A New Consistency Coefficient in the Multi-Criteria Decision Analysis Domain

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Abstract. The logical consistency of decision making matrices is an important topic in developing each multi-criteria decision analysis (MCDA) method. For instance, many published papers are addressed to the decisional matrix's consistency in the Analytic Hierarchy Process method (AHP), which uses Saaty's seventeen-values scale.
This work proposes a new approach to measuring consistency for using a simple three-value scale (binary with a tie). The paper's main contribution is a proposal of a new consistency coefficient for a decision matrix containing judgments from an expert. We show this consistency coefficient based on an effective MCDA method called the Characteristic Objects METhod (COMET). The new coefficient is explained based on the Matrix of Expert Judgment (MEJ), which is the critical step of the COMET method. The proposed coefficient is based on analysing the relationship between judgments from the MEJ matrix and transitive principles (triads analysis). Four triads classes have been identified and discussed. The proposed coefficient makes it easy to determine the logical consistency and, thus, the expert responses' quality is essential in the reliable decision-making process. Results are presented in some short study cases.

Keywords: Decision analysis · decision making · decision theory · consistency coefficient · inconsistency coefficient

1 Introduction

Multi-criteria decision analysis (MCDA) is one of the branches of operational research, whose main objective is to support the decision-maker in solving multi-criteria problems [30]. MCDA methods are widely used in many practical decision-making problems, e.g. medicine [14, 16], engineering [28], transport [3], energy [17, 22, 31], management [2], and others [4, 10, 15]. The quality of the solutions obtained in such cases depends on the specific MCDA method's algorithm and the error of the judgments in pairwise comparison by the expert.

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The most popular technique, which requires judgments in a pairwise comparison, is the Analytic Hierarchy Process (AHP) [19]. Unfortunately, the most critical AHP method's shortcoming is the ranking reversal phenomenon [23, 24], which may strongly discredit the results' reliability. For this approach, Saaty proposed the most popular inconsistency index in [18] and his research was continued in [1, 6, 8, 26]. The AHP uses a multi-valued Saaty scale with a consistency coefficient based on the matrix's eigenvalue, which is not suitable for determining the decisional matrices' logical consistency used three-value scale's. Kendall and Babington [11] proposed their consistency coefficient, which allows the consistency degree of a binary pairwise comparison set. It was also the inspiration to continue the research for many researchers [5, 7, 12, 25].

The rank reversal paradox has become the beginning of proposing a new method called the Characteristic Objects METhod (COMET) [20, 29]. Earlier works have shown that it is a method resistant to this paradox, and the obtained solutions are more accurate than with the AHP method [16, 23, 21]. However, once the pairwise comparison judgments have been received, this method lacks a coefficient to check the matrix's logical consistency.

Both methods use a pairwise comparison to create a comparison matrix. A different scale is used in both cases, i.e., three levels in the COMET and 17 degrees or more in the AHP. Due to an expert's possible mistakes with a series of similar questions, the AHP method uses an inconsistency coefficient based on the eigenvector method. It is helpful for an expert to assess if his answers are sufficiently consistent [13].

In this work, the most significant contribution is a new consistency coefficient for decision matrices with a simple three value scale. This coefficient is presented by using an example of a Matrix of Expert Judgment (MEJ), a crucial step in the COMET method. Currently, there is not possible to verify whether or not the decision matrix received is logically consistent. The proposed coefficient allows us to check how strongly the MEJ matrix is consistent. It is crucial because the decisional model will be as good as an expert is. Therefore, the essential characteristics of this coefficient will be examined in some experiments. The coefficient design will be based on Kendall's work [11] and relate to work [12].

The rest of the paper is organized as follows: In section 2, some preliminary MEJ concepts are presented. Section 3 introduces a new consistency coefficient. In Section 4, the discussion on simple experiments shows the most important properties of the presented consistency coefficient. In Section 5, we present the summary and conclusions.

2 Matrix of Expert Judgment (MEJ)

The first three steps of the COMET method should be presented to show the whole procedure for creating the MEJ matrix [20, 9]. In the first step, we should define the space of the problem. An expert determines the dimensionality of the problem by selecting the number r of criteria, $C_1, C_2, ..., C_r$. Then, the set of

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fuzzy numbers for each criterion C_i is selected (1):

$$\begin{array}{l} C_1 = \{\tilde{C}_{11}, \tilde{C}_{12}, ..., \tilde{C}_{1c_1}\} \\ C_2 = \{\tilde{C}_{21}, \tilde{C}_{22}, ..., \tilde{C}_{2c_1}\} \\ .... \\ C_r = \{\tilde{C}_{r1}, \tilde{C}_{r2}, ..., \tilde{C}_{rc_r}\} \end{array} \tag{1}$$

where $c_1, c_2, ..., c_r$ are numbers of the fuzzy numbers for all criteria.

As a second step, we generate characteristic objects. The characteristic objects (CO) are obtained by using the Cartesian Product of fuzzy numbers cores for each criteria as follows (2):

$$CO = C(C_1) \times C(C_2) \times ... \times C(C_r) \tag{2}$$

As the result, the ordered set of all CO is obtained (3):

$$\begin{array}{l} CO_1 = \{C(\tilde{C}_{11}), C(\tilde{C}_{21}), ..., C(\tilde{C}_{r1})\} \\ CO_2 = \{C(\tilde{C}_{11}), C(\tilde{C}_{21}), ..., C(\tilde{C}_{r2})\} \\ .... \\ CO_t = \{C(\tilde{C}_{1c_1}), C(\tilde{C}_{2c_2}), ..., C(\tilde{C}_{rc_r})\} \end{array} \tag{3}$$

where t is the number of COs (4):

$$t = \prod_{i=1}^{r} c_i \tag{4}$$

The third and final step is that we rank the characteristic objects. In the first part of this step, an expert determines the Matrix of Expert Judgment (MEJ). It is a result of pairwise comparison of the COs by the expert. The MEJ structure is presented (5):

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & ... & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & ... & \alpha_{2t} \\ ... & ... & ... & ... \\ \alpha_{t1} & \alpha_{t2} & ... & \alpha_{tt} \end{pmatrix} \tag{5}$$

where α_{ij} is the result of comparing CO_i and CO_j by the expert. The more preferred characteristic object gets one point and the second object gets zero points. If the preferences are balanced, both objects get a half point. It depends solely on the knowledge of the expert and can be presented as (6):

$$\alpha_{ij} = \begin{cases} 0.0, \ f_{exp}(CO_i) < f_{exp}(CO_j) \\ 0.5, \ f_{exp}(CO_i) = f_{exp}(CO_j) \\ 1.0, \ f_{exp}(CO_i) > f_{exp}(CO_j) \end{cases} \tag{6}$$

where f_{exp} is the expert mental judgment function. The some interesting properties are described by equations (7) and (8):

$$\alpha_{ii} = 0.5 \tag{7}$$

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$$\alpha_{ji} = 1 - \alpha_{ij} \tag{8}$$

Based on (7) and (8), the number of comparisons is reduced from t^2 cases to p cases (9):

$$p = \binom{t}{2} = \frac{t(t-1)}{2} \tag{9}$$

3 Consistency coefficient

This section is divided into two parts. In section 3.1, we analyze characteristic objects in the MEJ and discusses possible triad. It should be noted that the upper triangular matrix is in close relation to the lower triangular matrix. This is important due to the triad analysis, as only the upper triangular matrix will be analyzed. In section 3.2, we propose the consistency coefficient, which is based on the analysis results.

3.1 Triads analysis

Let suppose that we have four objects which are pairwise comparison, i.e., A, B, C, and D. Based on this pairwise comparison, and we obtain the following judgment matrix (10):

$$MEJ = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{pmatrix} \end{array} \tag{10}$$

In this case, an expert needs answering to six questions on preferences of the following pairs: (A, B), (A, C), (A, D), (B, C), (B, D), and (C, D). Triad is called a collection consisting of three objects. For this example, we can listed four triads: (A, B, C), (A, B, D), (A, C, D), and (B, C, D). In general, the number of all possible triads (T) from the $t - element$ set can be determined from the formula (11):

$$T = \frac{t!}{(t-3)!3!} \tag{11}$$

Assuming that each characteristic objects have a certain unknown evaluation (constant over time), the expert's preferences must be a transitive relation. If we take the triad (A, B, C) then we can formulate seven rules of transitivity (12):

$$\begin{array}{llllllll} if & A > B & and & B > C & then & A > C \\ if & A > B & and & B = C & then & A > C \\ if & A = B & and & B > C & then & A > C \\ if & A = B & and & B < C & then & A < C \\ if & A < B & and & B < C & then & A < C \\ if & A < B & and & B = C & then & A < C \\ if & A = B & and & B = C & then & A = C \end{array} \tag{12}$$

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Equation (11) presents the relationship between the number of characteristic objects (t) and the number of all possible triads (T). The number of all possible triads is much higher than the number of all upper triangular matrix elements. However, equation (12) presents only seven rules, and we have 27 possible. The term that another 20 rules mean inconsistent triads is not right. Therefore, all 27 rules will be analysed in the next subsection concerning the MEJ matrix.

3.2 Consistency coefficient

Based on (10) and (12), we are determined a set of consistent triads (CO_i, CO_j, CO_k) for which one of the seven conditions is met (13). The number of all consistent triads is written as T_{con}.

$$\begin{array}{llllllll} if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 1.0 \\ if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 1.0 \\ if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 1.0 \\ if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.0 \\ if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.0 \\ if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.0 \\ if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.5 \end{array} \tag{13}$$

More interesting are the triads, for which it is impossible to determine whether their relationship is logically consistent. At the same time, their inconsistency cannot be demonstrated. Let us assume that for 3 objects CO_i, CO_j and CO_k we know their preference values as $f_{CO_i} = 0.67$, $f_{CO_j} = 0.47$ $f_{CO_k} = 0.52$. Therefore, we get $\alpha_{ij} = 1$ $(0.67 > 0.47)$, $\alpha_{jk} = 0$ $(0.47 < 0.52)$ and $\alpha_{ik} = 1$ $(0.67 > 0.62)$. For these triads, a binding conclusion cannot be established. Therefore, these triads will be referred to as unknown. It is worth noting that they cannot influence the decrease of the matrix's consistency because, as the example above shows, they may result from real expert knowledge. The number of all unknown triads will be written as T_{unk}, and each unknown triad must be satisfied one of the following rules (14):

$$\begin{array}{llllllll} if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.0 \\ if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.5 \\ if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 1.0 \\ if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.0 \\ if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.5 \\ if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 1.0 \end{array} \tag{14}$$

The next group of triads is inconsistent triads, which we can divide into two subgroups: weak inconsistent and strong inconsistent triads. One more again, let us assume that for 3 objects CO_i, CO_j and CO_k we know their preference

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values as $f_{CO_i} = 0.67$, $f_{CO_j} = 0.66$ $f_{CO_k} = 0.65$. Then $\alpha_{ij} = 1$, $\alpha_{jk} = 1$ and $\alpha_{ik} = 1$. Let suppose that the expert gives the answer that $\alpha_{ik} = 0.5$. This answer is inconsistent, but if the expert answers that $\alpha_{ik} = 0$ it will be a bigger mistake. Both situations describe inconsistent triads. The weak inconsistent, we can describe as the following rules (15):

$$\begin{array}{llllll}
if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.5 \\
if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.5 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.5 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 1.0 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.0 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.5 \\
if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.5 \\
if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 0.5
\end{array} \tag{15}$$

The number of all weak inconsistent triads is called T_{inc}^{weak} (16). Finally, the last group is the strong inconsistent triads, which can be identify by using the following rules (16):

$$\begin{array}{llllll}
if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.0 \\
if & \alpha_{ij} = 1.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 0.0 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 1.0 & then & \alpha_{ik} = 0.0 \\
if & \alpha_{ij} = 0.5 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 1.0 \\
if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.5 & then & \alpha_{ik} = 1.0 \\
if & \alpha_{ij} = 0.0 & and & \alpha_{jk} = 0.0 & then & \alpha_{ik} = 1.0
\end{array} \tag{16}$$

The number of all strong inconsistent triads is denoted as T_{inc}^{strong}. Why are we showing two groups of inconsistent triads? It is more likely for very similar assessment values that an error will be classified as weak, inconsistent triads than as strong inconsistent triads. In this work, both groups will be represented as (17):

$$T_{inc} = T_{inc}^{weak} + T_{inc}^{strong} \tag{17}$$

Figure 1 shows triads percentage distributions for random the MEJ matrix, where consistent, unknown, weak inconsistent, and strong inconsistent triads are analyzed. For each of the six cases the MEJ matrix was drawn 10,000 times and then the distributions were determined. The draw was conducted with a uniform probability.

For cases (a) to (f), it can be said with 99% probability that at random selection we keep $[0.2669, 0.2900]$ consistent triads; $[0.1390, 0.1574]$ unknown triads; $[0.3879, 0.4132]$ weak inconsistent triads; and $[0.1631, 0.1827]$ strong inconsistent triads. In general, we can said that randomly obtained matrix has $[0.5510, 0.5959]$ inconsistent triads from the whole number of triads.

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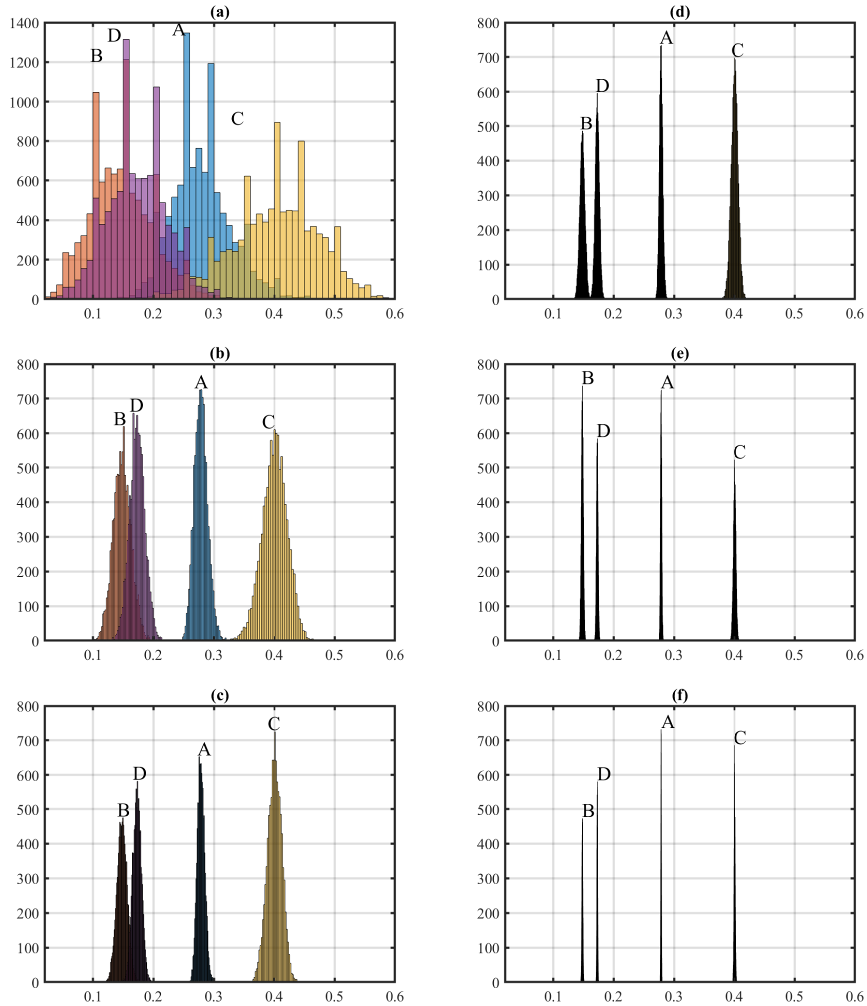

Fig. 1: Triads percentage distributions for 10000 randomly generated samples, where: A logically coherent triads; B triads unknown; C triads are slightly incoherent; D triads are strongly incoherent; (a) for t=10; (b) for t=30; (c) for t=50; (d) for t=100; (e) for t=250; (f) for 500.

Finally, we call ξ the coefficient of consistence for the MEJ matrix, and it can be obtained as (18):

$$\xi = 1 - \frac{T_{inc}}{T} \tag{18}$$

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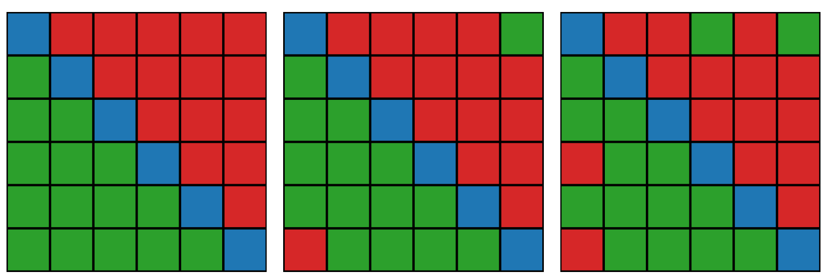
4 Consistency coefficient - study cases

Let's analyze a simple experiment for six characteristic objects. First, we will generate a MEJ matrix for elements in the following preference relation:

$$P_{CO_1} < P_{CO_2} < P_{CO_3} < P_{CO_4} < P_{CO_5} < P_{CO_6} \tag{19}$$

where P_{CO_i} means the preference for CO_i. In that way, we obtained the matrix, which is visualised in Fig. 2a. For this matrix, we get consistency coefficient $\xi = 1.0000$ because all 20 triads are consistent. Let us turn the value into a α_{16} cell from 0 to 1 as it shows in Fig. 2b. As a result, the consistency coefficient will decrease to $\xi = 0.8000$ because we obtain 16 consistent triads and four triads are strongly inconsistent. One more again, let us turn the value into a α_{14} cell from 0 to 1. We get a matrix which is presented in Fig. 2c. As a result, the consistency coefficient decrease to $\xi = 0.7500$, where we have 13 consistent triads and two triads are unknown, and five triads are strongly inconsistent. Each matrix in Fig. 2a-2c provides an order of characteristic objects. The Spearman correlation between reference matrix in Fig. 2a and analyzed matrices Fig. 2b and Fig. 2c is respectively $\rho = 0.9$ and $\rho = 0.8$.



(a) (b) (c)

Fig. 2: The MEJ matrices visualization for exemplary model with six characteristic objects, green 1.0, blue 0.5 and red 0.0 points.

The second experiment consists of drawing a group of 10 characteristic objects and then calculating several values of the consistency coefficient in response to changes in the original matrix. Table 1 presents the random selected ten characteristic objects with their preference values and an obtained ranking, where one is the highest rank, and ten is the lowest. The MEJ matrix in Fig. 3a was generated based on data from Table 1.

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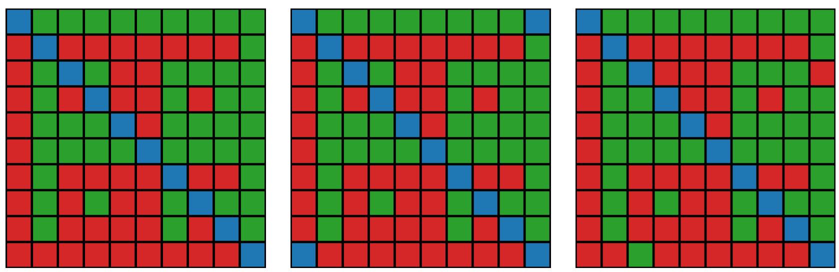
Table 1: Random ten characteristic objects (CO_i) with their preference values (P_{CO_i}) and a determined ranking rank(CO_i)

CO_i	P_{CO_i}	rank(CO_i)
CO_1	0.91	1
CO_2	0.22	9
CO_3	0.71	4
CO_4	0.55	6
CO_5	0.72	3
CO_6	0.82	2
CO_7	0.37	8
CO_8	0.63	5
CO_9	0.46	7
CO_{10}	0.10	10



(a) (b) (c)

Fig. 3: The MEJ matrices visualization for exemplary model with ten characteristic objects, green 1.0, blue 0.5 and red 0.0 points.

For the matrix in Fig. 3a, the consistency coefficients is $\xi = 1.0000$. More interesting is that only 49 triads were consistent triads, and the rest were unknown triads. This matrix was indeed generated from the data. Thus, despite a large number of unknown triads, it is with all certainty a logically consistent matrix what was also shown by the ξ coefficient. Subsequently, a change was made to the matrix in the first row and the last column on 0.5, see Figure 3b. In this case, we obtain $\xi = 0.9333$ with 41 consistent, 71 unknown, and eight weak inconsistent triads. The order of characteristic objects is not changed ($\rho = 1$). Now let us go back to the reference matrix in Fig. 3a and make another change. The two elements have been changed, i.e., $\alpha_{34} = 0$ and $\alpha_{310} = 0$. The new matrix is presented in Fig. 3c.

For matrix on Fig. 3c, we obtain $\xi = 0.9583$ with 44 consistent, 71 unknown, and 5 strong inconsistent triads. The order of characteristic objects is change. Now, the Spearman correlation between reference matrix in Fig. 3a and modified in Fig. 3c is equal $\rho = 0.9573$. Despite the high consistency coefficient, the final

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result is worse than in the matrix in Fig. 3b. A high degree of consistency in the expert response is a prerequisite for good results, but it is insufficient. In real examples, we rely on the expert's knowledge because data is not available. Therefore, the expert's answers' consistency is crucial, but only if the expert's knowledge is sufficient and up-to-date.

The last example will come from a previous work where the identified model showed good computational properties. The decision matrix for 27 characteristic objects took the following form Fig. 4.

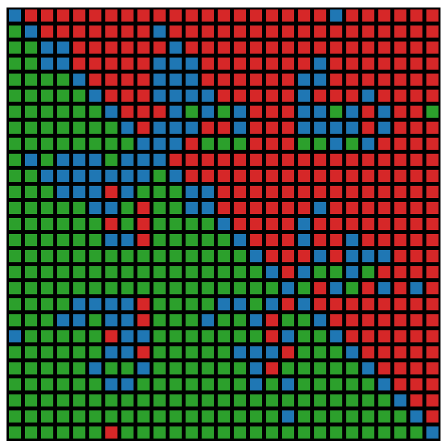

Fig. 4: MEJ matrix for example model, where green 1.0, blue 0.5 and red 0.0 points.

The designated consistency coefficient was $\xi = 0.8711$ with 2277 consistent, 271 unknown, 309 weak inconsistent, and 68 strong inconsistent triads. The coefficient developing is an important step to improve MCDA method performance (in this case, the COMET method). With a series of often similar questions, it is easy to pick a wrong answer, and therefore our task is to identify how consistent the expert-created matrix is. Now with a new coefficient, it is feasible.

5 Conclusions

This study proposes a new consistency coefficient. The proposed ξ coefficient is based on triads analysis. In the study, four groups of 27 presented rules were distinguished, i.e., consistent, unknown, weak inconsistent, and strong inconsistent triads. Triads percentage distributions for 10,000 randomly generated samples were used to determine the confidence interval for which the consistency range of random MEJ matrices was determined with 99% probability.

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Three case studies were presented to show how this coefficient works. The consistency is a necessary but not sufficient condition. If we have a strongly consistent matrix created by an inadequate expert, the model identified will still be inadequate. Therefore, the proposed coefficient could not determine the final results' quality, but only the MEJ matrix's consistency. Attempts should now be made to introduce it actively into the COMET procedure and other methods where a three values scale of pairwise comparison matrix is used.

Future directions of research are primarily:

- computer simulations to establish the minimum acceptable level of consistency for further calculation procedure;
- based on erroneously determined triads, the possibility of repairing the matrix by asking the expert again;
- considering the options of improving the proposed formula taking into account the differences between weak and strong inconsistencies;
- considering extended the proposed approach for the incomplete pairwise comparisons matrices [27].

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