed	67.4	Proposed	98.0

Table 2 lists the average computational time of our algorithm compared with some of the methods in Table 1 whose source codes were made available by the authors. This test was carried out over UIUC images (size 640×480 pixels). The computational setup corresponds to a portable computer with a CPU Intel Core i7-7700HQ (2.8GHz), 16GB of RAM, GPU NVIDIA Geforce GTX 1050Ti, Matlab R2017a 64 bits, Windows 10 Education 64 bits. In general, our method is among the fastest ones in terms of raw computational processing.

Generally speaking, the proposed method provided results in texture classification that confirm its potential as a texture image model. Indeed that was theoretically expected from its ability of smoothing spurious noise at the same

Table 2. Average computational time for an UIUC texture image (640×480).

Method	time (s)
CLBP	1.269
LBP	0.713
LBP _{riu2} /VAR	1.216
SIFT + BoVW	1.974
SIFT + VLAD	1.406
SIFT+LLC	2.939
SIFT + KCB	5.384
FC-CNN AlexNet	0.579
Proposed	0.406

time that preserves relevant discontinuities on the original image. The combination with a powerful yet simple local encoder like CLBP yielded interesting and promising performance neither requiring large amount of data for training nor advanced computational resources. Another remarkable point was its ability to be competitive with learning-based approaches that rely on transfer learning over huge datasets. Even though this is an interesting approach when we have limited data for training, it may become an infeasible strategy, for example, when the problem involves significant domain shift with respect to general purpose databases like ImageNet. Here our method achieved good generalization on heterogeneous textures without any external pre-training. In general, such great performance combined with the straightforwardness of the model, that allows some interpretation of the texture representation based on local homo/heterogeneous patterns, make the proposed descriptors a candidate for practical applications in texture analysis, especially when we have small to medium datasets and excessively complicated algorithms should be avoided.

6 Conclusions

In this study, we investigated the performance of a nonlinear PDE model (pseudo-parabolic) as an operator for the description of texture images. The operator was applied for a number of iterations (time evolution) and a local encoder was collected from each transformed image. The use of a basic histogram to pooling the local encoders was sufficient to provide competitive results.

The proposed descriptors were evaluated over a practical task of texture classification on benchmark datasets and the accuracy was compared with other approaches from the state-of-the-art. Our method outperformed several other local descriptors that follow similar paradigm and even some learning-based algorithms employing complex versions of convolutional neural networks.

The obtained results confirmed our expectations of a robust texture descriptor, explained by its ability of nonlinearly smoothing out spurious noise and unnecessary details, but preserving relevant information, especially those conveyed by sharp discontinuities. In general, the results and the confirmation of

the theoretical formulation suggest the suitability of applying such model in practice, in tasks of texture recognition that require simple models, easy to be interpreted and that do not require much data for training. This is a common situation in areas like medicine and several others.

Finally, such promising results also open space for future works. Particularly, the PDE operator described here can be interpreted as a preconditioner for image analysis algorithms. In this context, it can also be explored, for instance, in combination with the modern deep learning approaches, especially in the typical scenario where deep learning excels, i.e., when there is sufficient amount of data for training and computational power and when model interpretation is not priority. Application to other types of images beyond textures would also be possible in this scenario.

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