Socio-cognitive Evolution Strategies*

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Abstract. Socio-cognitive computing is a paradigm developed for the last several years, it consists in introducing into metaheuristics mechanisms inspired by inter-individual learning and cognition. It was successfully applied in hybridizing ACO and PSO metaheuristics. In this paper we have followed our previous experiences in order to hybridize the acclaimed evolution strategies. The newly constructed hybrids were applied to popular benchmarks and compared with their referential versions.

Keywords: metaheuristics · socio-cognitive computing · hybrid algorithms · evolution strategies.

1 Introduction

Tackling difficult optimization problems requires using metaheuristics [21], very often it is needed to create new ones [34], e.g. hybridizing the existing algorithms [30]. It is a well-known fact, that metaheuritics are very often inspired by nature, therefore their hybridizations often put together different phenomena observed in the real-world.

An interesting theory, which already has become a basis for efficient hybrid algorithms, is Social Cognitive Theory introduced by Bandura [2]. This theory is used in psychology, education, and communication and assumes that portions of an individual's acquisition of knowledge can be directly related to observing others in the course of their social interactions, their experiences, and outside media influences [3]. Thus, the individuals use this gathered information to guide their behaviors, not solely learning them by themselves (e.g., during the course of trials and errors). They can replicate others' deeds (trial and error) and predict the consequences based on observations, thus possibly reaching their goals sooner (cf. Bandura's Bobo Doll experiment [4]).

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Of course many social metaheuristics, processing a number of individuals, especially in the case when the individuals can be perceived as somewhat autonomous (e.g. EMAS [10, 23]) already use certain socio-cognitive inspirations, however we already introduced dedicated mechanisms rooted in Social-Cognitive Theory to selected metaheuristics (socio-cognitive ACO [11] and socio-cognitive PSO [8]), obtaining good results comparing to the reference algorithms.

This paper is devoted to hybridization of the socio-cognitive ideas with classic evolution strategies by Rechenberg and Schwefel [29]. Up-to-now we have researched hybridizing popular algorithms with strong social component (ACO [13], PSO [17]). In this and subsequent papers, we would like to try hybridizing well-known classic metaheuristics, such as genetic algorithm, evolution strategies, clonal selection algorithm, differential evolution and many others [30]. Our work is motivated (among others) by the famous “no free lunch” theorem by Wolpert and Macready [34] – its main conclusion is that we have to tune our metaheuristics, but it is to note that sometimes seeking new metaheuristics is also necessary (especially when no theoretical work has shown that the constructed metaheuristic actually is able to find anything (e.g. Michael Vose has proven that Simple Genetic Algorithm is a feasible computing method [33], Gunter Rudolph researched theoretically Evolution Strategies (see, e.g. [5]) and we have also applied methods similar to the ones used by Michael Vose to prove feasibility of agent-based computing methods [9]).

The main contribution of this paper is a novel hybrid of Evolution Strategies utilizing socio-cognitively inspired mechanism which makes possible to exchange the information among the individuals. The efficiency and efficacy of the novel algorithms are tested using well-known high dimensional, multi-modal benchmark functions.

2 Classic and hybrid Evolution Strategies

Evolution strategies (ES) are the classic approach in biologically-inspired computing discipline. Devised in the Technical University of Berlin by Rechenberg and Schwefel [25, 28] has been developed and applied for more than fifty years. Due to its universality in solving difficult problems, evolution strategy is considered as one of classic metaheuristics. The classic ES incorporate two nature-inspired mechanisms: mutation and selection. Even though there are notions of parents (μ) and offspring (λ), the offspring can be a result of crossover between selected pair of parents but also just a mutation of a selected parent. After the mutation, parents can be either excluded from the pool for next-generation selection (referred ','/comma strategies) or can be saved to be part of a new generation ('+'/plus strategies). The simplest version is 1+1 strategy, were two individuals are compared and the one with better fitness becomes a parent for the next generation. This is an instance of μ+λ, where both μ and $\lambda = 1$, but they can equal any greater number on the condition that μ is equal or less than λ. In the case of μ,λ ES, offspring population ought to be greater then parent in order to have a sufficient number of individuals to select from.

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During its long history ES have been modified and hybridized in many ways in order to improve performance in hard problem's solutions. Based on Talbi's flat taxonomy for hybrid metaheuristics [31] we can differentiate between homogeneous and heterogeneous hybrids. The latter ones join two or more types of metaheuristics and in such hybrids ES can serve either as major metaheuristic that is enriched by other one or as a supplementary algorithm [6]. Very often heterogeneous hybrid metaheuristics are designed to solve certain problem (e.g., [16, 18, 24], and thus are hard to be generalised.

On the other hand, homogeneous hybrids use only one type of metaheuristic and emerge from combining parts of algorithm with different parameters. In the case of evolution strategies, such control parameters as mutation strength or population size (step-size) can be manipulated. One of such versions is CMA-ES[15], but many other can be found in the literature, e.g., [1, 7, 32]).

The existing hybrids Evolution Strategies are often focused on particular application, e.g. vehicle routing problem [26] or optimization of engineering and construction problems [19, 27]. At the same time, general-purpose hybrids of Evolution Strategy exists, e.g. CMA-ES hybridization is proposed in [35], a hybrid ES for solving mixed (continuous and discrete) problems was proposed in [22]. Apparently the number of hybrid metaheuristics based on Evolution Strategies is not high, thus it seems that exploring the possibilities of creative hybridization of those algorithms might be interesting and advantageous. Therefore, based on our previous experiences in hybridizing ACO [11] and PSO [8], we propose to introduce socio-cognitive mechanisms into Evolution Strategies.

3 Socio-cognitive hybridization of the Evolution Strategies

Striving towards better exploration of socio-cognitive inspired hybridization of metaheuristics, we would like to present a first step towards verification of such possibilities, focusing on evolution strategies.

The classic self-adaptive version algorithm for evolution strategies can be described as follows.

1. Initialize parent population $P_\mu = \{i_1, \dots, i_\mu\}$. Each of the individuals can be described as follows: $I \ni i_k = \{g_{k,1}, \dots, g_{k,d}, s_{k_1}, \dots, s_{k,d}\}, k, d \in \mathbb{N}$ stands for an individual containing a genotype ($g_{k,l}$ is $l-th$ gene of $k-th$ genotype). The dimensionality of the considered problem is d. The $s_{k_1}, \dots, s_{k,d}$ are mutation strategy parameters that will be adapted in order to guide the search.
2. Generate λ offspring individuals forming the offspring population $P_\lambda = \{i_1, \dots, i_\lambda\}$ in the following procedure:
   - Randomly select ϱ parents from P_μ (if $\varrho = \mu$ take all of them of course).
   - Recombine the ϱ selected parents (traditionally a pair) to form a recombinant individual i_r, using any possible recombination means (traditionally averaging crossover operator was used).

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  - Mutate the strategy parameter set $s_{r,1}, \ldots, s_{r,d}$ of the recombinant i_r (adapting e.g. the mutation diversities for the next mutation). Traditionally mutation is realized by applying a distortion based on e.g. uniform or Gaussian random distribution, adding or substracting a certain value to (from) a selected gene.
  - Mutate the objective parameter set $g_{r_1}, \ldots, g_{r_d}$ of the recombinant i_r using the mutated strategy parameter set to control the statistical properties of the object parameter mutation.
3. Select new parent population (using deterministic truncation selection) from either the offspring population P_λ (this is referred to as comma-selection, usually denoted as "(μ, λ)-selection"), or the offspring P_λ and parent $P\mu$ population (this is referred to as plus-selection, usually denoted as $P(\mu+\lambda)$.
4. Goto 2. until termination criterion fulfilled.

We have decided to introduce the socio-cognitive mechanisms to all basic versions of evolution strategies, namely: $(1+1)$, (μ, λ), $(\mu+\lambda)$.

In the novel, socio-cognitive version of Evolution Strategy, we try to increase the exchange of the knowledge among the individuals, so they can get information not only during the mutation and adaptation of their mutation parameters, but also observe others. So inside the second step of the algorithm depicted above, we introduce the following changes:

1. The algorithm stores historically α_{best} and α_{worst} individuals.
2. During the mutation, one of the following three different mutations are realized:
   - Classic mutation realized with γ probability.
   - Modification of the individual towards the historically α_{best} best individuals with probability γ_{good}. This mutation sets the current gene copying the gene from one of the historically best individuals: Assume, $i_b = \{g_{b,1}, \ldots, g_{b,d}, s_{b_1}, \ldots, s_{b,d}\}$ is a randomly picked individual from the historically best ones. The individual about to be mutated is $i_m = \{g_{m,1}, \ldots, g_{m,d}, s_{m_1}, \ldots, s_{m,d}\}$. Let us randomly pick one of the genes of i_m. Assume $1 \leq p \leq d$ is this random value, so the picked gene is $g_{m,p}$. Now we will simply assign the value of this gene to the value of a correspondent gene in i_b, that is $g_{m,p} \leftarrow g_{b,p}$.
   - Modification of the individual trying to avoid the historically α_{bad} worst individuals with probability γ_{bad}. This mutation computes the difference between the current gene and one gene of the historically worst individuals, computes a fraction of this value, multiplying it by β, and adds it to the current gene. The procedure of randomly choosing one of the historically worst individual is similar as it was described above. Let us go to the final step, we have the individual to be mutated i_m, randomly chosen individual belonging to the historically worst i_w. $1 \leq p \leq d$ stands for the random index of the gene. The following assignment is realized: $i_{m,p} \leftarrow \beta \cdot (i_{m,p} - i_{w,p})$.
3. When better or worse (historically) individuals are generated, the lists of α_{best} and α_{worst} individuals are updated.

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Three basic versions of Evolution Strategies were modified using this mechanism. Thus the mutation is realized not only in a fully stochastic way, but also reuses the knowledge about the previous generations. Of course such mechanism is not complex, but as it will be shown in the next section, it already produced interesting results.

4 Study design

The main aim of the experiments was verification of efficacy of global optimization (minimization) of the novel algorithms for the selected benchmark functions (Ackley, De Jong, Rastrigin, and Griewank [12] visualized in Fig. 1) in $d = 10$, 50, 100, 500 and 1000 dimensions. Both the value obtained in the last iteration, and the trajectory of the fitness functions improvements were considered – in certain situations it is desirable to have a relatively fast convergence earlier, in other situations the focus is put on the final result. The equations used are as follows:

- Ackley: $f(x) = -ae^{-b\sqrt{1/n\sum_{i=1}^{n}(x_i^2)}} - e^{1/n\sum_{i=1}^{n}cos(cx_i)} + a + e; a = 20; b = 0.2; c = 2\pi; i \in [1:n]; -32.768 \leq x(i) \leq 32.768.$ $f(x^{opt}) = 0,\ x_i^{opt} = 0.$
- De Jong: $f(x) = \sum_{i=1}^{n} x_i^2, i \in [1, n]; -5.12 \leq x_i \leq 5.12.$ $f(x^{opt}) = 0,\ x_i^{opt} = 0.$
- Rastrigin: $f(x) = 10n + \sum_{i=1}^{n}(x_i^2 - 10cos(2\pi x_i)), i \in [1, n]; -5.12 \leq x_i \leq 5.12.$ $f(x^{opt}) = 0,\ x_i^{opt} = 0.$
- Griewank: $f(x) = \sum_{x=1}^{n} x_i^2/4000 - \prod cos(x_i/\sqrt{i}) + 1, i \in [1, n]; -600 \leq x_i \leq 600,\ f(x^{opt}) = 0,\ x_i^{opt} = 0.$

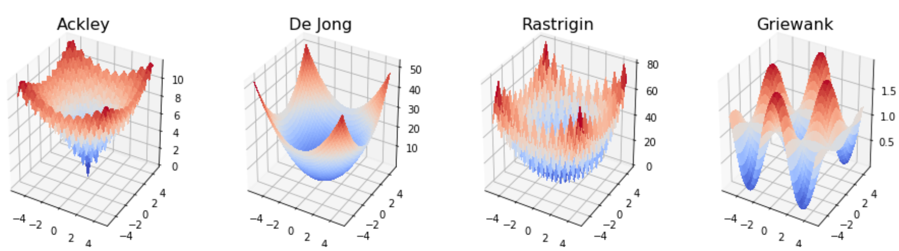

Fig. 1. 3-dimensional visualization of the benchmarks used in the study.

The following algorithms were benchmarked:

- original Evolution Strategy $(1+1)$, (μ, λ) or $(\mu + \lambda)$,
- hybrid Evolution Strategy – with the mechanism of getting closer to the historically best results,
- hybrid Evolution Strategy – with the mechanism of going farther from the historically worst results,

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– hybrid Evolution Strategy – with both the above-mentioned mechanism.

The stopping criteria was reaching maximum number of evaluations of fitness function (set as 25,000 for all the experiments). Number of the individuals in the population was μ (1 in the case of (1+1) Evolution Strategy, 20 in the case of other Strategies).

The following settings were used for the algorithms:

- $\mu = 1$, $\lambda = 1$ in the case of $(1 + 1)$ Evolution Strategy.
- $\mu = 20$, $\lambda = 140$ in other cases.
- $\gamma_{good} = 0.4$, $\gamma_{bad} = 0.04$, $\beta = 0.01$.
- $\gamma = 1/(\text{number of dimensions})$,
- number of the historically best or worst individuals: 5.
- polynomial mutation [20].

Each experiment was replicated 20 times and the mean value of fitness function was taken for the reference. In order to check whether the observed sample had normal distribution we have applied Shapiro-Wilk test with significance threshold of 0.05. Kruskal-Wallis test was used in order to check whether their cumulative distribution functions differed, and finally Dunn's test in order to check which ones were significantly different.

5 Experimental results

The algorithms were implemented using jMetalPy[1] computing framework. The source code is available on request. The computations were conduced on a Microsoft Windows 10 machine with AMD Ryzen 9 5900X (3.7GHz) CPU, NVIDIA GeForce RTX 3080 GPU, and 2x8GB RAM. We have analyzed the speed of improvement of the fitnesses depending on the number of evaluations and compared statistical differences between the algorithms after 25,000 iterations.

5.1 $(1 + 1)$ Evolution Strategy and its socio-cognitive hybrids

The $(1 + 1)$ Evolution Strategy was compared with its hybrids for Rastrigin benchmark in 1000 dimensions (see Fig. 2). A metaheuristic algorithm operating only on one individual (so-called trajectory method) as expected is not too versatile, especially in the case of high-dimensional benchmarks. So, even the original algorithm does not reach the vicinity of the global optimum, stopping around 4000. Its hybrid version with the mechanism of getting closer to the best results is very similar to the original one. The two remaining algorithms perform significantly worse, apparently getting stuck in a local sub-optimum. It seems that in the case of (1+1) Evolution Strategy, adding the mechanism of avoiding worse individuals does not help or requires significant improvements.

The more detailed observation of the results presented in Table 1 confirms the findings. The original implementation of Evolution Strategy produces similar results to its hybrid, while the two remaining hybrids are significantly worse.

[1] https://github.com/jMetal/jMetalPy

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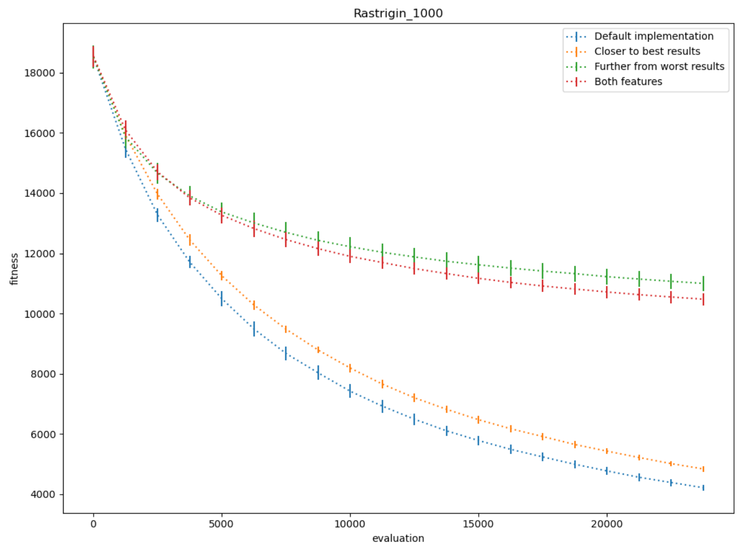

Fig. 2. Trajectory of changes of mean fitness function value (standard deviation whiskers are also shown) for 1000-dimensional Rastrigin problem and (1 + 1) Evolutionary Strategy, depending on the number of fitness function evaluation.

Table 1. Mean and standard deviation of fitness value after 25,000 evaluations of $(1, 1)$ Evolution Strategy and its hybrids for 10, 50, 100, 500 and 1000 dim. problems.

Dimension	10		50		100		500		1000	
	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.
Ackley										
Default	0,27	0,12	2,19	0,28	3,43	0,20	9,89	0,31	14,25	0,21
Closer to best	0,42	0,18	2,63	0,34	4,00	0,24	10,90	0,29	15,15	0,24
Farther from worst	**0,21**	**0,11**	2,88	0,25	5,01	0,33	17,72	1,04	20,67	0,12
Both	0,24	0,09	2,90	0,31	5,23	0,36	19,02	0,92	20,62	0,09
De Jong										
Default	0,00	0,00	0,07	0,02	0,58	0,12	69,97	7,37	490,89	36,76
Closer to best	0,00	0,00	0,11	0,04	0,94	0,18	95,67	6,23	627,27	43,34
Farther from worst	0,00	0,00	0,13	0,04	1,41	0,16	157,25	11,94	1108,31	69,49
Both	0,00	0,00	**0,06**	**0,02**	0,89	0,10	151,14	12,08	1039,64	42,48
Griewank										
Default	0,01	0,01	0,04	0,09	0,08	0,13	0,36	0,15	0,87	0,10
Closer to best	0,01	0,01	**0,02**	**0,04**	**0,03**	**0,04**	**0,35**	**0,07**	0,97	0,05
Farther from worst	0,01	0,01	0,03	0,05	0,08	0,10	0,53	0,08	1,24	0,03
Both	0,01	0,01	0,03	0,06	0,04	0,11	0,48	0,04	1,21	0,02
Rastrigin										
Default	0,10	0,05	10,96	2,29	50,78	4,66	1178,87	69,39	4027,59	132,27
Closer to best	0,16	0,09	15,02	2,37	64,07	6,08	1394,98	53,55	4661,99	114,84
Farther from worst	0,08	0,05	23,31	2,51	132,28	11,58	3797,29	136,07	10912,88	193,63
Both	0,09	0,05	16,29	3,03	106,35	10,34	3401,20	127,62	10459,14	309,85

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5.2 (μ, λ) Evolution Strategy and its socio-cognitive hybrids

Moving to population-based methods, let us start with (μ, λ) Evolution Strategy. Fig. 3 shows the curves of fitness functions for 1000-dimensional Ackley problem. Both the hybrids utilizing the mechanisms of getting toward the best results turned out to be significantly better than the original algorithm and its hybrid going towards the worst results. Again, it seems that the mechanism of avoiding the worst results might be further improved. It is to note, that the hybrid version with the mechanism going to the best results got close to the global optimum (around 18.0) still having the potential of improving the result, while all the other versions of the algorithms got stuck in local sub-optima. One should remember about specific features of Ackley function – in 3 dimensions it has a steep peak around the global optimum, while it is quite flat in other areas. Probably some of these features scale up to higher dimensions. So it is important to search broadly in this case, and when good results are found, it is necessary to explore them intensively.

Good efficacy of this algorithm is confirmed for Ackley benchmark in all tested dimensions (see Table 2). The hybrids are also significantly better in several other cases of De Jong, Griewank and Rastrigin functions.

5.3 $(\mu + \lambda)$ Evolution Strategy and its socio-cognitive hybrids

Finally, $(\mu + \lambda)$ Evolution Strategy is examined along with its hybrids, applied to optimization of 1000-dimensional Griewank problem. In Fig. 4 we can see that both hybrids utilizing the mechanism of getting closer to the best results prevailed. They come very close to the optimum and apparently do not loose the diversity. Other two algorithms also do not seem to be stuck, however they are significantly slower than the winning hybrids. However, this part of the experiment show, that $(\mu + \lambda)$ Evolution Strategy was the best starting point for the hybridization.

This observation is further confirmed when looking at Table 3. The final results produced by the hybrids getting closer to the best results actually prevailed for almost all of the tested instances of the problems.

In order to make sure that we have really different average results, we have used the Dunn's test (see Table 4) for excluding the null hypotheses that cumulative distribution function of the sampled final values (the fitness obtained when the stopping condition is met). As it can be seen, a significant majority of the tests were finished with excluding the null-hypothesis.

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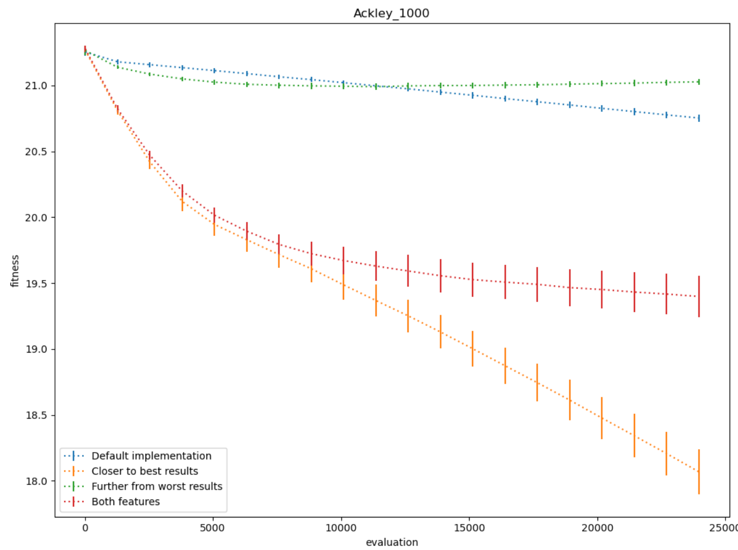

Fig. 3. Trajectory of changes of mean fitness function value (standard deviation whiskers are also shown) for 1000-dimensional Ackley problem and (μ, λ) Evolutionary Strategy, depending on the number of fitness function evaluation.

Table 2. Mean and standard deviation of fitness value after 25,000 evaluations of (μ, λ) Evolution Strategy and its hybrids for 10, 50, 100, 500 and 1000 dim. problems.

Dimension	10		50		100		500		1000	
	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.
Ackley										
Default	0,54	0,19	6,86	0,47	12,52	0,56	20,10	0,07	20,72	0,03
Closer to best	**0,30**	**0,11**	**2,84**	**0,27**	**5,00**	**0,23**	**15,01**	**0,29**	**17,89**	**0,19**
Farther from worst	0,29	0,13	7,28	0,63	14,40	0,97	20,85	0,05	21,04	0,02
Both	0,13	0,05	2,87	0,25	5,64	0,41	16,87	0,40	19,48	0,21
De Jong										
Default	0,00	0,00	2,77	0,64	33,59	5,17	1937,85	65,87	5797,31	119,30
Closer to best	0,00	0,00	0,15	0,05	1,90	0,33	**315,97**	**26,29**	**1646,45**	**88,98**
Farther from worst	0,00	0,00	2,78	0,58	37,02	4,56	2227,48	86,29	6538,94	118,44
Both	0,00	0,00	**0,01**	**0,01**	**1,05**	**0,25**	354,54	14,56	1787,58	109,50
Griewank										
Default	0,01	0,01	0,09	0,08	0,42	0,05	1,49	0,02	2,44	0,02
Closer to best	0,01	0,01	0,01	0,01	0,03	0,01	**0,79**	**0,06**	**1,39**	**0,02**
Farther from worst	0,01	0,01	0,10	0,05	0,46	0,08	1,55	0,02	2,63	0,02
Both	0,01	0,01	0,00	0,01	**0,02**	**0,01**	0,84	0,05	1,45	0,02
Rastrigin										
Default	0,34	0,17	62,43	7,90	314,27	19,86	5689,33	113,58	14276,61	141,83
Closer to best	0,15	0,11	16,26	3,02	80,71	6,91	**2188,21**	**74,69**	**7190,55**	**146,55**
Farther from worst	0,11	0,23	64,49	6,24	356,50	30,52	6292,99	131,96	15158,68	161,31
Both	**0,03**	**0,02**	**12,22**	**2,80**	**66,64**	**7,91**	2197,37	93,43	7521,77	170,56

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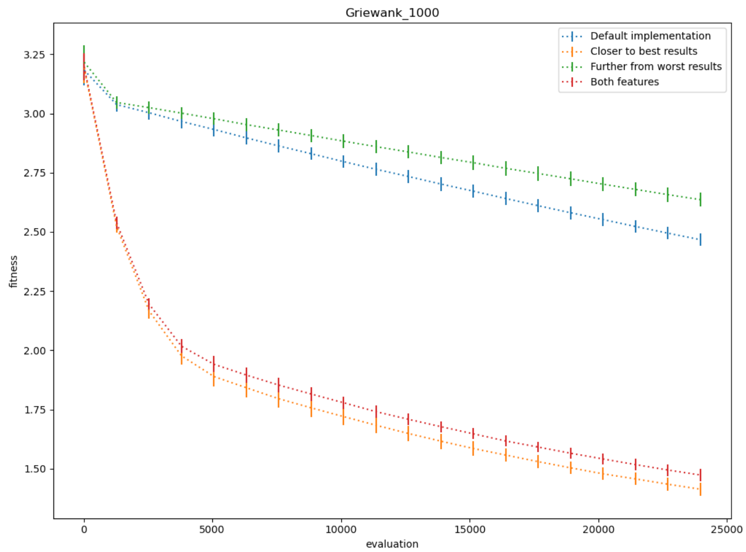

Fig. 4. Trajectory of changes of mean fitness function value (standard deviation whiskers are also shown) for 1000-dimensional Griewank problem and $(\mu + \lambda)$ Evolutionary Strategy, depending on the number of fitness function evaluation.

Table 3. Mean and standard deviation of fitness value after 25,000 evaluations of $(\mu+\lambda)$ Evolution Strategy and its hybrids for 10, 50, 100, 500 and 1000 dim. problems.

Dimension	10		50		100		500		1000	
	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.	Fit.	Std.
Ackley										
Default	0,56	0,17	6,47	0,60	11,92	0,57	20,10	0,07	20,72	0,02
Closer to best	0,40	0,16	**2,87**	**0,19**	**4,83**	**0,26**	**15,02**	**0,27**	**18,03**	**0,16**
Farther from worst	0,27	0,24	6,95	0,71	13,81	0,78	20,63	0,04	20,87	0,02
Both	**0,14**	**0,04**	3,08	0,39	5,94	0,37	17,21	0,52	19,58	0,17
De Jong										
Default	0,00	0,00	2,16	0,45	32,26	3,86	1945,75	63,55	5788,75	120,45
Closer to best	0,00	0,00	0,17	0,05	1,94	0,36	**326,75**	**24,01**	**1647,68**	**80,94**
Farther from worst	0,00	0,00	2,24	0,48	34,10	4,56	2143,40	64,08	6469,60	116,27
Both	0,00	0,00	**0,01**	**0,02**	**1,12**	**0,39**	363,40	23,09	1846,96	95,48
Griewank										
Default	0,02	0,01	0,09	0,10	0,41	0,05	1,48	0,02	2,44	0,02
Closer to best	0,01	0,01	0,03	0,08	**0,04**	**0,04**	**0,80**	**0,05**	**1,41**	**0,04**
Farther from worst	0,01	0,01	0,08	0,08	0,41	0,08	1,54	0,01	2,62	0,03
Both	0,01	0,01	**0,01**	**0,04**	0,04	0,08	0,83	0,05	1,46	0,02
Rastrigin										
Default	0,19	0,10	54,27	7,80	305,64	17,42	5677,59	93,41	14323,20	148,70
Closer to best	0,12	0,06	17,92	3,42	82,88	7,69	**2235,61**	**93,00**	**7261,88**	**216,26**
Farther from worst	0,07	0,04	49,53	6,56	318,51	22,35	6123,40	118,54	15003,41	221,74
Both	**0,03**	**0,03**	**10,80**	**3,05**	**67,64**	**9,09**	2268,98	108,03	7613,46	234,01

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Table 4. P-values generated by Dunn's test for excluding the null hypotheses that cumulative distribution function of the sampled final values (the fitness obtained when the stopping condition is met). The cases when the p-value is lower than 0.05 were enhanced with bold font.

(1+1) Evolution Strategy				
Ackley	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**3.90e-02**	**1.40e-12**	**2.03e-09**
Cl. b.			**2.46e-05**	**2.24e-03**
Far. w.				1.00e+00
De Jong	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**4.59e-02**	8.92e-14	**2.69e-08**
Cl. b.			**3.08e-06**	**8.30e-03**
Far. w.				4.09e-01
Griewank	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		4.48e-01	**8.46e-12**	**1.39e-07**
Cl. b.			**6.93e-07**	**8.56e-04**
Far. w.				8.06e-01
Rastrigin	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**3.90e-02**	**1.68e-14**	**6.83e-08**
Cl. b.			**1.35e-06**	**1.69e-02**
Far. w.				1.71e-01
(mu, lambda) Evolution Strategy				
Ackley	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**3.14e-07**	**3.90e-02**	**3.90e-02**
Cl. b.			**1.93e-15**	3.90e-02
Far. w.				**3.14e-07**
De Jong	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**3.08e-06**	**3.90e-02**	**1.00e-02**
Cl. b.			**5.82e-14**	3.62e-01
Far. w.				**2.69e-08**
Griewank	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**6.20e-07**	3.90e-02	**2.67e-02**
Cl. b.			**5.28e-15**	7.96e-02
Far. w.				**1.57e-07**
Rastrigin	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**9.33e-07**	**3.90e-02**	**2.11e-02**
Cl. b.			**9.70e-15**	1.20e-01
Far. w.				**1.02e-07**
(mu+lambda) Evolution Strategy				
Ackley	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**3.14e-07**	**3.90e-02**	**3.90e-02**
Cl. b.			**1.93e-15**	**3.90e-02**
Far. w.				**3.14e-07**
De Jong	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**7.20e-07**	3.90e-02	**2.45e-02**
Cl. b.			**6.59e-15**	9.25e-02
Far. w.				**1.34e-07**
Griewank	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**1.67e-06**	**3.90e-02**	**1.48e-02**
Cl. b.			**2.33e-14**	2.10e-01
Far. w.				**5.37e-08**
Rastrigin	**Def.**	**Cl. b.**	**Far. w.**	**Both**
Def.		**1.93e-06**	**4.59e-02**	**1.12e-02**
Cl. b.			**4.45e-14**	2.73e-01
Far. w.				**4.57e-08**

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6 Conclusions

In this paper we have presented a novel hybrid algorithms based on classic Evolution Strategies by Schwefel and Rechenberg. The method was tested on a set of well known multi-modal benchmark problems. Three different evolution strategies were analyzed: $(1 + 1)$, (μ, λ) and $(\mu + \lambda)$ and efficacy of the new socio-cognitive hybrids was compared to the baseline model.

The proposed herein socio-cognitive hybrids increase the possibility to exchange information between the individuals by introducing a set of historically best and historically worst solutions, affecting the way in which the individuals are mutated. In the case of the hybrid utilizing the historically worst solutions, an attempt has been made to avoid them during mutation. Conversely, when considering the historically best solutions, during the mutation some of their information was copied.

The $(1+1)$ Evolution Strategy turned out not to be an effective solution for hybridization purposes, as the baseline algorithm outperformed hybrids in most of the examined cases. In the case of (μ, λ) Evolution Strategy, the best results were obtained for Ackley benchmark where 'Closer to best' strategy converged much faster and to better solutions. As for $(\mu + \lambda)$ Evolution Strategy was actually better in the case of mechanism going closer to the best up-to-date solutions, for all the considered problem instances.

We conclude, that using information from the best performing individuals may increase the convergence speed of the evolutionary strategies and significantly improve over the baseline model. Both hybrids, that used this feature, namely 'Closer to best' and 'Both', performed usually better than the baseline model. This is actually an expected outcome, as those strategies focus on further local optimizations, thus are more likely to fine-tune better solutions. It is also in line with other research in evolutionary computation (e.g. [14]). Avoiding worst individuals implemented in 'Further from worst' turned out to be somewhat helpful, but only for low dimensional spaces (up to 100 dimensions). When the dimensionality of the problem increased, this hybrid was slowing down convergence and eventually led to less effective solutions.

In future, besides the necessary enhancement, we would like to further explore the social component in the Evolution Strategies hybrids, by testing certain parameters (e.g. the length of the history), but also by introducing new and updated social mechanisms (e.g. different species and relations among them). Other population-based metaheuristics will be also considered.

References

1. Arnold, D.V.: Weighted multirecombination evolution strategies. Theoretical Computer Science **361**(1), 18–37 (2006), foundations of Genetic Algorithms
2. Bandura, A.: Self-efficacy: Toward a unifying theory of behavioral change. Psychological Review **84**(2), 191–215 (1977)
3. Bandura, A.: Social foundations of thought and action: a social cognitive theory. Prentice-Hall, Englewood Cliffs, N.J. (1986)

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4. Bandura, A., Ross, D., Ross, S.: Transmission of aggression through the imitation of aggressive models. J. of Abnormal and Social Psychology **63**(3), 575–582 (1961)
5. Beume, N., Rudolph, G.: Faster s-metric calculation by considering dominated hypervolume as klee's measure problem. In: Kovalerchuk, B. (ed.) Proceedings of the Second IASTED International Conference on Computational Intelligence, San Francisco, California, USA, November 20-22, 2006. pp. 233–238. IASTED/ACTA Press (2006)
6. Blum, C., Puchinger, J., Raidl, G.R., Roli, A.: Hybrid metaheuristics in combinatorial optimization: A survey. Applied Soft Computing **11**(6), 4135–4151 (2011). https://doi.org/https://doi.org/10.1016/j.asoc.2011.02.032, https://www.sciencedirect.com/science/article/pii/S1568494611000962
7. Brockhoff, D., et al.: Mirrored sampling and sequential selection for evolution strategies. In: Schaefer, R., Cotta, C., Kołodziej, J., Rudolph, G. (eds.) Parallel Problem Solving from Nature, PPSN XI. pp. 11–21. Springer Berlin Heidelberg, Berlin, Heidelberg (2010)
8. Bugajski, I., Listkiewicz, P., Byrski, A., Kisiel-Dorohinicki, M., Korczynski, W., Lenaerts, T., Samson, D., Indurkhya, B., Nowé, A.: Enhancing particle swarm optimization with socio-cognitive inspirations. In: Connolly, M. (ed.) International Conference on Computational Science 2016, ICCS 2016. Procedia Computer Science, vol. 80, pp. 804–813. Elsevier (2016)
9. Byrski, A., Schaefer, R., Smołka, M., Cotta, C.: Asymptotic guarantee of success for multi-agent memetic systems. Bulletin of Polish Academy of Sciences: Technical Sciences **61**(1), 257–278 (2013)
10. Byrski, A., Drezewski, R., Siwik, L., Kisiel-Dorohinicki, M.: Evolutionary multi-agent systems. Knowl. Eng. Rev. **30**(2), 171–186 (2015)
11. Byrski, A., Swiderska, E., Lasisz, J., Kisiel-Dorohinicki, M., Lenaerts, T., Samson, D., Indurkhya, B., Nowé, A.: Socio-cognitively inspired ant colony optimization. J. Comput. Sci. **21**, 397–406 (2017)
12. Dieterich, J., Hartke, B.: Empirical review of standard benchmark functions using evolutionary global optimization. Applied Mathematics **3**(18A) (2012)
13. Dorigo, M., Stutzle, T.: Ant Colony Optimization. MIT Press (2004)
14. Du, H., Wang, Z., Zhan, W., Guo, J.: Elitism and distance strategy for selection of evolutionary algorithms. IEEE Access **6**, 44531–44541 (2018). https://doi.org/10.1109/ACCESS.2018.2861760
15. Hansen, N., Ostermeier, A.: Completely derandomized self-adaptation in evolution strategies. Evolutionary Computation **9**(2), 159–195 (2001)
16. Jamasb, A., Motavalli-Anbaran, S.H., Ghasemi, K.: A novel hybrid algorithm of particle swarm optimization and evolution strategies for geophysical non-linear inverse problems. Pure and Applied Geophysics **176** (04 2019)
17. Kennedy, J., Eberhart, R.: Particle swarm optimization. In: Proceedings of ICNN'95 - International Conference on Neural Networks. vol. 4, pp. 1942–1948 vol.4 (1995)
18. Klose, A.D., Hielscher, A.H.: Hybrid approach for diffuse optical tomography combining evolution strategies and gradient techniques. In: Chance, B., Alfano, R.R., Tromberg, B.J., Tamura, M., Sevick-Muraca, E.M. (eds.) Optical Tomography and Spectroscopy of Tissue IV. vol. 4250, pp. 11 – 19. International Society for Optics and Photonics, SPIE (2001)
19. Koulocheris, D., Vrazopoulos, H., Dertimanis, V.: Hybrid evolution strategy for the design of welded beams. In: Proc. of Int. Congress on Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems EUROGEN 2003. CIMNE Barcelona (2003)

ICCS Camera Ready Version 2021
To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77964-1_26

20. Liagkouras, K., Metaxiotis, K.: An elitist polynomial mutation operator for improved performance of moeas in computer networks. In: 2013 22nd International Conference on Computer Communication and Networks (ICCCN). pp. 1–5 (2013). https://doi.org/10.1109/ICCCN.2013.6614105
21. Michalewicz, Z., Fogel, D.: How to Solve It: Modern Heuristics. Springer (2004)
22. Moreau-Giraud, L., Lafon, P.: A hybrid evolution strategy for mixed discrete continuous constrained problems. In: Fonlupt, C., Hao, J.K., Lutton, E., Schoenauer, M., Ronald, E. (eds.) Artificial Evolution. pp. 123–135. Springer Berlin Heidelberg, Berlin, Heidelberg (2000)
23. Placzkiewicz, L., Sendera, M., Szlachta, A., Paciorek, M., Byrski, A., Kisiel-Dorohinicki, M., Godzik, M.: Hybrid swarm and agent-based evolutionary optimization. In: Shi, Y., Fu, H., Tian, Y., Krzhizhanovskaya, V.V., Lees, M.H., Dongarra, J., Sloot, P.M.A. (eds.) Computational Science – ICCS 2018. pp. 89–102. Springer International Publishing, Cham (2018)
24. Rabiej, M.: A hybrid immune–evolutionary strategy algorithm for the analysis of the wide-angle X-ray diffraction curves of semicrystalline polymers. Journal of Applied Crystallography **47**(5), 1502–1511 (Oct 2014)
25. Rechenberg, I.: Cybernetic solution path of an experimental problem. Royal Aircraft Establishment Library Translation 1122 (1965), https://ci.nii.ac.jp/naid/10000137330/en/
26. Repoussis, P., Tarantilis, C., Bräysy, O., Ioannou, G.: A hybrid evolution strategy for the open vehicle routing problem. Computers & Operations Research **37**(3), 443–455 (2010), hybrid Metaheuristics
27. dos Santos Coelho, L., Alotto, P.: Electromagnetic device optimization by hybrid evolution strategy approaches. International journal for computation and mathematics in electrical and electronic engineering **26**(2), 269–279 (2007)
28. Schwefel, H.P.: Numerische Optimierung von Computer-Modellen mittels der Evolutionsstrategie: mit einer vergleichenden Einführung in die Hill-Climbing-und Zufallsstrategie, vol. 1. Springer (1977)
29. Schwefel, H.P.: Evolution and Optimum Seeking. Wiley (1995)
30. Talbi, E.G.: Metaheuristics: From Design to Implementation. Wiley (2009)
31. Talbi, E.G.: A taxonomy of hybrid metaheuristics. Journal of Heuristics **8**, 541–564 (01 2002). https://doi.org/10.1023/A:1016540724870
32. Tao-Yuan Huang, Yung-Yaw Chen: Modified evolution strategies with a diversity-based parent-inclusion scheme. In: Proceedings of the 2000. IEEE International Conference on Control Applications. Conference Proceedings (Cat. No.00CH37162). pp. 379–384 (2000)
33. Vose, M.D.: The simple genetic algorithm - foundations and theory. Complex adaptive systems, MIT Press (1999)
34. Wolpert, D.H., Macready, W.G.: No free lunch theorems for optimization. Trans. Evol. Comp **1**(1), 67–82 (Apr 1997)
35. Zhang, G., Shi, Y.: Hybrid sampling evolution strategy for solving single objective bound constrained problems. In: 2018 IEEE Congress on Evolutionary Computation (CEC). pp. 1–7 (2018). https://doi.org/10.1109/CEC.2018.8477908

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