Dual formulation of the TV-Stokes denoising model for multidimensional vectorial images*

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Abstract. The TV-Stokes denoising model for a vectorial image defines a denoised vector field in the form of the gradient of a scalar function. The dual formulation naturally leads to a Chambolle-type algorithm, where the most time consuming part is application of the orthogonal projector onto the range space of the gradient operator. This application can be efficiently executed by the fast cosine transform taking advantage of the fast Fourier transform. Convergence of the Chambolle-type iteration can be improved by Nesterov's acceleration.

Keywords: Total variation · Denoising of a vector field · Chambolle's algorithm.

1 Introduction

Let $\tilde{u}(x)$ be a scalar function, or a continuous grayscale image, defined in a domain $\Omega \subset \mathbb{R}^2$, which is corrupted with an additive noise, i.e., $\tilde{u} = u + \eta$, where $u(x)$ is an unknown true function, or image, and $\eta(x)$ is noise. A classical variational model for image denoising is the Rudin-Osher-Fatemi (ROF) variational model introduced in [8],

$$\min_u \int_\Omega |\nabla u| + \frac{1}{2\lambda}\|u - \tilde{u}\|_2^2, \tag{1}$$

where $\nabla u = (u_{x_1}, u_{x_2})$ is the gradient of $u(x)$ and $|\nabla u| = \sqrt{u_{x_1}^2 + u_{x_2}^2}$. The term $\int_\Omega |\nabla u|$ is called the total variation of $u(x)$ in Ω. The term $\|u - \tilde{u}\|_2^2 = \int_\Omega (u - \tilde{u})^2$ is the data fitting term. A suitable regularization parameter $\lambda > 0$ depends on statistical properties of the noise η. Solution of (1) gives an approximation to the true function such that sufficiently large discontinuities available in $u(x)$ are well preserved. A classical numerical method for solving (1) is Chambolle's algorithm from [2]. A recent survey of the most efficient numerical algorithms for solving (1) is found in [3]. These algorithms belong to the class of local methods.

Modern image denoising techniques are dominated by the non-local patch-based algorithms; see survey in [6]. Nevertheless, the ROF model should not be entirely discarded because the model and its special variants can be useful in

* Supported by the Research Executive Agency of the European Commission, grant 778035 - PDE-GIR - H2020-MSCA-RISE-2017.

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DOI: 10.1007/978-3-030-50426-7_43

some cases, for example, when smoothing an image until a cartoon-looking result or when denoising an image subject to geometric constraints. Our study below is devoted to the latter case.

The model (1) is trivially extended to the case, when $\tilde{u}(x)$ and $u(x)$ are vector functions, by applying the model (1) separately to each component of $\tilde{u}(x)$ and $u(x)$. Since the trivial extension is not always satisfactory, other approaches to the vectorial images have been proposed. For example, the so called TV-Stokes model, which is restricted to two-dimensional vectorial images $v(x) = (v_1(x), v_2(x))$, satisfies the Stokes constraint $\operatorname{div} v = \partial v_1/\partial x_1 + \partial v_2/\partial x_2 = 0$ and reads

$$\min_{v:\,\operatorname{div} v=0} \int_\Omega \lambda|\nabla v| + \frac{1}{2}\|v-\tilde{v}\|_2^2, \tag{2}$$

where $|\nabla v| = \sqrt{(v_1)_{x_1}^2 + (v_1)_{x_2}^2 + (v_2)_{x_1}^2 + (v_2)_{x_2}^2}$; see [7, 10, 4] for more details and a great deal of numerical illustrations.

The present note introduces a multidimensional TV-Stokes model and derives its dual formulation similar to that of [4]. The dual formulation allows us to propose a Chambolle-type algorithm for numerical solution of the TV-Stokes model. Most of the arithmetical work at each iteration of this algorithm is required for application of the orthogonal projector to the linear subspace defined by the Stokes constraint. We propose an efficient implementation of this operation via the fast Fourier transform. We also note that the original Chambolle algorithm can be improved by means of Nesterov's acceleration as in [1], and similar acceleration may be applied to the Chambolle-type algorithm following the recipes given in [3].

2 TV-Stokes model for multidimensional images and its dual formulation

We consider real-valued functions defined in $\Omega = [0, L_1] \times \cdots \times [0, L_n] \subset \mathbb{R}^n$ for arbitrary $n = 1, 2, \ldots$. The gradient operator ∇ is applied only to functions with homogeneous Neumann boundary conditions. First of all, we use the gradient field of a scalar function $u(x)$, $x \in \Omega$, which is the vector function $\nabla u(x) = [u_{x_1}(x), u_{x_2}(x), \ldots, u_{x_n}(x)]^T$. We also apply the gradient operator to n-dimensional vector fields $v(x) = [v_1(x), v_2(x), \ldots, v_n(x)]$ in Ω and label it with the bar as $\bar{\nabla} v(x)$ in order to distinguish from the scalar case. The object $\bar{\nabla} v(x)$ is the tensor field $\partial v_i(x)/\partial x_j$, $i, j = 1, 2, \ldots, n$.

Given an n-dimensional vector field $\widetilde{v}(x) \in \mathbb{R}^n$ corrupted with an additive noise, a constrained variant of the ROF model defines the gradient field $v(x) = [v_1(x), v_2(x), \ldots, v_n(x)] \in \mathbb{R}^n$ satisfying the variational problem

$$\min_{v=\nabla u} \left(|\bar{\nabla} v|_1 + \frac{1}{2\lambda}\|v - \widetilde{v}\|_2^2 \right), \tag{3}$$

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DOI: 10.1007/978-3-030-50426-7_43

where $\lambda > 0$ is a suitable scalar parameter and $\|v - \widetilde{v}\|_2^2 = \int_\Omega \sum_{i=1}^n (v_i - \widetilde{v}_i)^2(x)$. The seminorm $|\bar{\nabla} v|_1$ is the total variation

$$|\bar{\nabla} v|_1 = \int_\Omega |\bar{\nabla} v| = \int_\Omega \sqrt{\sum_{i,j=1}^n (\partial v_i(x)/\partial x_j)^2}.$$

The inner product of vector functions $v, w \in \mathbb{R}^n$ is

$$\langle v, w\rangle = \int_\Omega \sum_{i=1}^n v_i(x) w_i(x)$$

so that the norm $\|v\|_2$ satisfies $\|v\|_2^2 = \langle v, v\rangle$.

Solution $v(x)$ of the variational model (3) is constrained to the linear subspace $V = \{v \in \mathbb{R}^n \colon v = \nabla u,$ where $u(x)$ is a scalar function$\}$. When $n = 2$, the constraint $v = \nabla u$ is equivalent to the constraint $\operatorname{div} v = 0$, which participates in the 2D TV-Stokes model from [7, 10, 4]. Following this observation, we will call (3) the TV-Stokes model too. More specifically, (3) is a primal formulation of the multidimensional TV-Stokes model.

Note that the continuous functional $\mathcal{F}(v) = |\bar{\nabla} v|_1 + \frac{1}{2\lambda}\|v - \widetilde{v}\|_2^2$ is strictly convex. Therefore, its minimum in V is unique and attained in the closed ball $\{v \colon \|v - \widetilde{v}\|_2 \le \|\widetilde{v}\|_2\}$.

Let us equip tensor fields $p(x)$ having the components $p_{ij}(x)$, $i, j = 1, 2, \ldots, n$, with the two norms

$$\|p\|_\infty = \left\| \sqrt{\sum_{i,j=1}^n p_{ij}^2(x)} \right\|_\infty, \quad \|p\|_2 = \left\| \sqrt{\sum_{i,j=1}^n p_{ij}^2(x)} \right\|_2 .$$

The total variation can be rewritten in the form

$$|\bar{\nabla} v|_1 = \max_{\|p\|_\infty \le 1} \langle \bar{\nabla} v, p\rangle \tag{4}$$

using the tensor $p(x)$ as a dual variable; see arguments in [3]. Thus, the model (3) is equivalently reduced to the primal-dual formulation

$$\min_{v \in V} \max_{||p||_\infty \le 1} F(v, p), \quad \text{where} \quad F(v, p) = \langle \bar{\nabla} v, p\rangle + \frac{1}{2\lambda}\langle v - \widetilde{v}, v - \widetilde{v}\rangle. \tag{5}$$

The order of the operations min and max in (5) may be interchanged due to

Theorem 1 ([9]). *Let X be a convex subset of a linear topological space, Y be a compact convex subset of a linear topological space, and $f \colon X \times Y \to \mathbb{R}$ be lower semicontinuous on X and upper semicontinuous on Y. Suppose that f is quasiconvex on X and quasiconcave on Y. Then*

$$\inf_{x \in X} \max_{y \in Y} f(x, y) = \max_{y \in Y} \inf_{x \in X} f(x, y).$$

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DOI: 10.1007/978-3-030-50426-7_43

Owing to Theorem 1, $\min_{v\in V}\max_{|p|\le 1}F(v,p)=\max_{|p|\le 1}\min_{v\in V}F(v,p)$, and we arrive at the primal-dual max-min formulation

$$\max_{||p||_\infty\le 1}\min_{v\in V}\left[\langle\bar{\nabla}v,p\rangle+\frac{1}{2\lambda}\langle v-\widetilde{v},v-\widetilde{v}\rangle\right]. \tag{6}$$

Further derivations make use of the conjugate to ∇ operator denoted by ∇^*. In particular, $\langle\bar{\nabla}v,p\rangle=\langle v,\bar{\nabla}^*p\rangle$ and

$$F(v,p)=\langle v,\bar{\nabla}^*p\rangle+\frac{1}{2\lambda}\langle v-\widetilde{v},v-\widetilde{v}\rangle. \tag{7}$$

Replacing v by ∇u in (7) and further rearrangements yield

$$\begin{aligned}F(v,p)&=\langle\nabla u,\bar{\nabla}^*p\rangle+\frac{1}{2\lambda}\langle\nabla u-\widetilde{v},\nabla u-\widetilde{v}\rangle\\&=\langle u,\nabla^*\bar{\nabla}^*p\rangle+\frac{1}{2\lambda}\left[\langle\nabla^*\nabla u,u\rangle-2\langle u,\nabla^*\widetilde{v}\rangle+\langle\widetilde{v},\widetilde{v}\rangle\right].\end{aligned}$$

The necessary condition for $\min_{v=\nabla u}F(v,p)$ in terms of the first variation of u is the equality

$$\lambda\nabla^*\bar{\nabla}^*p+\nabla^*\nabla u-\nabla^*\widetilde{v}=0. \tag{8}$$

Solution of (8) is not unique because of the homogeneous Neumann boundary conditions. However, all solutions differ only by a constant, i.e. if $u_I(x)$ and $u_{II}(x)$ are two solutions, then $u_I-u_{II}\equiv\text{const}$.

Let us choose the linear least squares solution

$$u=(\nabla^*\nabla)^\dagger\nabla^*\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right), \tag{9}$$

where $\dagger$ denotes the Moore-Penrose pseudoinverse, i.e. the solution of (8) with the minimum 2-norm. The vector field $v=\nabla u$ is determined uniquely as

$$v=\nabla(\nabla^*\nabla)^\dagger\nabla^*\left(\widetilde{v}-\lambda\nabla^*p\right)=\mathit{\Pi}\left(\widetilde{v}-\lambda\nabla^*p\right). \tag{10}$$

Recall that the symmetric operator $\mathit{\Pi}=\nabla(\nabla^*\nabla)^\dagger\nabla^*$ is an orthogonal projector, i.e., $\mathit{\Pi}^2=\mathit{\Pi}$ and $\mathit{\Pi}^*=\mathit{\Pi}$.

In order to find $\max_{|p|\le 1}F(v,p)$ subject to (10), we insert the representation $v=\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right)$ into (7) and perform equivalent transformations:

$$\begin{aligned}F(v,p)&=\frac{1}{2\lambda}\left[2\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right),\lambda\bar{\nabla}^*p\rangle+\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right)-\widetilde{v},\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right)-\widetilde{v}\rangle\right]\\&=\frac{1}{2\lambda}\left[2\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right),\lambda\bar{\nabla}^*p\rangle+\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right),\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right)\rangle\right.\\&\qquad\left.-2\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right),\widetilde{v}\rangle+\langle\widetilde{v},\widetilde{v}\rangle\right]\\&=\frac{1}{2\lambda}\left[\langle\widetilde{v},\widetilde{v}\rangle-\langle\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right),\mathit{\Pi}\left(\widetilde{v}-\lambda\bar{\nabla}^*p\right)\rangle\right]\\&=\frac{1}{2\lambda}\|\widetilde{v}\|_2^2-\frac{1}{2\lambda}\|\mathit{\Pi}(\widetilde{v}-\lambda\bar{\nabla}^*p)\|_2^2.\end{aligned}$$

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DOI: 10.1007/978-3-030-50426-7_43

Therefore, the problem $\max_{|p|\leq 1} F(v,p)$ subject to (10) is equivalently reduced to the constrained minimum distance problem

$$\max_{||p||_\infty \leq 1} \|\Pi(\bar{\nabla}^* p - \widetilde{v}/\lambda)\|_2. \tag{11}$$

We formulate the above proven facts in the form of

Theorem 2 (Dual formulation of the TV-Stokes model). *The unique solution to the TV-Stokes variational problem*

$$\min_{v=\nabla u} \left(|\bar{\nabla} v|_1 + \frac{1}{2\lambda} \|v - \widetilde{v}\|_2^2 \right)$$

is the vector field

$$v = \Pi \left(\widetilde{v} - \lambda \nabla^* p \right),$$

where $\Pi = \nabla(\nabla^*\nabla)^\dagger \nabla^*$ *is an orthogonal projector, and the tensor field* $p(x)$ *solves the dual variational problem*

$$\max_{||p||_\infty \leq 1} \|\Pi(\bar{\nabla}^* p - \widetilde{v}/\lambda)\|_2.$$

3 The Chambolle-type iteration

Following the derivation of Chambolle's algorithm in [2], we write the Karush-Kuhn-Tucker conditions for (11) as the equation

$$\bar{\nabla}\Pi \left(\bar{\nabla}^* p - \widetilde{v}/\lambda \right) + \|\bar{\nabla}\Pi \left(\bar{\nabla}^* p - \widetilde{v}/\lambda \right)\|_\infty p = 0. \tag{12}$$

Hence the solution of (11) can be approximated by the projected gradient iteration

$$p^0 = 0, \quad p^{k+1} = \frac{p^k - \tau \bar{\nabla}\Pi \left(\bar{\nabla}^* p^k - \widetilde{v}/\lambda \right)}{\max(1, \left\| p^k - \tau \bar{\nabla}\Pi \left(\bar{\nabla}^* p^k - \widetilde{v}/\lambda \right) \right\|_\infty)}. \tag{13}$$

where $\tau > 0$ is a step parameter.

Nesterov's acceleration for the iteration (13) and other numerical methods such as the primal-dual methods can be found in [1, 3].

4 The singular value decomposition of the differentiation matrix

We approximate the partial differentiation operators $\partial/\partial x_d$ by the $N \times N$ differentiation matrices of order $N = N_d$:

$$D = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \\ & & & & 0 \end{bmatrix}. \tag{14}$$

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DOI: 10.1007/978-3-030-50426-7_43

The discrete cosine transform is defined by the orthogonal $N \times N$ matrix C with the entries

$$C_{1j} = \sqrt{\frac{1}{N}}, \quad C_{ij} = \sqrt{\frac{2}{N}} \cos \frac{\pi(i-1)(2j-1)}{2N}, \quad i = 2, \dots, N, \quad j = 1, \dots, N.$$

The discrete sine transform is defined by the orthogonal $(N-1) \times (N-1)$ symmetric matrix S with the entries

$$S_{ij} = \sqrt{\frac{2}{N}} \sin \frac{\pi i j}{N}, \quad i, j = 1, \dots, N-1.$$

The singular value decomposition (SVD) of D is

$$D = -\begin{bmatrix} 0 & S \\ 1 & 0 \end{bmatrix} \Sigma C,$$

where the diagonal matrix Σ has the entries $\Sigma_{ii} = 2 \sin \frac{\pi(i-1)}{2N}$, $i = 1, \dots, N$.

5 Discrete gradient operators

Discretization of a scalar function $u(x)$ on a rectangular grid, which is equidistant along each of n dimensions, is given by the components $u_{\alpha_1 \alpha_2 \dots \alpha_n}$, $1 \le \alpha_d \le N_d$. The set of components is often called the grid function. The r-norm of a grid function u is defined as $\|u\|_r = \left(\sum_{\alpha_1 \alpha_2 \dots \alpha_n} |u_{\alpha_1 \alpha_2 \dots \alpha_n}|^r \right)^{1/r}$.

The product of an $N_d \times N_d$ matrix A with a grid function u along dimension d is denoted by $A_{\times d} u$ such that the product $w = A_{\times d} u$ has the components $w_{\alpha_1 \dots \alpha_{d-1} \beta \alpha_{d+1} \dots \alpha_n} = \sum_{\gamma=1}^{N_d} A_{\beta\gamma} u_{\alpha_1 \dots \alpha_{d-1} \gamma \alpha_{d+1} \dots \alpha_n}$.

The discrete gradient operator ∇ is defined by means of the differentiation matrices D introduced in the previous section. For example, the gradient of a scalar grid function u is the set of n grid functions

$$\nabla u = \{D_{\times 1} u, D_{\times 2} u, \dots, D_{\times n} u\}.$$

The discrete gradient $\bar{\nabla}$ is defined for a vectorial grid function $v = [v_1, v_2, \dots v_n]$ as the set of $n \times n$ grid functions $D_{\times j} v_i$, $i, j = 1, 2, \dots, n$.

It is rather straightforward to introduce discrete analogs of the norms $\|\nabla u\|_2$, $\|\bar{\nabla} v\|_2$, seminorms $|\bar{\nabla} v|_1$ and so on.

The singular value decomposition of D allows us to prove that

$$\|\bar{\nabla}\|_2 = \|\nabla\|_2 < 2\sqrt{n}. \tag{15}$$

Let us consider the iteration (13) for grid functions and with the discrete gradient operators. The following lemma determines the range of steps τ, for which the iteration (13) is stable.

Lemma 1. *The iteration (13) is 1-Lipschitz if*

$$\tau \le 2/\|\bar{\nabla} \Pi \bar{\nabla}^*\|_2. \tag{16}$$

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Proof. Each step of (13) consists of two mappings: $p \mapsto p - \tau\bar{\nabla}\Pi\left(\bar{\nabla}^* p - \widetilde{v}/\lambda\right)$ and $q \mapsto q/\max(1, \|q\|_\infty)$. The first mapping is linear and 1-Lipschitz if and only if $\|I - \tau\bar{\nabla}\Pi\bar{\nabla}^*\|_2 \leq 1$, where I is the identity transformation. The second mapping is always 1-Lipschitz.

The estimate (15) implies that stability of (13) holds when $\tau \leq 1/(2n)$. The fastest convergence occurs for $\tau = 1/(2n)$.

6 Computation of $(\nabla^*\nabla)^\dagger$ by the fast cosine transform

Using the differentiation matrices D of order N_d along each dimension d, a discretization of the operator $\nabla^*\nabla$ is applied to a grid function u as follows,

$$\nabla^*\nabla u = (D^T D)_{\times 1} u + (D^T D)_{\times 2} u + \ldots + (D^T D)_{\times n} u.$$

By the aid of the SVD of each differentiation matrix D, the discretized equation $\nabla^*\nabla u = f$ is equivalently reduced to the diagonal system of linear equations

$$\Sigma^2_{\times 1}\widehat{u} + \Sigma^2_{\times 2}\widehat{u} + \ldots + \Sigma^2_{\times n}\widehat{u} = \widehat{f},$$

where $\widehat{u} = C_{\times n} \ldots C_{\times 2} C_{\times 1} u$ and $\widehat{f} = C_{\times n} \ldots C_{\times 2} C_{\times 1} f$. Recall that $C_{\times d}$ are the matrices of the discrete cosine transform of order N_d. The components of $\widehat{u}$ and $\widehat{f}$ are related by the equalities

$$\widehat{u}_{\alpha_1 \cdots \alpha_n} \Psi_{\alpha_1\alpha_2\ldots\alpha_n} = \widehat{f}_{\alpha_1\cdots\alpha_n}, \quad \alpha_d = 1, \ldots, N_d,$$

where $\Psi_{\alpha_1\alpha_2\ldots\alpha_n} = \Sigma^2_{\alpha_1\alpha_1} + \Sigma^2_{\alpha_2\alpha_2} + \ldots + \Sigma^2_{\alpha_n\alpha_n}$ Note that $\Psi_{\alpha_1\alpha_2\ldots\alpha_n} = 0$ if and only if $\alpha_1 = \ldots = \alpha_n = 1$ and is positive otherwise. Hence the least squares solution of $\nabla^*\nabla u = f$ has the components

$$\widehat{u}_{1\cdots 1} = 0, \qquad \widehat{u}_{\alpha_1\cdots\alpha_n} = \widehat{f}_{\alpha_1\cdots\alpha_n}/\Psi_{\alpha_1\alpha_2\ldots\alpha_n} \quad \text{if } \alpha_1 + \ldots + \alpha_n > n.$$

Recall that $u = C^T_{\times 1} C^T_{\times 2} \cdots C^T_{\times n}\widehat{u}$. Note that multiplication of grid functions by the matrices C and C^T can be efficiently implemented by means of the fast Fourier transform (FFT); see, e.g. [12].

Alternatively, fast application of the operator $(\nabla^*\nabla)^\dagger$ can also be computed by the multigrid method [11].

7 Conclusion

In this note, we propose the variational model (3) for denoising of multidimensional vectorial images satisfying the Stokes constraint. Theorem 2 gives the dual formulation of this model. The dual formulation is used for construction of the Chambolle-type iteration (13), which solves the problem (3). Faster convergence is achieved by applying Nesterov's acceleration to (13) as in [1].

While the potential applicability of the new model (3) is not wide, we hope that the TV-Stokes model will become a useful instrument for denoising of images of hydrodynamical flows and for image inpainting.

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DOI: 10.1007/978-3-030-50426-7_43

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DOI: 10.1007/978-3-030-50426-7_43