Out-plant milk-run-driven mission planning subject to dynamic changes of date and place delivery

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Abstract. We consider a dynamic vehicle routing problem in which a fleet of vehicles delivers ordered services or goods to spatially distributed customers while moving along separate milk-run routes over a given periodically repeating time horizon. Customer orders and the feasible time windows for the execution of those orders can be dynamically revealed over time. The problem essentially entails the rerouting of routes determined in the course of their proactive planning. Rerouting takes into account current order changes, while proactive route planning takes into account anticipated (previously assumed) changes in customer orders. Changes to planned orders may apply to both changes in the date of services provided and emerging notifications of additional customers. The considered problem is formulated as a constraint satisfaction problem using the ordered fuzzy number (OFN) formalism, which allows us to handle the fuzzy nature of the variables involved, e.g. the timeliness of the deliveries performed, through an algebraic approach. The computational results show that the proposed solution outperforms the commonly used computer simulation methods.

Keywords: out-plan milk-run system, dynamic planning, delivery uncertainty

1 Introduction

In the paper an out-plant Dynamic Milk-run Routing Problem (DMRP), which consists of designing vehicle routes in an online fashion as orders executed in supply networks are revealed incrementally over time, is considered. In real-life settings, the Out-plant Operating Supply Networks (O^2SNs) [12], apart from randomly occurring disturbances (changes in the execution of already planned requests/orders and the arrival of new ones, traffic jams, accidents, etc.), an important role is played by the imprecise nature of the parameters which determine the timeliness of the services/deliveries performed [18]. This is because the time of carrying out the operations of transport and service delivery depends on both the transport infrastructure and the prevailing weather conditions as well as on human factors. The imprecise nature of these parameters is implied

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due to the operator's psychophysical disposition (e.g. manifested in various levels of stress, distraction, fatigue), disturbances in the flow of traffic, etc. resulting in delays in deliveries and unloading / loading operations. Therefore, the time values of the operations performed vary and are uncertain.

The non-stationary nature of the uncertainty of the parameters mentioned, and the usually small set of available historical samples in practice, limits the choice of a formal data model to a fuzzy-numbers-driven one. It means the uncertainty of O^2SN data connected with traffic disturbances as well as the changes in service delivery dates require the use of a model based on the formalism of fuzzy sets. However, it is worth noting that the specificity of the process involved in the course of the services' delivery schedule planning results in the need to determine the sequentially cumulative uncertainty in the performance of the operations involved in it. The question that arises concerns the method for avoiding additional uncertainty introduced in the combinations of summing up uncertainties of cyclically executed operations, e.g. in cyclic production [3] or distribution [2]. In this context, in contrast to standard fuzzy numbers, the support of a fuzzy number obtained by algebraic operations performed on the Ordered Fuzzy Numbers (OFNs) domain does not expand. In turn, however, the possibility of carrying out algebraic operations is limited to selected domains of the computability of these supports. This is a reason why this contribution focuses on the development of sufficient conditions implying the calculability of arithmetic operations that guarantee the interpretability of the results obtained. Consequently, the objective is to develop an algebraic model aimed at fast calculation of fuzzy schedules for vehicles as well as for planning of time buffers enabling the adjustment of currently fuzzy schedules to baseline schedules assuming the deterministic nature of operation times (i.e. their crisp values).

The present study is a continuation of our previous work [1, 2, 3, 20] on methods for fast online prototyping of supply schedules and transport routes of a tugger train fleet making adjustments for the tradeoff points between fleet size and storage capacity [21] as well as problems regarding the planning and control of production flow in departments of automotive companies [3]. Its main contribution is threefold:

- An OFN algebra allows the possibility to plan vehicle fleet scenarios aimed at requests while taking into account the uncertainty of the deliveries' operation times.
- In contrast to standard fuzzy numbers, the support of a fuzzy number does not expand as a result of algebraic operations performed on the OFN domain.
- The objective is to maximize the number of new transportation requests, which are inserted dynamically throughout the assumed time horizon.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 provides the OFNs' framework. Section 4 contains the problem formulation supported by illustrative examples. The model, the methodology used in solving the problem and the conclusions are provided in Sections 5, 6, and 7, respectively.

2 Literature Review

Most of the problems appearing in O^2SNs are aimed at searching for an optimal distribution policy, i.e., a plan of whom to serve, how much to deliver, and what routes to travel by what fleet of vehicles. Examples of such problems [4, 21] include both simple

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ones, such as the Mix Fleet VRP, Multi-depot VRP, Split-up Delivery VRP, Pick-up and Delivery VRP, VRP with Time Windows, and more complex ones. Many works are devoted to the Periodic Vehicle Routing Problems (PVRP) aimed at searching for an optimal periodic distribution policy providing a set of routes assigning customers to vehicles that minimize the total travel cost while satisfying vehicle capacity and the time periods when customers should be visited [9]. In turn, the Multi-Depot PVRP with Due Dates and Time Windows [6] being its extension while determining regularly repeated routes to travel by each vehicle in order to satisfy the customer demands can be seen as a kind of the Milk-run Vehicle Routing problem with Time Windows [11]. Since milk-run routing and scheduling problems are NP-hard VRPs, they are solved by using heuristic methods [10,16]. Regardless of problems typical for in-plant or out-plant milk-run systems [12] or problems that accentuate the dynamic or static character of vehicle routing [10, 15], the goal is to search for optimal solutions.

The Dynamic VRPs that arise when new customers appear in the tours after the starting visit are among the more important and more challenging extensions of VRP [8,19]. Solving such problems comes down to setting proactive and/or reactive routing strategies [14]. Proactive routing strategies are based on a certain knowledge about demand and are used to anticipate a possible order from a new customer. In turn, reactive routing strategies try to reschedule used fleets due to the occurrence of a new order when a new one arrives. In focusing on the search for such solutions, it is usually assumed that planned routings and schedules are robust to assumed disturbances [15, 16].

It is worth noting that relatively few studies are devoted to the problems of out-plant milk-run dynamic routing and systems in which services are provided by appointment. In systems of this type, the dynamic multi-period VRP is solved, which involves scheduling services in a rolling horizon fashion, in which new requests received, but unfulfilled, during the first period together with the set of customer requests preplanned for the next period constitute a new portfolio of orders [7]. An exhaustive review of VRP taxonomy for milk-run systems can be found in [4].

In many real-life situations, due to the uncertainty of DMRP data caused by traffic disturbances (uncertain travel times, daily changes in traffic intensity, etc.) the service provided cannot be estimated in a precise way. However, the majority of models of the so-called Fuzzy VRP only assume vagueness for fuzzy customer demands to be collected and fuzzy travel times. The literature on these issues is very scarce [5], despite the rapidly growing demand for predictive oriented service providers. The quickly developing enterprise servitization indicates a growing need for this type of service [13].

To summarize, the review shows a gap in the literature in terms of analytical approaches for the assessment of possible rerouting and rescheduling scenarios. The current paper aims to fill this gap by providing the method combining the declarative modeling paradigm with OFN algebra.

3 An Ordered Fuzzy Numbers framework

The milk-run routing and scheduling problems developed so far have limited use due to the data uncertainty observed in practice. The values describing parameters such as

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transport time or loading/unloading times depend on the human factor, which means they cannot be determined precisely. It is difficult to account for data uncertainty by using fuzzy variables due to the imperfections of the classic fuzzy numbers algebra [2]. Equations which describe the relationships between fuzzy variables using algebraic operations do not meet the conditions of the Ring. This means that no matter what algebraic operations are used, the support of the fuzzy number, which is the result of these operations, expands. An example of imperfections of the classic fuzzy numbers algebra is shown in Fig. 1, where the uncertainty increases with successive cycles.

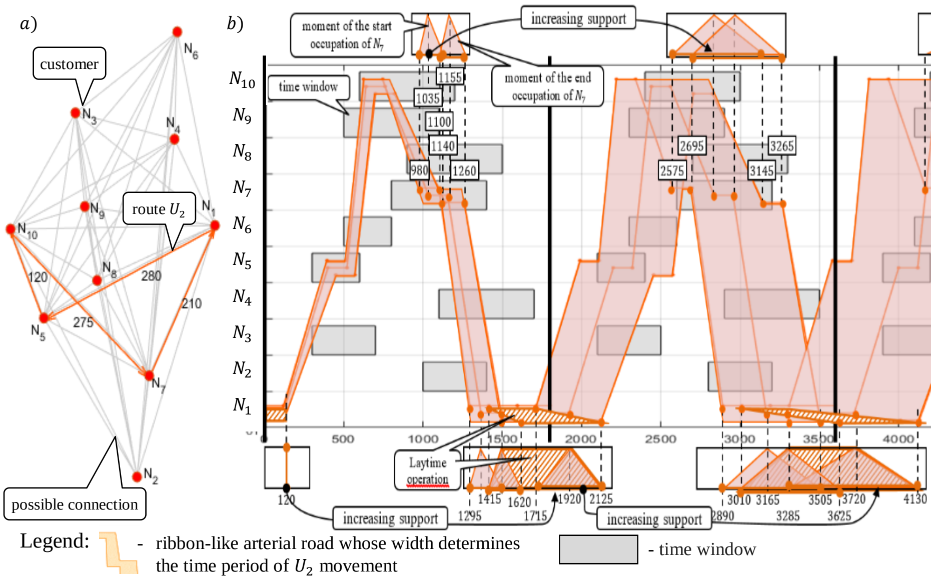

Fig. 1. Route connecting customers N_1, N_5, N_{10}, N_7 served by deliveries a), fuzzy schedule b)

Fig. 1 distinguishes the fuzzy values of the start/end moments of service operations carried out on nodes N_7 and N_1 located along the route selected for U_2 (orange line). The attainable values of these moments are characterized by an increasing support in subsequent cycles (the level of uncertainty increases).

In the case of classic fuzzy numbers $\hat{a}$, $\hat{b}$, $\hat{c}$ (marked with the symbol $\hat{}$), it is impossible to solve a simple equation $\hat{A} + \hat{X} = \hat{C}$. This fact significantly hinders the application of approaches based on declarative models. Therefore, we propose a model based on OFN algebra in which it is possible to solve algebraic equations. OFNs can be defined [17] as a pair of continuous real functions (f_A – "up"; g_A- "down") i.e.:

$$\hat{A} = (f_A, g_A), \text{ where: } f_A, g_A \colon [0,1] \to \mathbb{R}. \tag{1}$$

Assuming that f_A is increasing and g_A is decreasing as well as that $f_A \leq g_A$, the membership function μ_A of the OFN $\hat{A}$ is as follows (see OFN rows in Table 1):

$$\mu_A(x) = \begin{cases} f_A^{-1}(x) & when\ x \in UP_A \\ g_A^{-1}(x) & when\ x \in DOWN_A \\ 1 & when\ x \in CONST_A \\ 0 & \text{in the remaining cases} \end{cases} \tag{2}$$

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where, $UP_A = (l_{A0}, l_{A1})$, $CONST_A = (l_{A1}, p_{A1})$ and $DOWN_A = (p_{A1}, p_{A0})$. OFNs are two types of orientation [17]: **positive**, when $\hat{A} = (f_A, g_A)$; **negative**, when $\hat{A} = (g_A, f_A)$. The algebraic operations used in the proposed model are as follows:

Definition 1. Let $\hat{A} = (f_A, g_A)$ and $\hat{B} = (f_B, g_B)$ be OFNs. $\hat{A}$ is a number equal to $\hat{B}$ ($\hat{A} = \hat{B}$), $\hat{A}$ is a number greater than $\hat{B}$ or equal to or greater than $\hat{B}$ ($\hat{A} > \hat{B}$; $\hat{A} \geq \hat{B}$), $\hat{A}$ is less than $\hat{B}$ or equal to or less than $\hat{B}$ ($\hat{A} < \hat{B}$, $\hat{A} \leq \hat{B}$) if: $\forall_{x\in[0,1]}\ f_A(x) * f_B(x)\ \wedge\ g_A(x) * g_B(x)$, where: the symbol $*$ stands for: =, >, ≥, <, or ≤. ■

Definition 2. Let $\hat{A} = (f_A, g_A)$, $\hat{B} = (f_B, g_B)$, and $\hat{C} = (f_C, g_C)$ be OFNs. The operations of addition $\hat{C} = \hat{A} + \hat{B}$, subtraction $\hat{C} = \hat{A} - \hat{B}$, multiplication $\hat{C} = \hat{A} \times \hat{B}$ and division $\hat{C} = \hat{A}/\hat{B}$ are defined as follows: $\forall_{x\in[0,1]}\ f_C(x) = f_A(x) * f_B(x)\ \wedge\ g_C(x) = g_A(x) * g_B(x)$, where: the symbol $*$ stands for +, −, ×, or ÷; The operation of division is defined for $\hat{B}$ such that $|f_B| > 0$ and $|g_B| > 0$ for $x \in [0, 1]$. ■

The ordered fuzzy number $\hat{A}$ is a proper OFN [QW] when one of the following conditions is met: $f_A(0) \leq f_A(1) \leq g_A(1) \leq g_A(0)$ (for positive orientation) or $g_A(0) \leq g_A(1) \leq f_A(1) \leq f_A(0)$ (for negative orientation). They allow us to specify the conditions which guarantee that the result of algebraic operations is a proper OFN [2]:

Theorem 1. Let $\hat{A}$ and $\hat{B}$ be proper OFNs with different orientations: $\hat{A}$ (positive orientation), $\hat{B}$ (negative orientation). If one of the following conditions holds:

- $(|UP_A| - |UP_B| \geq 0) \wedge (|CONST_A| - |CONST_B| \geq 0) \wedge (|DOWN_A| - |DOWN_B| \geq 0)$,
- $(|UP_B| - |UP_A| \geq 0) \wedge (|CONST_B| - |CONST_A| \geq 0) \wedge (|DOWN_B| - |DOWN_A| \geq 0)$,

then the result of the operation $\hat{A} + \hat{B}$ is a proper OFN $\hat{C}$.

where: UP_X – an image (codomain) of function f_X, $CONST_X = \{x \in X: \mu_X = 1\}$, $DOWN_X$ – an image of function g_X, $|a|$ – length of the interval a. ■

The fulfillment of the conditions underlying the above theorem may lead to a reduction in the fuzziness of the sum of OFNs with different orientations. This is because algebraic operations (in particular sums) take values which are proper OFNs , i.e. are fuzzy numbers which are easy to interpret.

4 Problem formulation

Let us consider the graph $G = (N, E)$ modeling an O[2]SN. The set of nodes $N = \{N_1, \dots, N_\lambda, \dots, N_n\}$, (where $n = |N|$) includes one node representing distribution center N_1 and $\{N_2, \dots, N_n\}$ nodes representing customers. The set of edges $E = \{(N_i, N_j)|\ i, j \in \{1, \dots, n\}, i \neq j\}$ determines the possible connections between nodes. Given is a fleet of vehicles $\mathcal{U} = \{U_1, \dots, U_k, \dots, U_K\}$. The customers $\{N_2, \dots, N_n\}$ are cyclically visited (with period T) by vehicles U_k traveling from node N_1. Variable Q_k denotes the payload capacity of vehicles U_k. Execution of the ordered delivery z_λ by the customer N_λ takes place in the fuzzy period $\widehat{t_\lambda}$ (represented by an OFN). The moment when the vehicle U_k starts delivery to the customer N_λ is indicated by fuzzy variable $\widehat{y_\lambda^k}$ (represented by an OFN). The deliveries ordered by the customer N_λ are carried out in the delivery time interval (the time window for short) $TW_\lambda = [ld_\lambda; ud_\lambda]$, i.e. $\widehat{y_\lambda^k} \geq ld_\lambda$ and $\widehat{y_\lambda^k} + \widehat{t_\lambda} \leq ud_\lambda$. It is assumed that the fuzzy variable $\widehat{d_{\beta,\lambda}}$ (taking the form of an OFN) determines traveling time between nodes N_β, N_λ, where: $(N_\beta, N_\lambda) \in E$. The

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routes of U_k are represented by sequences: $\pi_k = \left(N_{k_1}, \dots, N_{k_i}, N_{k_{i+1}}, \dots, N_{k_\mu}\right)$, where: $k_i \in \{1,\dots,n\}$, $\left(N_{k_i}, N_{k_{i+1}}\right) \in E$. Moreover, the following assumptions are met: Z denotes a sequence of required amounts of goods z_λ $(\lambda = 1, \dots, n)$; Π denotes a set of routes π_k, $k = 1, \dots, K$; node N_1 representing the distribution center occurs only once in each route of the set Π; node representing the customer N_λ $(\lambda > 1)$ occurs only once in the route belonging to the set Π; the amount of goods transported by U_k cannot exceed payload capacity Q_k, deliveries are being made over a given periodically repeating time horizon with period T.

In that context, typical proactive planning of goods distribution fundamentally involves the question: *Given a fleet $\mathcal{U}$ providing deliveries to the customers allocated in a network G (ordering assumed amounts of goods z_λ). Does there exist the set of routes Π guaranteeing timely execution of the ordered services following time windows TW_λ?*

For the purpose of illustration, let us consider network G shown in Fig. 2a), where 10 nodes (1 distribution center and 9 customers) are serviced by fleet $\mathcal{U} = \{U_1, U_2, U_3\}$.

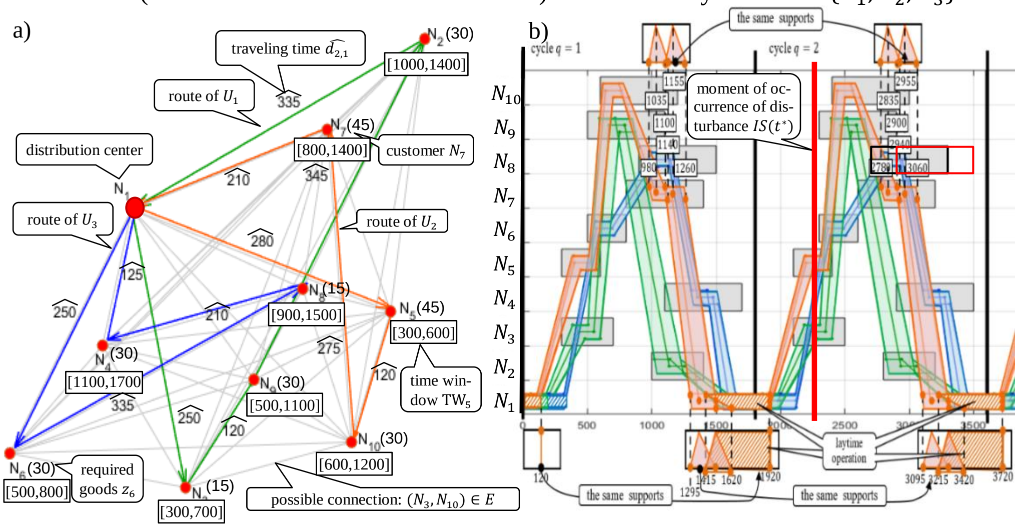

Fig. 2. Graph G modeling the considered O^{2}SN a) and corresponding fuzzy cyclic schedule b)

Table 1. Assumed traveling time values

$\widehat{d_{1,3}}$	$\widehat{d_{3,9}}$	$\widehat{d_{9,2}}$	$\widehat{d_{2,1}}$	$\widehat{w_1^1}$	$\widehat{d_{1,5}}$	$\widehat{d_{5,10}}$	$\widehat{d_{10,7}}$	$\widehat{w_1^2}$	$\widehat{d_{1,6}}$	$\widehat{d_{6,8}}$	$\widehat{d_{8,4}}$	$\widehat{d_{4,1}}$	$\widehat{w_1^3}$
				115 270 435				300 435 505					330 400 465

The following routes: $\pi_1 = (N_1, N_3, N_9, N_2)$ (green line), $\pi_2 = (N_1, N_5, N_{10}, N_7)$ (orange line), $\pi_3 = (N_1, N_6, N_8, N_4)$ (blue line) guarantee the delivery of the required services related to the fulfillment of the ordered amount of goods to all customers cyclically (within the period T = 1800). The solution was determined assuming that the vehicle payload capacity Q_k is equal to 120 and required amounts of goods are equal $Z = (0, 30, 15, 30, 45,\ 30, 45, 15, 30, 30)$. The corresponding fuzzy cyclic schedules are shown in Fig. 2b). This solution assumes that travelling times $\widehat{d_{\beta,\lambda}}$ (in Table 1) are

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represented by an OFNs and times of node occupation $\widehat{t_\lambda}$ are singletons ($\widehat{t_\lambda} = 120, \lambda = 1, \dots, 10$). The implemented routes are determined in the process of proactive planning. However, other disruptions may occur in the process of implementing such planned routes. An example of such a disturbance IS concerns changes of delivery time windows or new order notification from a customer located outside of a given route.

Such a disturbance is presented in Fig. 2b) where the dispatcher receives information on $TW_8^* = [2900; 3500]$ changing the delivery time window being located at the node N_8 (from $TW_8 = [2700; 3300]$ to) – see the second ($q = 2$) cycle of schedule (moment $t^* = 2300$ when U_1 occupies N_3, U_2 occupies N_5 and U_3 is moving to N_7). Due to this change, the adopted routes do not guarantee the implementation of maintenance services on the set dates – the handling of N_8 according to the new $TW_8^* = [2900; 3500]$ prevents the timely handling of the customer N_8 and vice versa. In such a situation, it becomes necessary to answer to the following question:

Given a vehicle fleet $\mathcal{U}$ providing deliveries to the customers allocated in a network G. Vehicles move along a given set of routes Π according to a cyclic fuzzy schedule $\widehat{\mathbb{Y}}$. Given is a disturbance $IS(t^*)$ related either to changing from TW_λ to TW_λ^* or occurrence of a new order from the customer located in the place N_λ outside of a given route. Does there exist a rerouting ${}^*\Pi$ and rescheduling $\widehat{{}^*\mathbb{Y}}$ of vehicles, which guarantee timely execution of the ordered amounts of goods but not at the expense of the already accepted orders?

The possibility of reactive (dynamic) planning of vehicle missions in the event of a disturbance occurrence is the subject of the following chapters.

5 OFN-Based Constraint Satisfaction Problem

In general the problem under consideration can be formulated in the following way:

Given a fleet $\mathcal{U}$ providing deliveries to the customers allocated in a network G (customers are serviced by prescheduled time windows TW*). Vehicles move along a given set of routes Π according to the cyclic fuzzy schedule $\widehat{\mathbb{Y}}$. Assuming the appearance of the disturbance $IS(t^*)$ (which changes TW to TW^* and/or location of customer N_λ to N_λ^* at the moment t^*), a feasible way of rerouting (${}^*\Pi$) and rescheduling ($\widehat{{}^*\mathbb{Y}}$) of MSTs, guaranteeing timely execution of the ordered services, is sought.*

Parameters:

G: graph of a transportation network $G = (N, E)$,

$\mathcal{U}$: set of vehicles: $\mathcal{U} = \{U_1, \dots, U_k, \dots, U_K\}$, U_k is the *k*-th vehicle,

K: size of vehicle fleet,

TW: set of delivery time windows: $TW = \{TW_1, \dots, TW_\lambda, \dots, TW_n\}$, where $TW_\lambda = [ld_\lambda; ud_\lambda]$ is a deadline for service at the customer N_λ (see example in Fig. 2),

$IS(t)$: state of vehicle fleet mission at the moment t: $IS(t) = (M(t), {}^*TW(t), {}^*E(t))$ where:

$M(t)$ is an allocation of vehicles at the moment t: $M(t) = (N_{a_1}, \dots, N_{a_k}, \dots, N_{a_K})$, where $a_k \in \{1, \dots, n\}$ determines the node N_{a_k} occupied by U_k (or the node the U_k is headed to).

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$^{*}TW(t)$ is the set of time windows TW_{λ}^{*} at the moment t: $^{*}TW(t) = \{TW_1^{*}, \ldots, TW_{\lambda}^{*}, \ldots, TW_n^{*}\}$, where $TW_{\lambda}^{*} = [ld_{\lambda}^{*}; ud_{\lambda}^{*}]$.

$^{*}E$(t) is a set of edges (with traveling times $\widehat{d_{\beta,\lambda}}$) of network G (customer location) at the moment t.

The state $IS(t^{*})$ such that the following condition $[^{*}TW(t^{*}) \neq TW] \vee [^{*}E(t^{*}) \neq E]$ holds is called the **disturbance** occurring at the moment t^{*}.

T : period in which all customers should be serviced (see Fig. 2b) – $T = 1800$),

Π: set of routes π_k for the network G, when there is no disturbance, where π_k is a route of U_k:

$\pi_k = \left(N_{k_1}, \ldots, N_{k_i}, N_{k_{i+1}}, \ldots, N_{k_\mu}\right)$, where: $x_{k_i,k_{i+1}}^{k} = 1$ for $i = 1, \ldots, \mu - 1$

$$x_{\beta,\lambda}^{k} = \begin{cases} 1 & \text{if } U_k \text{ travels from node } N_\beta \text{ to node } N_\lambda \\ 0 & \text{otherwise} \end{cases},$$

z_λ: customer's N_λ demand,

U_k: maximum loading capacity of U_k,

c_λ^k: weight of goods delivered to N_λ by U_k, when there is no disturbance,

$\widehat{d_{\beta,\lambda}}$: fuzzy traveling time along the edge (N_β, N_λ) – defined as positive OFNs,

$\widehat{t_\lambda}$: fuzzy time of node N_λ occupation (represented by an OFN),

$\widehat{\mathbb{Y}}$: fuzzy schedule of fleet $\mathcal{U}$, $\widehat{\mathbb{Y}} = (\widehat{Y}, \widehat{W}, C)$ when there is no disturbance:

- $\widehat{Y}$: set of $\widehat{Y^k}$, where $\widehat{Y^k}$ is a sequence of moments $\widehat{y_\lambda^k}$: $\widehat{Y^k} = \left(\widehat{y_1^k}, \ldots, \widehat{y_\lambda^k}, \ldots, \widehat{y_n^k}\right)$, $\widehat{y_\lambda^k}$ is fuzzy time at which U_k arrives at node N_λ,
- $\widehat{W}$: set of $\widehat{W^k}$, where $\widehat{W^k}$ is a sequence of laytimes $\widehat{w_\lambda^k}$: $\widehat{W^k} = \left(\widehat{w_1^k}, \ldots, \widehat{w_\lambda^k}, \ldots, \widehat{w_n^k}\right)$, $\widehat{w_\lambda^k}$ is laytime at node N_λ for U_k.
- C: set of C^k, where C^k is a sequence of delivered goods c_λ^k: $C^k = (c_1^k, \ldots, c_\lambda^k, \ldots, c_n^k)$, c_λ^k is weight of goods delivered to node N_λ by U_k.

Variables:

$^{*}x_{\beta,\lambda}^{k}$: binary variable indicating the travel of U_k between nodes N_β, N_λ after occurrence of the disturbance $IS(t^{*})$:

$$^{*}x_{\beta,\lambda}^{k} = \begin{cases} 1 & \text{if } U_k \text{ travels from node } N_\beta \text{ to node } N_\lambda \\ 0 & \text{otherwise} \end{cases},$$

$^{*}c_\lambda^k$: weight of goods delivered to node N_λ by U_k, after occurrence of $IS(t^{*})$,

$^{*}\widehat{y_\lambda^k}$: fuzzy time at which U_k arrives at node N_λ, after occurrence of $IS(t^{*})$,

$^{*}\widehat{w_\lambda^k}$: laytime at node N_λ for U_k, after occurrence of the disturbance $IS(t^{*})$,

$^{*}\widehat{s^k}$: take-off time of U_k.

$^{*}\pi_k$: route of U_k, after occurrence of the disturbance $IS(t^{*})$,: $^{*}\pi_k = \left(N_{k_1}, \ldots, N_{k_i}, N_{k_{i+1}}, \ldots, N_{k_\mu}\right)$, where: $^{*}x_{k_i,k_{i+1}}^{k} = 1$ for $i = 1, \ldots, \mu - 1$ and $^{*}x_{k_\mu,k_1}^{k} = 1$; $^{*}\Pi$ is a set of routes $^{*}\pi_k$.

$^{*}C^k$: set of $^{*}c_\lambda^k$, payload weight delivered by U_k; $^{*}C$ is family of $^{*}C^k$

$^{*}\widehat{W^k}$: sequence of laytimes $^{*}\widehat{w_\lambda^k}$: $\widehat{W^k} = \left(^{*}\widehat{w_1^k}, \ldots, ^{*}\widehat{w_n^k}\right)$; $^{*}\widehat{W}$ is a set of $^{*}\widehat{W^k}$,

$^{*}\widehat{Y^k}$: sequence of moments $^{*}\widehat{y_\lambda^k}$: $^{*}\widehat{Y^k} = \left(^{*}\widehat{y_1^k}, \ldots, ^{*}\widehat{y_n^k}\right)$; $^{*}\widehat{Y}$ is a set of $^{*}\widehat{Y^k}$,

$^{*}\widehat{\mathbb{Y}}$: fuzzy schedule of fleet $\mathcal{U}$, after occurrence of $IS(t^{*})$: $^{*}\widehat{\mathbb{Y}} = (^{*}\widehat{Y}, ^{*}\widehat{W}, ^{*}C)$.

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Constraints:

$$\widehat{{}^*s^k} \geq 0\,;\; k = 1 \dots K \tag{3}$$

$$\left(\widehat{s^k} \leq t^*\right) \Rightarrow \left(\widehat{{}^*s^k} = \widehat{s^k}\right);\; k = 1 \dots K \tag{4}$$

$$\left(\widehat{y_j^k} \leq t^*\right) \Rightarrow \left({}^*x_{i,j}^k = x_{i,j}^k\right);\; j = 1 \dots n;\; i = 2 \dots n;\; k = 1 \dots K \tag{5}$$

$$\left(\widehat{y_j^k} \leq t^*\right) \Rightarrow \left(\widehat{{}^*y_j^k} = \widehat{y_j^k}\right);\; j = 2 \dots n;\; k = 1 \dots K \tag{6}$$

$$\left(\widehat{y_j^k} \leq t^*\right) \Rightarrow \left(\widehat{{}^*w_j^k} = \widehat{w_j^k}\right);\; j = 2 \dots n;\; k = 1 \dots K \tag{7}$$

$$\textstyle\sum_{j=1}^{n} {}^*x_{1,j}^k = 1\,;\; k = 1 \dots K \tag{8}$$

$$\left({}^*x_{1,j}^k = 1\right) \Rightarrow \left(\widehat{{}^*y_j^k} = \widehat{{}^*s^k} + \widehat{d_{1,j}}\right);\; j = 1 \dots n;\; k = 1 \dots K \tag{9}$$

$$\left({}^*x_{i,j}^k = 1\right) \Rightarrow \left(\widehat{{}^*y_j^k} = \widehat{{}^*y_i^k} + \widehat{d_{i,j}} + \widehat{t_i} + \widehat{{}^*w_i^k}\right);\; j = 1 \dots n;\; i = 2 \dots n;\; k = 1 \dots K \tag{10}$$

$$\left(\widehat{y_i^k} \leq t^*\right) \Rightarrow \left({}^*c_i^k = c_i^k\right);\; i = 1 \dots n;\; k = 1 \dots K \tag{11}$$

$${}^*c_i^k \leq Q \times \textstyle\sum_{j=1}^{n} {}^*x_{i,j}^k;\; i = 1 \dots n;\; k = 1 \dots K \tag{12}$$

$$\left({}^*x_{i,j}^k = 1\right) \Rightarrow {}^*c_j^k \geq 1;\; k = 1 \dots K;\; i = 1 \dots n;\; j = 2 \dots n \tag{13}$$

$$\textstyle\sum_{k=1}^{K} {}^*c_i^k = z_i;\; i = 1 \dots n \tag{14}$$

$$\widehat{{}^*s^k} + T = \widehat{{}^*y_1^k} + \widehat{t_1} + \widehat{{}^*w_1^k};\; k = 1 \dots K \tag{15}$$

$$\widehat{{}^*y_j^k} \geq 0;\; i = 1 \dots n;\; k = 1 \dots K \tag{16}$$

$$\textstyle\sum_{j=1}^{n} {}^*x_{i,j}^k = \sum_{j=1}^{n} {}^*x_{j,i}^k\,;\; i = 1 \dots n;\; k = 1 \dots K \tag{17}$$

$$\widehat{{}^*y_i^k} \leq T, i = 1 \dots n;\; k = 1 \dots K \tag{18}$$

$${}^*x_{i,i}^k = 0;\; i = 1 \dots n;\; k = 1 \dots K \tag{19}$$

$$\widehat{{}^*y_i^k} + \widehat{t_i} + c \times T \;\leq\; ud_\lambda^*, i = 1 \dots n;\; k = 1 \dots K \tag{20}$$

$$\widehat{{}^*y_i^k} + c \times T \;\geq\; ld_\lambda^*, i = 1 \dots n;\; k = 1 \dots K \tag{21}$$

It is assumed that the arithmetic operations contained in the above constraints meet the conditions of Def. 1 and Theorem 1. The rescheduling and rerouting of the vehicles then resulting in a new plan of delivery are the result of the disturbance $IS(t^*)$. In that context, when disturbance $IS(t^*)$ occurs, the new set of routes ${}^*\Pi$ and a new schedule $\widehat{{}^*\mathbb{Y}}$, which guarantee timely servicing of customers, are determined by solving the following Ordered Fuzzy Constraint Satisfaction (OFCS) Problem (22):

$$\widehat{FCS}(\widehat{\mathbb{Y}}, \Pi, IS(t^*)) \;= \left(\left(\widehat{\mathcal{V}}, \widehat{\mathcal{D}}\right), \widehat{\mathcal{C}}(\widehat{\mathbb{Y}}, \Pi, IS(t^*))\right), \tag{22}$$

where:

$\widehat{\mathcal{V}} = \left\{\widehat{{}^*\mathbb{Y}}, {}^*\Pi\right\}$ – a set of decision variables: $\widehat{{}^*\mathbb{Y}}$ – a fuzzy cyclic schedule guaranteeing timely provision of service to customers in the case of disturbance IS, and ${}^*\Pi$ – a set of routes determining the fuzzy schedule $\widehat{{}^*\mathbb{Y}}$;

$\widehat{\mathcal{D}}$ – a finite set of decision variable domains: $\widehat{{}^*y_\lambda^k}, \widehat{{}^*w_\lambda^k} \in \mathcal{F}$ ($\mathcal{F}$ is a set of OFNs (1)), ${}^*x_{\beta,\lambda}^k \in \{0,1\}$, ${}^*c_\lambda^k \in \mathbb{N}$;

$\widehat{\mathcal{C}}$ – a set of constraints which take into account the set of routes Π, fuzzy schedule $\widehat{\mathbb{Y}}$ and disturbance $IS(t^*)$, while determining the relationships that link the operations occurring in vehicles fleet cycles (5)–(21).

To solve $\widehat{FCS}$ (22), the values of the decision variables from the adopted set of domains for which the given constraints are satisfied must be determined.

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6 Dynamic mission planning

The idea standing behind the proposed reaction to occurring disruption $IS(t^*)$ can be reduced to dynamic adaptation (i.e., rerouting and rescheduling) of previously adopted routes Π, and schedules $\widehat{\mathbb{Y}}$, i.e. adjusting them (if possible) to the changes in time windows TW^* or occurrence of a new order from the customer located outside of a given route E^*. Let $\widehat{\mathbb{Y}}(q)$ denote the output fuzzy schedule of the q-th cycle defined as:

$$\widehat{\mathbb{Y}}(q) = \left(\widehat{Y}(q), \widehat{W}(q), C(q)\right) \tag{23}$$

where $\widehat{Y}(q)$, $\widehat{W}(q)$, $C(q)$ are families of following sets ($q = 1,2 \dots, Q$):

$\widehat{Y^k}(q) = \left(\widehat{y_1^k}(q), \dots, \widehat{y_\lambda^k}(q), \dots, \widehat{y_n^k}(q)\right)$ and $\widehat{y_\lambda^k}(q) = \widehat{y_\lambda^k} + (q-1) \times T$,

$\widehat{W^k}(q) = \left(\widehat{w_1^k}(q), \dots, \widehat{w_\lambda^k}(q), \dots, \widehat{w_n^k}(q)\right)$ and $\widehat{w_\lambda^k}(q) = \widehat{w_\lambda^k} + (q-1) \times T$,

$C^k(q) = \left(c_1^k(q), \dots, c_\lambda^k(q), \dots, c_n^k(q)\right)$ and $c_\lambda^k(q) = c_\lambda^k + (q-1) \times T$.

In that context the schedule's sequence: $\widehat{\mathbb{Y}}(1)$, $\widehat{\mathbb{Y}}(2)$, …, $\widehat{\mathbb{Y}}(Q)$ determined by routes Π, presents the execution of adopted delivery plans in subsequent $q = 1,2 \dots, Q$. It is assumed that disturbance $IS(t^*)$ can occur in any time $t^* \in \{T \times (q-1), \dots, T \times q\}$.

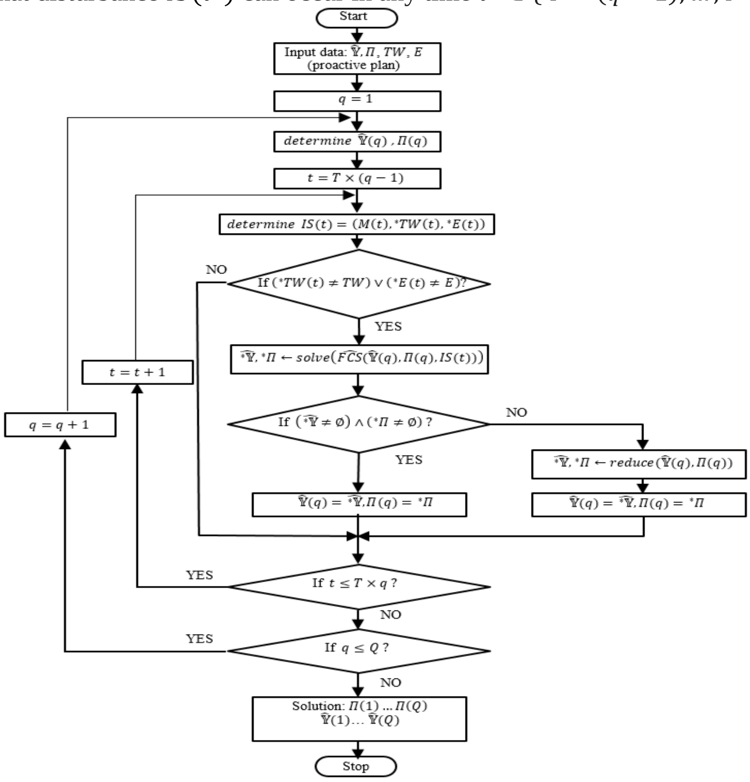

Fig. 3. A dynamic rerouting and rescheduling algorithm

An algorithm that supports dynamic planning, based on the proposed concept of $\widehat{FCS}$ (23), is shown in Fig. 3. The algorithm processes the successive customer service

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cycles $q = 1,2,\ldots,Q$. If there is a disturbance $IS(t^*)$ (i.e., the condition $({}^*TW(t) \neq TW) \vee ({}^*E(t) \neq E)$ holds) in a given cycle q (at moment t^*), then problem $\widehat{FCS}$ is solved ($solve$ function). The function *solve* represents algorithms implemented in declarative programming environments (responsible for the search for admissible solutions to the problems considered). The existence of an admissible solution (i.e.$({}^*\widehat{\mathbb{Y}} \neq \emptyset) \wedge ({}^*\Pi \neq \emptyset)$) means that there are routes which ensure that customers are serviced on time when disturbance $IS(t^*)$ occurs in the cycle q. If an admissible solution does not exist, then the currently used routes and the associated vehicle schedule should be modified ($reduce$ function) in such a way that it removes the delivery operation at node N_λ at which disturbance $IS(t^*)$ occurs.

The presented algorithm generates in reactive mode (in situations of occurrence $IS(t^*)$) alternative corrected versions of the assumed customer delivery plan. The computational complexity of the algorithm depends on methods used to solve the problem $\widehat{FCS}$ (function *solve*). This problem was implemented in the IBM ILOG environment.

7 Computational Experiments

Consider the network from Fig. 2a), in which the three vehicles $\mathcal{U} = \{U_1, U_2, U_3\}$ cyclically service customers $N_2 – N_{10}$ with period $T = 1800$ [u.t]. The fuzzy traveling times between nodes $\widehat{d_{\lambda,\beta}}$ and the time windows TW are as shown in Table 1 and Fig. 2a), respectively. Routes $\pi_1 = (N_1, N_3, N_9, N_2)$, $\pi_2 = (N_1, N_5, N_{10}, N_7)$, $\pi_3 = (N_1, N_6, N_8, N_4)$ determine the fuzzy schedule $\widehat{\mathbb{Y}}$, as shown in Fig. 4a). It is easy to see (Fig. 4a) that in the second cycle ($q = 2$) of the fuzzy schedule (at the moment $t^* = 2300$ for the location ${}_1M(t^*) = (N_3, N_5, N_6)$), the disturbance ${}_1IS(t^*)$ concerning unplanned changes in time window $TW_8^* = [2900; 3500]$ (instead $TW_8 = [2700; 3300]$) on node N_8 is announced (customer locations remain unchanged, i.e. ${}_1^*E(t^*) = E$).

Given this, an answer to the following question is sought: Does there exist a set of routes ${}_1^*\Pi$ operated by vehicles U_1, U_2 and U_3 for which the fuzzy cyclic schedule ${}_1^*\widehat{\mathbb{Y}}$ will guarantee that all customers are serviced on time when disturbance ${}_1IS(t^*) = ({}_1M(t^*)), {}_1^*TW(t^*), {}_1^*E(t^*))$ occurs?

In order to find the answer to this question, we used the algorithm shown in Fig. 3. The problem $\widehat{FCS}$ (22) was then implemented in the constraint programming environment IBM ILOG (Windows 10, Intel Core Duo2 3.00 GHz, 4 GB RAM). The solution time for problems of this size does not exceed 30 s. The following routes were obtained: ${}_1^*\pi_1 = (N_1, N_3, N_7, N_4)$, ${}_1^*\pi_2 = (N_1, N_5, N_9, N_2)$, ${}_1^*\pi_3 = (N_1, N_6, N_{10}, N_8)$ (see Fig. 4c). It should be noted that the route change occurs when the vehicles U_1, U_2 and U_3 are serving customers N_3, N_5, N_7, respectively. New routes allow for timely delivery to the rest of customers despite changing the time window at the customer N_8.

An example of another type of disturbance ${}_2IS(t^*)$ is presented in the third cycle ($q = 3$). At the moment $t^* = 4200$ (location ${}_2M(t^*) = (N_7, N_9, N_6)$) customer's N_{10} location change is signaled – see Fig. 3d). In this case the adopted plan does not guarantee timely deliveries. Similar to the previous case, an attempt was made to designate the routes ${}_2^*\Pi$ guaranteeing the timely servicing of all customer caused by ${}_2IS(t^*) = ({}_2M(t^*), {}_2^*TW(t^*), {}_2^*E(t^*))$ occurrence.

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The following routes ${}_{2}^{*}\pi_1 = (N_1, N_3, N_7, N_8)$, ${}_{2}^{*}\pi_2 = (N_1, N_5, N_9, N_2)$, ${}_{2}^{*}\pi_3 = (N_1, N_6, N_{10}, N_4)$ (see Fig. 4d) were obtained as a solution to the problem $\widehat{FCS}$ (22). In the case under consideration, the change of routes only applies to U_1 and U_3 vehicles. Taking over the customer N_8 by U_1 (see Fig. 4d) allows for timely handling of N_{10} despite changing its location.

In fuzzy schedule ${}^{*}\widehat{\mathbb{Y}}$ (Fig. 4a), the operations are represented as ribbon-like "arterial roads", whose increasing width shows the time of vehicle movement resulting from the growing uncertainty. It is worth noting that the uncertainty is reduced at the end of each time window as a result of the operation of vehicles waiting at node N_1.The increasing uncertainty is not transferred to the subsequent cycles of the system. Uncertainty is reduced as a result of the implementation of the OFN formalism. The vehicles' waiting time at node N_1 has a negative orientation (laytimes ${}^{*}\widehat{w_1^1}$, ${}^{*}\widehat{w_1^2}$ and ${}^{*}\widehat{w_1^3}$- Table 1). Taking the above factors into account, the proposed method of dynamic planning vehicle missions in cyclic delivery systems is unique, due to the possibility of taking into account the reduction of uncertainty in subsequent work cycles of the considered system.

Moreover, the routes ${}^{*}\pi_1$, ${}^{*}\pi_2$, ${}^{*}\pi_3$ remain unchanged (see routes ${}^{*}\pi_1$, ${}^{*}\pi_2$, ${}^{*}\pi_3$ in Fig. 4a) until a disturbance occurs, and then they are rerouted, rescheduled and finally synchronized again so that all customers are serviced on time.

In addition to the above experiments, the effectiveness of the proposed approach was evaluated for distribution networks of different sizes (different numbers of customers and vehicles). The results are presented in Table 2. To summarize, the experiments were carried out for networks containing 6–18 nodes in which services were made by sets consisting of 1–4 vehicles. This means that the problems considered can be solved in online mode (<900s) when the size of service distribution network does not exceed 16 nodes. In the case of larger networks, the effect of combinatorial explosion limits the practical use of this method.

Table 2. Results of computational experiments carried out for selected instances of O^{2}SN.

Number of nodes n	Number of vehicles K	Disturbance IS	
		time window change for one customer	relocation of one customer
		Calculation time [s]	Calculation time [s]
6	1	<1	<1
6	2	<1	<1
6	3	1	<1
6	4	5	3
10	1	10	9
10	2	25	20
10*	3	30	27
10	4	67	60
14	1	150	132
14	2	321	294
14	3	554	410
14	4	>900	>900
18	1	>900	>900
18	2	>900	>900
18	3	>900	>900
18	4	>900	>900

* - solution from Fig. 4

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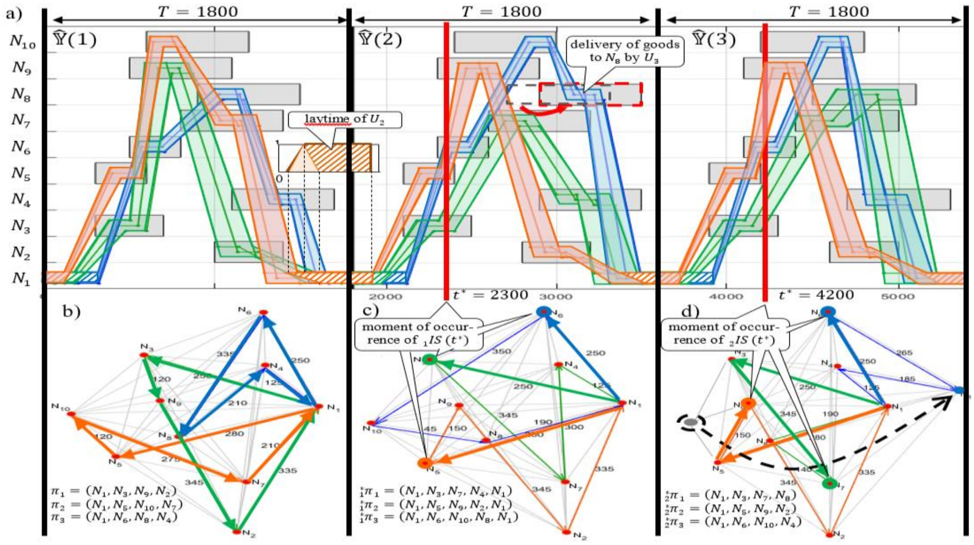

Fig. 4. Response schedules to disturbances $_1IS\,(t^*)$, $_2IS\,(t^*)$ in the network G from Fig. 2a)

8 Conclusions

The main contribution of this work in the field of dynamic vehicle routing and scheduling involves demonstrating the possibility of using of OFNs' algebra framework in the course of decision making following dynamic changes of date and place delivery. Besides of the capability to handle fuzzy nature of variables, the proposed approach can be used for rapid prototyping of time buffers' sizing and allocation underlying proactive and/or reactive routing strategies.

The results of experiments confirm the competitiveness of the analytical approach in relation to the computer-simulation-based solutions aimed at rerouting and rescheduling of a vehicle fleet following the milk-run manner. In this context a compromise between the sizes of delivery cycles and the size of time buffers, taking into account the uncertainty of the data characterizing the O^2SN will be recorded and streamlined into the proposed approach. Moreover, the related issues, concerning vehicle mission planning aimed at dynamic planning of multi-period outbound delivery-driven missions being implemented in a rolling horizon approach, will be the subject of future work.

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