An Adaptive Computational Network Model for Strange Loops in Political Evolution in Society

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Abstract. In this paper a multi-order adaptive temporal-causal network model is introduced to model political evolution. The computational network model makes use of Hofstadter's notion of a Strange Loop and was tested and validated succesfully to reflect political oscillations seen in presidential elections in the USA over time.

1   Introduction

Hofstadter [8] originally described a Strange Loop as a phenomenon that, after going through a hierarchy of levels, you would return to the starting level; see also [9, 10]. In his original literature, Holfstadter illustrates this for common domains such as graphical art (Escher), music (Bach), and logical paradoxes (Gödel) [13, 16]. Holfstadter theorised that the brain may also use Strange Loops in the creation of human intelligence and consciousness. Although at a conceptual level much literature can be found referring to Strange Loops in one way or the other, almost none of it actually shows a computational model for this phenomenon. An exception is [20], Ch. 8, where the concept of multi-order adaptive reified temporal-causal network is exploited to show some small toy examples of computational Strange Loop models.

In the current paper a more serious and more complex domain is addressed, namely of political evolution over time. A Strange Loop temporal-causal network was created, tested and validated to reflect political oscillations seen in presidential elections in the US. The temporal-causal network breaks a political system into 3 groups, the individual people, the politicians, and the laws. The individuals' combined unhappiness causes them to vote for politicians who align with their desires. The elected politicians in turn vote for the laws which they are aligned to. These laws then cause an effect on the individuals in the form of the weight for their unhappiness, which then begins the cycle again.

Once the network design was created, the parameters of the network were varied in order to obtain oscillations as predicted in empirical Social Science literature. Simulation were conducted for the model, changing the initial values of the individuals of the poor and rich groups to see if the predicted effects concerning different types of laws

were seen. The model was then tuned for specific empirical data from the popular votes from the USA elections over time. All these will be discussed in subsequent sections. Finally, the next steps for the network and an enhancement to the network to create an infinite reified network will be discussed.

2 Background: Domain Description

Since [8] many have applied this to various application areas such as advertising [7], self-representation in consciousness [11] and psychotherapeutic understanding [11, 18]. However, in this literature no computational models are proposed. After seeing how the brain and advertising might be modeled in such a loop, the idea to model a political system with a strange loop was considered. The original idea was that people have to follow laws, which are created by politicians, who are elected by the people. When considering this system, it can clearly be seen that there is a loop in the levels. The causal pathways affecting people's lives are affected by the laws, which are created by causal pathways for politicians; so, people are in effect indirectly voting for these laws by voting for politicians. Therefore, a literature review was conducted to determine if this observation had been made before and if any models of it existed.

The idea to create a Strange Loop out of a political system is based on observations made in the USA political system. The system in the USA can be seen to switch between Democratic and Republican leadership every few elections. This switching of power has caused the policy on a national and state level change over time, such as with abortion law and financial policy. The same kind of oscillations have been observed in England and in coalitions during war. This type of behavior has been noted as early as 1898, where Lowell [12] observed oscillations in elections in the USA. It was, and still is, easily observed when viewing the elections in the USA over time, as seen in Fig. 1 from the above paper.
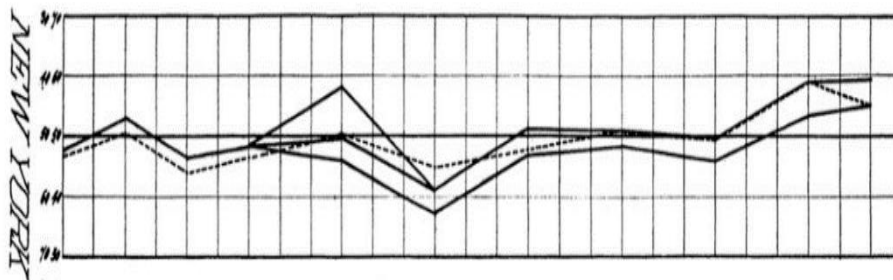

Fig. 1 Voting in New York between 1870 and 1897. The number of republicans is shown below the black lines, while the number of democrats is shown above. Expected values for these elections are shown by the dotted line. Adopted from [12].

The second type of feedback she references is the ability of state capacities to transform over time. "State capacities" refers to the ability of the states to implement and enforce their laws. She writes that "policies transform or expand the capacities of the

state. They therefore change the administrative possibilities for initiatives in the future, and affect later prospects for policy implementation". This can be seen as the effect that the laws have on the political structures. This second influence was considered for implementation in this model, but was disregarded as this first model was kept to its basic form to show that the theory was sound.

Pierson [14] notes that "politics produce politics", discussing how the policy affects its own creation and upkeep. He states that it has been "increasingly harder to deny that that public policies were not only outputs of but important inputs into the political process". He notes that interest groups often follow rather than proceed the adoption of public policy, referencing that Skocpol [15] identifies changes in "social groups and their political goals and capability" as one of the two major types of political feedback. This can be seen as the political power of the people affecting the laws that govern them, which is the centerpiece of the network which is introduced in the current paper.

More evidence of this phenomenon has been noted more recently by Baumgartner and Jones [1]. They noted that american policy is characterized by contrasting characteristics of stability and dramatic changes which can be expressed in positive and negative feedback loops. These loops can be seen between the politics and the individuals, leading to more support for this form of conceptualisation.

3   The Adaptive Network Modeling Approach Used

The adaptive computational model is based on the Network-Oriented Modelling approach based on reified temporal-causal networks described in [19, 20]. The *network structure characteristics* used are as follows. A full specification of a network model provides a complete overview of their values in socalled role matrix format.

- **Connectivity:** The strength of a connection from state X to Y is represented by weight $\omega_{X,Y}$
- **Aggregation:** The aggregation of multiple impacts on state Y by combination function $c_Y(..)$.
- **Timing:**   The timing of the effect of the impact on state Y by speed factor η_Y

Given initial values for the states, these network characteristics fully define the dynamics of the network. For each state Y, its (real number) value at time point t is denoted by $Y(t)$. Each of the network structure characteristics can be made adaptive by adding extra states for them to the network, called *reification states* [20]: states $\mathbf{W}_{X,Y}$ for $\omega_{X,Y}$, states $\mathbf{C}_Y$ for $c_Y(..)$, and states $\mathbf{H}_Y$ for η_Y. Such reification states get their own network structure characteristics to define their (adaptive) dynamics and are depicted in a higher level plane, as shown in Fig. 2. For example, using this, the adaptation principle called Hebbian learning [6], considered as a form of plasticity of the brain in cognitive neuroscience ("neurons that fire together, wire together") can be modeled. The concept of reification has been shown to provide substantial advantages in expressivity and transparency of models within AI; e.g., [2, 3, 4, 8, 17, 21]. The notion of network reification exploits this concept for the area of adaptive network modeling.

A dedicated software environment is available by which the conceptual design of an adaptive network model is automatically transformed into a numerical representation of the model that can be used for simulation; this is based on the following type of

(hidden) difference of differential equation defined in terms of the above network characteristics:

$$Y(t+\Delta t) = Y(t) + \eta_Y \, [\mathbf{aggimpact}_Y(t) - Y(t)] \, \Delta t \quad \text{or} \quad \mathrm{d}Y(t)/\mathrm{d}t = \eta_Y \, [\mathbf{aggimpact}_Y(t) - Y(t)] \quad (1)$$
$$\text{with } \mathbf{aggimpact}_Y(t) = \ \mathbf{c}_Y(\omega_{X_1,Y}X_1(t), \, \ldots, \, \omega_{X_k,Y}X_k(t))$$

where the X_i are all states from which state Y has incoming connections. Different combination functions are available in a library that can be used to specify the effect of the impact on a state (see [19, 20]). The following three of them are used here:

- *the identity* function for states with impact from only one other state $\quad \mathbf{id}(V) = V \quad (2)$
- *the scaled sum* with scaling factor $\lambda \qquad \mathbf{ssum}_\lambda(V_1, \, \ldots, \, V_k) \ = \ \dfrac{V_1 + \cdots + V_k}{\lambda} \qquad (3)$
- *th*e *advanced logistic sum* combination function with steepness σ and threshold τ

$$\mathbf{alogistic}_{\sigma,\tau}(V_1, \, \ldots, V_k) = [\frac{1}{1+e^{-\sigma(V_1+\cdots+V_k-\tau)}} \ - \ \frac{1}{1+e^{\sigma\tau}}](1+e^{-\sigma\tau}) \qquad (4)$$

4   Design of the Multi-Order Adaptive Network Model

The idea behind this model was the following scenario. There is a group of people who have a law which makes them unhappy. As the people get more unhappy, they vote more, electing politicians who will support the laws which will make their unhappiness less. The politicians then vote for the laws which they support. After some discussion between the groups of politicians, the law is agreed upon which is a combination of the desires of the groups, and then the law comes back to affect the individual people's unhappiness, starting the cycle again. In this scenario, causal pathways in society at three different interacting levels play a role:

(1) Causal pathways that determine the unhappiness of people
(2) Causal pathways that determine the politicians' positions
(3) Causal pathways that determine the laws

Here the effects resulting from the causal pathways of type (1) are the unhappiness of the people; these effects affect the causal pathways of type (2) by voting. In turn, the effects resulting from the causal pathways of type (2) affect the causal pathways of type (3). Finally, the effects resulting from the causal pathways of type (3) affect the causal pathways of type (1), which closes the Strange Loop.

For the scenario addressed by the designed network, it was decided to have two laws which would affect the individuals' lives. Two groups are considered, a group who benefit from one law, and a group that benefit from the other law, which, to help explain the model more succinctly, will be referred to as the rich group and the poor group. These individuals would then vote for the political party which favour the law that favour them. Therefore there are two political parties as well. For each of the 3 distinct levels, networks were created with mutual connections in mind. First the networks themselves will be discussed and then the connections between the levels.

The individuals subnetwork. The first subnetwork modeled addresses the individual level. Fig. 2 shows the individual level for 10 individuals. Each individual has a starting

value with represents the context in which they function. These are the odd nodes X_{2i-1} seen in the bottom of the network figure. In the simplest form of this network, this can be thought of as a context that generates some level of gross income. The unhappiness of the individual, which can be seen in the top of Fig. 2 as the even nodes X_{2i}, is determined through a one-step causal pathway by the starting context value X_{2i-1} multiplied by the weight of the connection from X_{2i-1} to X_{2i} which represents how much the current laws affect this person's life for that context. This connection weigh is represented by reification state X_{33} (for $i>5$) or X_{34} (for $i\leq5$). The way these weights are derived will be determined by the other subnetworks and their interaction. Again, in the simplest form it can be thought of as a tax on their income. As stated previously, the network has 2 groups of individuals which are in accordance with the different weights for them.
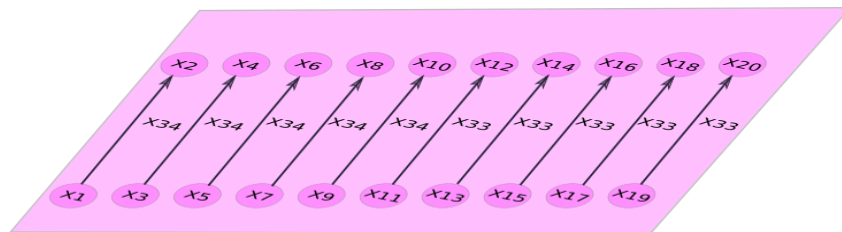

Fig. 2 Subnetwork for the individual level

The politicians subnetwork. The next subnetwork devised concerns the causal pathways for the politicians and their parties. Fig. 3 shows the politicians subnetwork. There is a limited resource of political power which is represented by an input node X_{21} for the politician level.
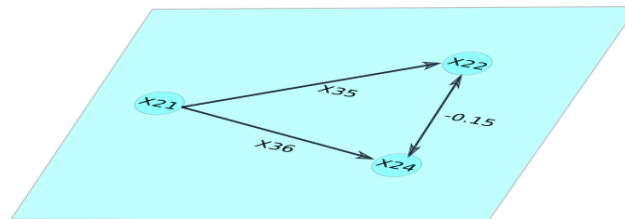

Fig. 3 Subnetwork for the politicians level

The people then vote for the political party they support, which then adjusts the causal pathway for the resulting power each party has, which can be seen in the effect nodes X_{22} and X_{24}. Within the causal pathways, the weights for each party (X_{35} and X_{36}) are determined by the previous level. A negative connection between the two parties, represents that the parties attempt to minimize the others influence.

The laws subnetwork. The final subnetwork devised was for the law level; it can be seen in Fig. 4. In this network, there is a limited budget for laws, which is the input node X_{26}. Given this budget, the political parties vote on either law 1 or law 2 (X_{27} or X_{28}). Here the weights X_{22} and X_{24} (and also the scaling factors) are determined by the previous level. After the vote, a logistic function is applied to the output of each of the laws individually with a weight of 1, determining the new power of each law which is seen in the network as X_{52} and X_{53}. Once the new power is determined, the effect of each law on the two groups is updated where the weights represent the effect of each law on each of the groups. Finally for each of the two groups, the effect of the new combination of laws is combined. These values for X_{33} and X_{34} become the new effects of the laws on the two types of individuals.
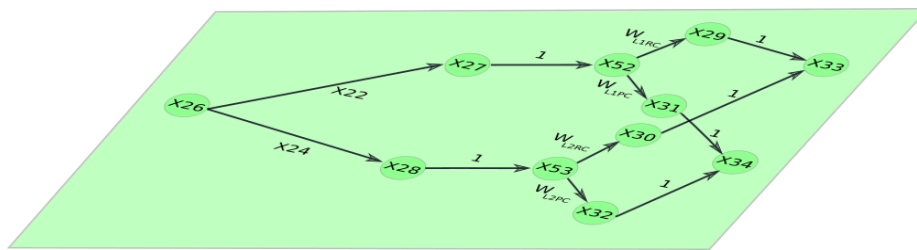

Fig. 4 Subnetwork for the law level

Connections between the subnetworks. Now the subnetworks have been defined, the connections between them can be discussed. A simplified version of the network can be seen in Fig. 5, which shows how the networks are connected. Beginning with the individual's levels connections, the weight for the causal pathway from the input of the individual to the unhappiness of the individual is determined by the laws. As discussed for the law subnetwork, nodes X_{33} and X_{34} represent how much an individual's causal pathway of each group (rich or poor) is impacted by the current law system; the values of these nodes X_{33} and X_{34} are used as the weights for how much the current laws affect an individual of the corresponding group. This can be seen in Fig. 5 as the blue connections going from the laws network (green) to the individuals network (pink).

Examining the connection from the individuals to the politician subnetwork, the unhappiness of voters determines the weight in the causal pathway from the input to the political powers for each party. This is shown by the blue arrows connecting the individuals network (pink) to the political power network (blue).

Finally, examining the connection from the politicians subnetwork to the laws subnetwork, the weight within the causal pathway which determines the vote for each law (X_{22} and X_{24}) comes from the politician subnetwork. This is the power for each of the individual parties which supports each law, which, in this model, is one party for each law and is shown by the blue arrows connecting the political power network (blue) to the laws network (green). More on how the values were determined can be found in Section 5.
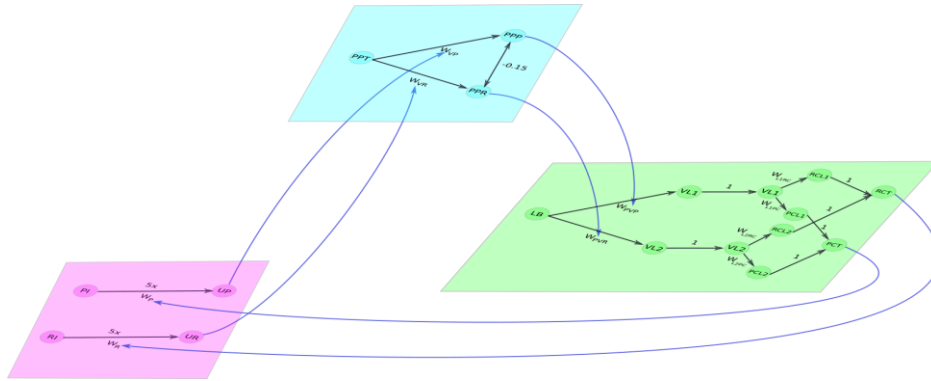
Fig. 5 Simplified picture of the overall network

5 Simulation Experiments

The network characteristics used can be found in the Appendix at https://www.re-searchgate.net/publication/340162169. For simulations, for all states the general equation (1) from Section 3 was used where the chosen combination functions were (see Section 3, formulae (2), (3), and (4)):

identity function	$\mathbf{id}(V)$	X_1 to X_{21}, X_{26}, X_{29} to X_{32}, X_{39} to X_{48}
scaled sum function	$\mathbf{ssum}_{\lambda}(V_1, \ldots, V_k)$	X_{22} to X_{25}, X_{33} to X_{38}, X_{49} to X_{51}
logistic function	$\mathbf{alogistic}_{\sigma,\tau}(V_1, \ldots, V_k)$	X_{27}, X_{28}, X_{52}, X_{53}

For the first simulation experiments, the steepness σ of the logistic functions was 16, and the threshold τ was 0.35 for X_{27}, X_{28} and 0.7 for X_{52}, X_{53}.

From the literature, it was seen that the system should oscillate, therefore in the first run of the model this behaviour was searched for. For the first simulation both groups were initialized with the same values (or worth). This meant the groups have the same unhappiness if their preferred law is not active. For some parameter settings the behaviour was observed as seen in Fig. 6. The unhappiness of the people can be seen to oscillate between the two groups, as well as the laws the political power. This figure actually shows the unhappiness of one representative person for each group, not the total unhappiness of the group. All persons in the group show the exact same behavior, since they are initialized the same and influenced by the same law. It can be seen that a rise in political power for a group closely follows the rise of unhappiness in that same group and that the laws preferred by a group follow slower, but they do rise when the political power of that group rises. This can be explained by the slower speed factors associated to the laws. All the oscillations now have the same amplitude, since all groups and laws are initialized either exactly the same or in the case of the laws at 1 for the poor law and 0 for the rich law.
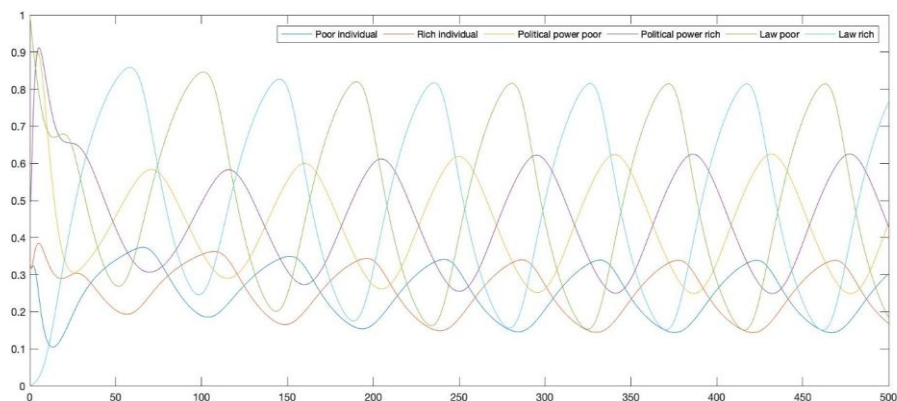
Fig. 6 Behavior for the first run with oscillations

To get the model to simulate real societies better, in the following simulation the two groups were initialized differently. The "rich" group was initialized with a score (or income) of 0.8 and the "poor" group with a score (or income) of 0.4. This meant that the rich people will have the ability to have a much higher unhappiness than the poor people, so it is expected the "rich" group will have a higher political power and get their preferred law more active than the law preferred by the "poor" people. The behavior resulting from this simulation can be seen in Fig. 7.
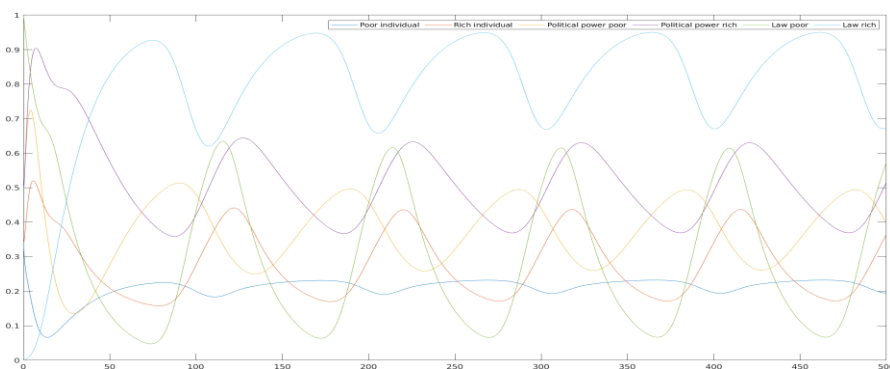

Fig. 7 Behaviour for different initial values of the rich and poor groups

The "rich" law is always more active than the "poor" law, and although there are still some oscillations, the "poor" group only gets influence when they are very unhappy and

are always less influential than the "rich" group, which is to be expected when there is a group which is more influential than the other with the same number of people.

6 Further Validation of the Network Model

Data from the popular votes of the United States presidential elections was collected from the USA archive (archives.gov), and plotted using the percentage of republican and democratic votes. This data was then used to validate the model. A graph of this data can be seen in Fig. 8. Oscillations between the two parties are clearly visible here. Both the initial simulation and the analysis of popular votes, shows the same trend, where oscillations between the two "parties" can be seen. The difference is in the size of the oscillations. The popular votes simulation has small oscillations between 0.65 and 0.35, while the initial model has oscillations between 0.8 and 0.2.

Parallels can be drawn between the behavior of the model with the groups initialized differently. Continuing to follow the rich and poor example, in real life there are less rich people, but they are still very influential. Looking at figure 7, it can be seen that the rich easily overpower the poor.
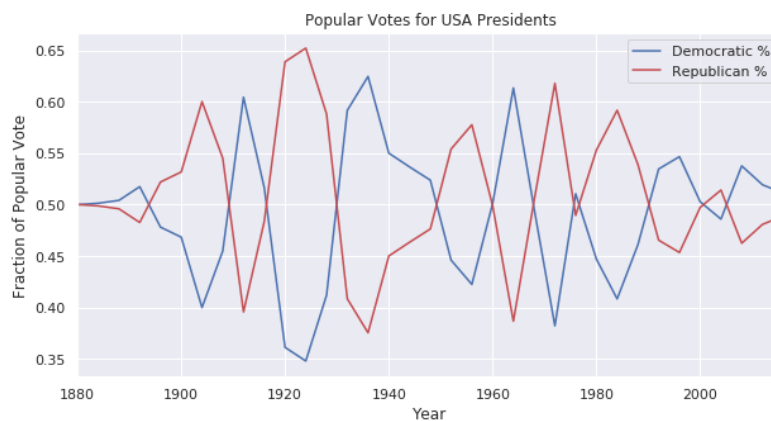

Fig. 8 Statistics for the US presidential elections

The model was tuned on these data from US elections. Speed factors were tuned for X_{21} until X_{48}, and for X_{52} and X_{53}. Furthermore, the sigma's and tau's for the alogistic functions of X_{27}, X_{28}, X_{52} and X_{53} were also tuned. These are the nodes for voting for and activation of the laws. In total there were 38 parameters that were tuned. All speed factors were initialized with a speed of 0.5 and the minimum and maximum were set at 0 and 2. Steepness parameters σ were initialized with 16 and thresholds τ at either 0.35 (for X_{27} and X_{28}) or at 0.7 (for X_{52} and X_{53}). Minima for both were set at 0 and maxima for the σ's was at 30, while the maxima for the τ's was 1. Simulated annealing was used for the tuning with a reannealing interval of 500 iterations and otherwise standard settings from the Global Optimization Toolkit from Matlab. After tuning a RMSE of

0.06736 was found. Values found during tuning can be found in Table 1 (order is the same as stated above). The behavior of the system can be seen in Fig. 9. As can be seen, the tuning did not work as expected. The RMSE is very low, which would normally mean the model fits the data really well, but the system shows no oscillations at all. This could probably be explained by the small differences between the republican and democratic votes in percentage. For most values the difference is around 5%. Because of this, it is expected that the model with these values fits so well, because it is the middle between the two values and the values are very close. Another possible reason could be the time frame. In the original simulation, the oscillations occur round every 100 time steps. In the data from the elections, 1 time step was set to be a year. Therefore the speed of the oscillations would have to be much higher, and its possible that the speed factors were not high enough to capture this completely.

Table 1 Tuned parameter values found

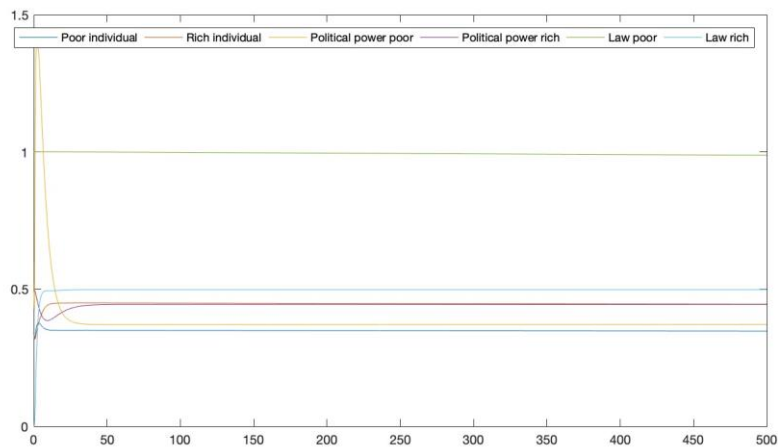| 1.61369	1.96366	1.10475	0.13759	1.00872
1.30867	0.08119	1.47845	1.80759	1.24313
0.72002	$6.84443\ 10^{-5}$	0.58717	0.98266	0.21999
$4.49622\ 10^{-7}$	0.97381	1.54367	1.99999	0.16781
0.77973	1.78931	0.09258	0.90414	1.40409
0.54539	1.03380	1.70694	$2.60123\ 10^{-5}$	0.62080
16.2610	0.95972	25.26481	0.15189	10.55741
0.01325	8.79826	0.99999		

Fig. 9 Behaviour after first parameter tuning

To overcome this, we exaggerated the empirical data and put it at 0.5, 1 and 0 in alternating order with 48 time steps between, which is 4 years in terms of months. Figure 10 shows the empirical data and the simulation data for this and as can be seen, oscillations did occur with the exaggerated data. RMSE behavior can be seen in figure

10 and shows that the lowest RMSE value was found at the beginning and after that never again. This could occur due to a couple of reasons. Either, this minimum score is difficult to reach and after leaving the optimum, it is unlikely to find back again due to the specificity of the values. It could be to do with the reannealing interval after 100 iterations, which makes the temperature rise again, so less optimal solutions are again accepted without giving time to search the space for more optimum values. Evidence for this could be seen in Fig. 10, since sometimes it seems to trend down as expected from simulated annealing and after which the RMSE rises again.
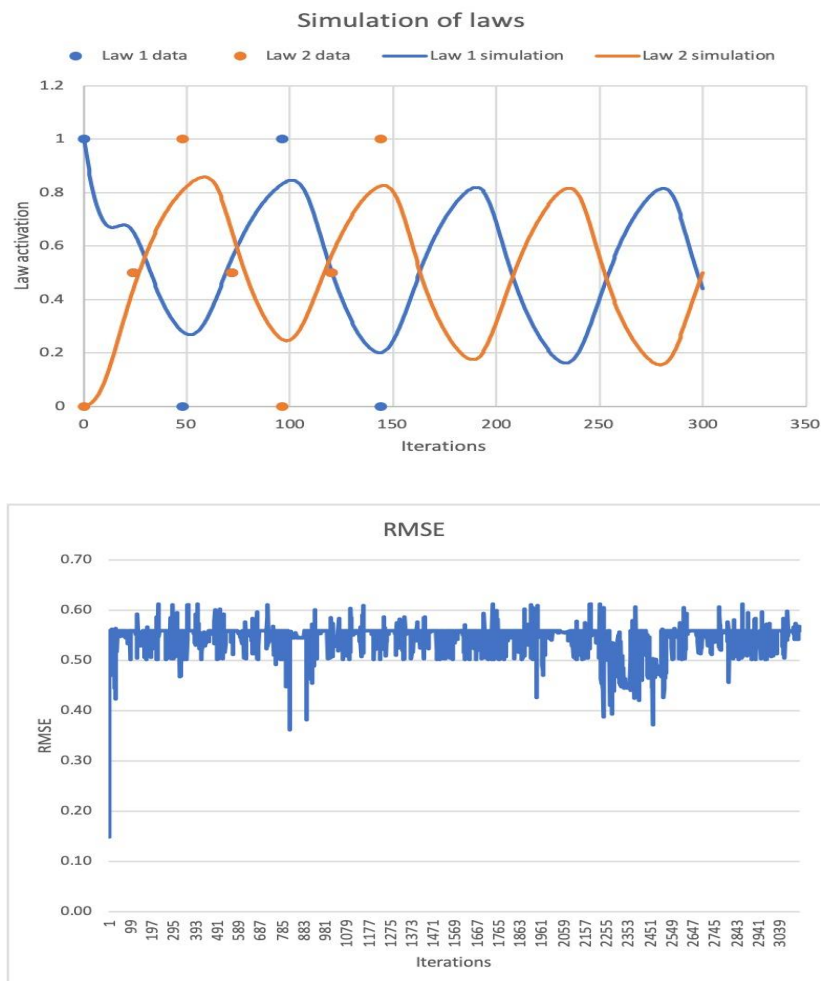

Fig. 10 Upper graph: Behaviour after second parameter tuning.
Lower graph: RMSE over iterations

It could also be due to trying to fit the wrong parameters, since less parameters were fitted for this tuning. Only the σ's and the τ's of the activation of the laws and their speed factors were tuned as it was thought that these would be the parameters that would affect the general shape the most.

7   Discussion and Future Work

In the first simulation run, in a qualitative sense the network behaved as expected from the research done, with the political powers oscillating between rich party being in power and the poor party being in power in periodic oscillations. In the initial simulation, seen in Fig. 6, it can be seen that as the poor parties unhappiness is rising, the political power of the poor group rises as well, then about 90 degrees out of phase, the poor law begins to increase. As the law increases, the unhappiness begins to decrease and the rich groups unhappiness increases as they are dissatisfied with the situation and begin to vote more. This same behaviour is also seen in the rich group, about 180 degrees out of phase. The laws oscillate around 0.5 for both the poor and the rich.

When initializing the individuals groups (rich and poor) with different starting values as seen in Fig. 7, the periodic oscillation behaviour is still seen, but the center of these oscillations for each group is different. The value at which the laws oscillate around is approximately 0.8 for the rich and 0.33 for the poor compared to those seen in the previous simulation at 0.5 each. This behaviour is expected as the rich group has more unhappiness since they have more wealth to lose. This means that they will be more active in ensuring that their law, which benefits them more, is in effect, where the poor people's unhappiness is relatively small compared to them so they don't have the ability to compete. This can be seen in real life politics, as the rich have more ability to influence politics due to the influence and money they have, where the poor often have to struggle and campaign much harder to get change.

When tuning the model to the numerical data from the USA, it was seen that the model showed no oscillations. One reason this could occur is that since the data does not oscillate much outside of 0.5, the mean is the best optimum that system can reach from those starting values. Another reason could be due to the levels of the parameters being tuned not being high enough, or the assumed number of steps for the model being too small, as one time step was set to a year. When observing the original network simulation, it can be seen that an oscillation occurred once approximately every 100 steps. Therefor, if the data was set to months rather than years (48 steps between oscillations rather than 4) or if the speed factors were allowed to increase above 2, the network may have converged to periodic oscillation.

In the future it would be interesting to see how increasing the number of poor people would affect the system. From observing politics in real life situations, if there are enough poor people, the activation of the rich law should decrease, as there is more reactive unhappiness coming from the poor group. This was not done in this experiment as there was not enough time to update and modify the network.

Another interesting addition to a future version of the model would be to add in multiple laws. This would require more complex individuals, with nodes for each of the

different issues and then a general unhappiness. The political level would also have to be updated to reflect the multiple laws each party could vote for. This would also open the model to have parties who voted for the laws in different ways and having the individuals vote for the parties who best reflected where the largest unhappiness was coming from.

Towards the end of the experiment, a network with add in media to the system was devised. In this network the upwards connections would be from the people to the media, where the people affect what the media talks about based off their interests and views. Then the media would affect the politicians by enhancing or detracting from how the people view them. The people would have an upward connection to politicians to vote for them as before. The politicians would then effect the laws in the same way they do now through voting and finally the laws would affect the people in a similar way.

The downward connections, starting with the laws, would be the laws affect the politicians through changing how the voting works and/or the speed factors. The politicians would affect the media through what equates to forcing them to talk positively or negatively about certain topics or suppression of others. The media would enhance or dampen the peoples reactions/care for the policies and laws. Finally the people would affect the laws by determining how quickly the laws come into effect due to how well they are followed/received by the population.

In future developments, a number of other relevant subtleties can be addressed as well. For example, for the US, the important roles of the hierarchy from cities to states to federal level, of competing lobby groups, and of the differences in access to information for different subpopulations can be addressed.

8 Conclusion

In the reported research an experiment of a strange loop adaptive temporal-causal network was created, tested and validated to reflect political oscillations as seen in presidential elections. The temporal network breaks a political system into 3 groups, the individual people, the politicians, and the laws where the individuals feed into the politicians, who feed into the laws, which feed into the individuals. In the initial simulation, the oscillatory behaviour which was expected from the literature review was observed. Next the network was modified to reflect an unbalanced political system with one group of individuals that were influenced more by the laws than the other. This cause the law which benefited those with more influence to be higher than the law which was beneficial for those with less influence, as expected. Finally the network was tuned to data from the USA presidential elections popular vote using simulated annealing, with both actual and simplified data. The simulated annealing did not perform as expected, giving a network which did not oscillate when using the real data, when using the simplified data, managed to reflect the behaviour which it was tuned on. The network not being able to tune on the real data could be due to the oscillations data being so close to 0.5, that the model found 0.5 as the ideal with the initial setting given and was unable to escape to another optimum. Another possible reason would be that the speed factors not being allowed to be tuned above 2 or due to the small number of steps between oscillations.

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