Estimation of tipping points for critical and transitional regimes in the evolution of complex interbank network *

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Abstract. We consider an agent-based model of dynamic interbank network, evolving under several influential factors. Co-evolution is formally performed by the connection between node states and topology of interaction, and vice versa. During the simulation, network evolves to critical regime, corresponding to cascading behaviour in the system, through transitional one. Results show these global regimes correspond to dynamics at micro-level with three types of node states. On the base of formal model of system evolution and regimes formal definitions we estimate the starting point of cascading behaviour and determine number of iterations before its early warning signal – the start of transitional regime. Experiment is made for the interbank market model, nevertheless, possible applications are not restricted by the case. We show, that the obtained estimations allow for appropriate prediction of starting points of critical and transitional regimes (which correspond to cascading behaviour and its early warning signal) and explanation of observed dynamics in the evolution of banking system model under fund infusion scenario.

Keywords: complex dynamic network · interbank market · analytical estimation · regimes · criticality · tipping point

1 Introduction

Complexity of systems, emerging from element interactions, their inner structure, and dynamic processes affecting network evolution, is hard to explore and predict due to wide number of affecting factors. Analytical methods are usually differential models, which assume fully connected graph for interactions between system elements. Effects of topology of nodes interaction on further system evolution is considered in simulation models. They allow for reproduction of local dynamics by means of agent-based approach in the combination with graph models. Agent-based models involve small changes and dynamics at node-level and consider their effect on network structure and further changes in node states arising from their interactions with other nodes. Nevertheless, the number of details

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and heterogeneities in structure and behaviour obstruct analytical prediction and analysis of complex systems.

Analytical estimations are useful for models verification and validation. The validation process of agent-based models is restricted by observed data segment. Large number of model parameters puts trajectory of system evolution to a multidimentional surface, containing numerous possible trajectories [6]. Therefore, the observed data segment, used for validation, can be in several different possible trajectories, leaving a system still unpredictable, despite model validity. Other usage of analytical estimations is early-warning signals. They are usually explored to predict catastrophe bifurcations and critical failures in time-evolving systems, and are associated with systemic characteristic, served as a predictor [18]. Banking systems stability is one of the applied issues related to emergent criticality in systems, which were studied from the points of network effects, default prediction [15], single bank stress-tests [14], early-warning signals.

Here, we make an attempt to predict critical failure of interbank market and corresponding early-warning signals analytically on the base of system organisation and local dynamics, resulting in interbank network evolution. We decompose initial agent-based model of interbank market into basic local actions, formalise them, and introduce structure of interactions between agents. Node states are real-valued, which allows for consideration of individual bank effects on systemic stability. Generality of system performance allows for application of the suggested approach to wide variety of systems and to generalise some models widely used in epidemiology, ecology, etc.

For the evolution process we consider three regimes, referred to as normal, transitional, and critical. Normal regime demonstrates no special warnings, but it can be unsteady and lead to criticality. Transitional regime is related to changes in dynamics of system state variable, before a cascade, when dramatical changes are not available for observation, but they are able to be. In literature this regime is associated with early warning signals. Critical regime is associate with cascading behaviour. We build mathematical model and estimate points of transitions between regimes analytically, in contrast to early-warning signals, based on observed state variable changes [21] (like "critical slowing down"). The suggested approach application is shown for the case of interbank network with Poison structure of connections and for interbank market simulation model with varied counterparty choice, resulting in various network structures. Transition to cascading behaviour of an interbank network is taken as a case.

2 Literature

Natural systems usually evolve under internal and external drivers. The combinational effects of these drivers may shift a system to qualitatively new states, so called phase transitions, while the corresponding points are called tipping points [16]. In biological systems these transitional processes are associated with homeostasis, when positive and negative feedback links lead a system back to its stable state (negative feedback) or amplify external effects allowing for the

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transition to a qualitatively new state or phase [12]. In the combination with interaction patterns between elements of a system this phenomena results in self-organisation [23], and, in special cases, to self-organised criticality [20]. This can be observed as default cascades in banking systems, as synchronisation in the case of oscilators, or as avalanche in a sandpile model. In general case, there are system elements characterised by states, some results of interactions affecting elements states, and a system state, related to the combination of element states, which we call critical and want to estimate.

Transitional behaviour (preceding critical one), as it is explored in this paper, is well-presented by the avalanche model of a sandpile. Learning sandpile behavior [5] shows the systemic stability question may be reduced to the criticality of individual grains. Bouchaud et al. considers two types of grains, stable and rolling, and find a critical angle when grain turn to instable state. Systemic properties of human-made, economical and socio-technical systems can also be reduced to the local dynamics of their elements. Nevertheless, their criticality may be human made which makes the dynamics and corresponding transitions dependent on verbal definitions, notions, and regulation mechanisms. For example, bank defaults in banking systems may initialize default cascades, and in this case banks correspond to grains and a default cascade to an avalanche. Nevertheless, regulatory restrictions, applied to banks, can be considered as a critical state vector, like the critical angle of a grain, and may affect systemic evolution. This interdependence and systemic variability give rise not only to a problem of systemic phase transitions, emerging from systemic elements states, but also to the inverse problem of regulatory restrictions optimisation. But in the second case functional properties of a system and corresponding formulations of criticality can also play an important influence on resulting requirements to systemic elements.

In this section, we observe existing methods of analytical estimation of agent-based systems and network dynamics, and results related to criticality in their evolution and early-warning signals.

2.1 Agent-based models estimation

Agent-based models tend to present accurate copies of observed reality and usually take many parameters and model combinations to reach the required accuracy. Nevertheless, such multidimentionality results in difficulties in model estimation, analysis, and calibration. Problem of formal description of agent-based models is not new in literature [11]. Hinkelman et. al. argue, agent-based models are usually poorly formalised and suggest field theory application for their formal representation. Nevertheless, the suggested framework is aimed at providing a uniform way of agent-based system description and does not imply predictability. Laubenbacher [17] suggest formal methods like sequential dynamical systems over Boolean set of node states. Both studies propose frameworks for agent-based systems formalisation, but there are no means for state estimation, and the set of element state variables is discrete. Formalisation and estimation of agent-based models are not so developed for the best of our knowledge.

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2.2 Early-warning signals

Early-warning signals are strongly related to questions of predictability and criticality, which are of great importance for wide range of applications. In particular, financial systems suffer from repeated crises, so authors claim, early warning signals are of great interest [25, 13]. Others show how dynamics of a system at micro-level results in critical behaviour at system-level, showing synchronisation [1] as mathematically explored phenomena, homeostasis as a complex of positive and negative feedback mechanisms, resulting in regime shifts [12], and applications to other areas, involving fractal nature of river and its evolution [24]. An attempt to estimate long-term influence of node state on criticality is performed in [8], where authors introduce early-warning signal on the base of change in real-valued node state.

2.3 Complex networks dynamics prediction/analysis

The most developed literature area, related to our research field, is prediction and analysis of the network evolution. Lambert and Vanni [16] explore changes of topological state variable in the dynamic graph model with edges addition and deletion at micro-level[1], and show that its fluctuations given by the derived master-equation are more significant than for the case of mean-field approximation. Nevertheless, the model does not capture influence of node state effects on micro-level dynamics.

Sole et. al. [22] provide a dynamical model of node state change with consideration to neighbourhood influence according to connectivity matrix. Element states are of two values $s_i \in \{0; 1\}$, for extinct and for alive; when $t = 0 \Rightarrow \forall i \; s_i = 1$. Hernandez-Urbina and Herrmann [10] provide evolution of state variable in the iterative form with consideration of external forces impact and the influence of node states in the combination with adjacent edge weights. Nevertheless, dynamics of a network and its relation to node states are not captured.

In contrast to these methods, we consider graduate change of real-valued node states and imply three kinds of dynamics of links creation at micro-level, depending on node state value. This results in three patterns of network formation affecting changes of node states and resulting in three regimes of systemic evolution.

3 Method

3.1 Interbank market model

We consider an interbank network as a directed graph where nodes correspond to banks and links correspond to interbank exposures. Each bank is characterised by its balance sheet, having interbank and external assets and liabilities, s. t. assets

[1] The Generators–Destroyers model [19, 26]

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correspond to output edges, liabilities correspond to input edges. External assets and liabilities reflect bank interactions with customers and firms, and interbank exposures are related to weighted sums of output and input edges, adjacent to a considered bank. The difference between assets and liabilities determines bank state and is referred to as net worth [2, 3, 7, 9]. Changes of bank state (resulting from balance sheet changes) may initiate activity on the interbank market, which is aimed at enhancing individual bank state. In this way, dynamics on node-level in the combination with counterparty choice strategy results in patterns of network topology evolution [4]. Lack of assets initiates output links creation, and lack of liabilities results in the creation of input links, adjacent to a node. Changes in network structure in turn affect bank states, resulting in further changes. In combination with initial parameters and network configuration, the dynamics of interbank network is affected by different factors, as it is illustrated in Figure 1.

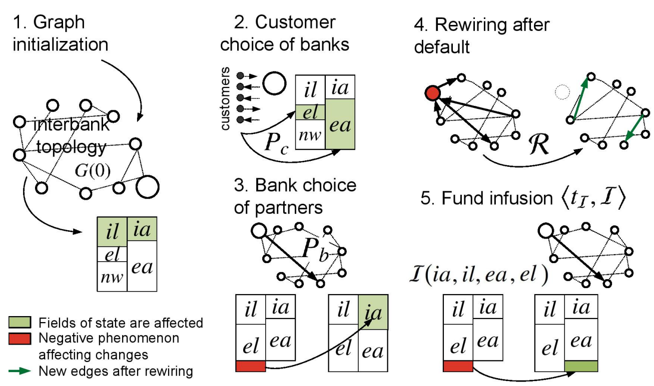

Fig. 1. Factors affecting the evolution of interbank network [9]

Therefore, the above mentioned dynamics can be decomposed to the following components: i) the bank model determines node state and is related to network structure (by definition) and activity triggers; ii) the network model connects banks and is related to the changes in their states; iii) the model of counterparty choice, resulting in emerging network topology; iv) triggers of link formation; v) bank default condition. These components provide two ways of evolution, driven by basic triggers of link formation and default condition. Prevalence of one basic mechanism over others provide transitions between normal, transitional, and critical regimes, associated with closeness to cascading behaviour. This will be formally shown further in the paper.

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3.2 Formal dynamics of interbank network evolution

Let $S(t) = \{s_i(t)\}, i \in \{1, \ldots, N\}$ be a state of an evolving system of N elements with their states $s_i \in \mathbb{R}$. States are related to bank balance sheet in the interbank market model, where banks represent system elements. They interact with each other, which is formally reflected by a directed evolving graph $\Gamma(t) = \{\gamma_{ij}\}$, where nodes correspond to elements and are attributed by states. Since the structure of interactions is connected to node states, addition of output edges positively contributes to a state, while input edges negatively contribute to it. In a static case, that means $s_i = \sum_{k=1}^{N} (\gamma_{ik} - \gamma_{ki}) + C$, where C is summarised contribution of factors not related to topology. Coming to dynamics, there are triggers related to node states and resulting in different patterns of topological changes.

Types of node states & corresponding triggers. Type of node states described in this section are met in literature related to interbank markets as stressed and defaulted banks [2]. Here, node states are taken real-valued to reflect the possibility of their graduate decrease. Nevertheless, number of possible reactions at micro-level, resulting in corresponding macro-level dynamic patterns, is restricted by three and fixed for each type of node state.

Let us take a node i having state s_i, fix $a, b \in \mathbb{R}$ dividing $\mathbb{R}$ into 3 semi-intervals. Without loss of generality let $a \leqslant b$.

- $s_i < a \Rightarrow$ the state is *critical* and node is removed from the network with the edges adjacent to it;
- $s_i \in [a; b) \Rightarrow$ the state is *transitional* and the node tends to enhance its state by creating new edges in the network, to make the state $s_i \geq b$;
- $s_i \in [b; \infty) \Rightarrow$ the state is stable, and the node does not create new edges actively, nevertheless, it can interact with other nodes if they need it.

Therefore,

$$s_i(t) < b \Rightarrow \frac{ds_i}{dt} = b - s_i(t) = \frac{\sum_{j=1}^{N} \gamma_{ij}}{dt} \tag{1}$$

$$s_i(t) < a \Rightarrow \forall s_j \in \mathcal{N}(s_i) : \frac{ds_j}{dt} = \gamma_{ij} - \gamma_{ji}; \forall j \gamma_{ij} = 0, \gamma_{ji} = 0, \tag{2}$$

where $\mathcal{N}(s_i)$ is a neighbourhood of s_i node in terms of connection graph.

Therefore, each kind of node states has a corresponding type of related dynamics at different scales (Table 1).

Network formation process, initiated by equation (1), is determined by the considered node strategy. In random case, edges are distributed equiprobably. Counterparty choice strategies, corresponding to preferencial patterns, can be

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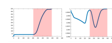
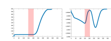
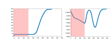
Table 1. Correspondence between dynamics and events at different scales. The rightest column show observable system states in the evolution trajectories, represented by number of nodes in a system and by entropy of graph Laplacian spectrum

	node-level	local formation pattern	network-level
critical	$(-\infty; a)$	del. node	
transitional	$[a; b)$	add edge	
stable	$[b; +\infty)$	—	

taken into account by implementing probability distribution over the all nodes for adjacency matrices of interaction. Here we suppose random connections, resulting in a network with Poisson degree distribution, as initial consideration and random choice strategy during simulation. Equation 2 can also be modified to consider rewiring process related to market clearing algorithm, nevertheless, this detail is out of this study consideration.

Co-evolution – feedback mechanisms. On one hand, changes in node states affect micro-scale dynamics, resulting in changes in network topology, on other, network topology contributes node states. In the system considered, each node state depends on adjacent edges and their attributes (related to $\Gamma(t)$), neighbouring nodes (system state $S(t)$), and external factors $g_i(t)$ affecting node i at time t:

$$s_i(t) = f(\Gamma(t),\ g_i(t)), \tag{3}$$

where $\Gamma(t) = \langle \{\gamma_{ij}(t)\}, V(t) = S(t) \rangle$ is a dynamic graph, reflecting the interactions between agents at the moment t. Then, following the components interplay (eq. 1), the change in node state per iteration can be rephrased as

$$\frac{ds_i}{dt} = \sum_{k=1}^{N} f\left(t, \frac{d\gamma_{ik}}{dt} - \frac{d\gamma_{ki}}{dt}, \frac{dS}{dt}\right) + g_i(t), \tag{4}$$

where f is the rule setting dependence of a node state on its neighborhood, and $g_i(t)$ is the aggregation of external effects on the system.

Let fix parameters: $N,\ a,\ b,\ \{g_k(t)\}$; $s_i \in \mathbb{R}$, a and b determines 3 types of states. Initial conditions are denoted as: $S(0) = \{s_k(0)\}$; $\Gamma(0) = \{\gamma_{ij}(0)\}$. Then, coming to iterative form and using conditions (1)–(4), we obtain eq. (5) and (6), determining dynamics in the system:

$$s_i(t+1) = s_i(t) \cdot \chi_{(-\infty;a)}(s_i(t)) + (b - s_i(t)) \cdot \chi_{[a;b)}(s_i(t)) - \tag{5}$$

$$-\chi_{[a;\infty)}(s_i(t)) \cdot \left[\sum_{s_j \in [a;b)} \frac{b - s_j(t)}{N} + \sum_{s_j < a} \big(\gamma_{ij}(t) - \gamma_{ji}(t)\big) - g_i(t+1) \right]$$

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$$\gamma_{ij}(t+1) = \gamma_{ij}(t) + \frac{b - s_i(t)}{N} \cdot \chi_{[a;b)}(s_i(t))\chi_{[a;+\infty)}(s_j(t)) - \tag{6}$$
$$-\gamma_{ij}(t)\left(1 - \chi_{[a;+\infty)}(s_i(t))\chi_{[a;+\infty)}(s_j(t))\right)$$

Since algorithms used for simulation are discrete, formulae contain indicator function $\chi_{set}(var)$, selecting addends depending on node state, which allows for consideration of node-level dynamics variations (from Table 1).

3.3 Regime durability estimation

Since the transitional phase is associated with the existence of node having $s_i \in [a;b)$, while others are $\geq b$, the length of first phase, i. e. expected time before start of transitional regime is estimated as minimal number of iterations before one of nodes reach b. That is $|\Phi I| = \left|\left\{ \min t : \exists i \in \mathbb{N}\ s_i(t) \in [a;b)\ \forall j \neq i\ s_j(t) \geq b\right\}\right|$. Similarly, $|\Phi II| = \left|\left\{ \min t : \exists i \in \mathbb{N}\ s_i(t) \in (-\infty;a)\ \forall j \neq i\ s_j(t) \geq a\right\}\right|$.

Consider a set of nodes $\mathbf{B} = \{b\}$. $\forall b \in \mathbf{B}$ with the corresponding state s_b fix $g_b(t)$. Say ea_b and el_b are external assets and liabilities, therefore they are related to node state, on one hand, and to external impact – on other. Let $\forall t > 0$ $g_b(t) = const > 0 \Rightarrow$

$$\forall t > 0 \quad \Delta|ea_b - el_b| = -g_b(t) \Leftrightarrow \sum_k \left[\Delta\gamma_{ik} - \Delta\gamma_{ki}\right] = -g_b(t) \tag{7}$$

Then, summarizing eq. 7 for the whole system:

$$\sum_{i=1}^{N}\sum_{k=1}^{N}\left[\Delta\gamma_{ik} - \Delta\gamma_{ki}\right] = \sum_{b\in\mathbf{B}} -g_b \tag{8}$$
$$\Leftrightarrow \quad 0 = \sum_{b\in\mathbf{B}} -g_b, \tag{9}$$

which is obvious. Then we simplify indicator function and further equations for the cases of ΦI and ΦII, summarise equation for all nodes, and modify sums in the consideration of Erdos-Renyi graph:

$$s_i(t+1) = s_i(0) + \sum_{k=1}^{t+1} g_i(t) - \sum_{k=1}^{t}\sum_{s_j(t)\in[a;b)} \frac{b - s_j(k)}{N} \tag{10}$$
$$\sum_{i=1} N s_i(t+1) = \sum_{i=1}^{N} s_i(0) + \sum_{i=1}^{N}\sum_{k=1}^{t+1} g_i(k) - \sum_{i=1}^{N}\sum_{k=1}^{t}\sum_{s_j(t)\in[a;b)} \frac{b - s_j(k)}{N} \tag{11}$$
$$\left[\sum_{i=1}^{N}\sum_{k=1}^{t+1} g_i(k) \approx N\cdot(t+1)\cdot g\right] \Rightarrow \tag{12}$$

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If we fix the number of initially stressed nodes as $\alpha_0 = \left|\left\{i | s_i \in [a;b)\right\}\right|$, then

$$|\Phi I| = \frac{s_i(0) - b}{g} \tag{13}$$

$$|\Phi II| \geq \frac{g + b - a}{g - \frac{\alpha_0}{N}(b - s(\bar{k}))} \tag{14}$$

4 Experiment results

4.1 Case study: Failures Prediction

For this experiment we made simulations of implemented system and check if our estimations of tipping points correspond to simulation results. We suppose random structure of interactions, s. t. initial network configuration is provided by Erdos-Renyi model, parametrised by 1000 nodes and 0.2 connection probability. Tipping points of node states are $a = -0.5$ and $b = 2.5$, and $\alpha = 0.5$. (Parameter values were chosen arbitrary and does not affect predictability.) Taking formulae (13) and (14) we obtain estimated number of iterations before starting point of an avalanche and its early warning signal, which is the point before transitional regime. In Figure 2 predicted values, evaluated by formulae (13) and (14) are displayed by vertical pink lines, which corresponds to dives in dynamics of node and edge counts. Right panel of Figure 2 is aimed at demonstrating inner processes, explaining the dynamics at left panel, and shows dynamics of nodes count being in transitional and critical states.

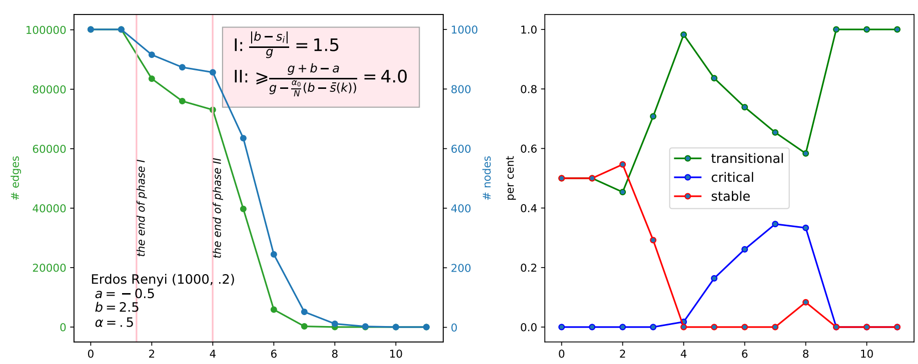

Fig. 2. Analitical estimations of transitional points and simulation results (on the **left**); dynamics of node count inside each category (on the **right**)

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Results demonstrate satisfactory prediction ability in the case of poisson network structure. The consideration of other topologies, formally, will require other approximation techniques for final estimation formulae. Nevertheless, for scenarios, when network structure plays minor role in contrast to external and node effects, this method will work.

4.2 Case study: funds infusion

The approach, provided in current paper, allows for explanation of dynamics of interbank network evolution, observed in simulations under several driving factors, like choice models, market clearing methods, and external effects [9]. Lines of different colours correspond to different combinations of counterparty choice models with choice models for external impacts (Fig. 3), and display the influence of funds infusion at different time moments.

The simulation scenario shows system evolution coming to cascading dynamics under different parameters and showing effects of funds infusion to the system. System state is observed with the number of removed nodes and with the entropy of Laplacian spectrum [27]. The left panel shows the begin of cascade when number of removed nodes increases sharply. Infusion stops this process temporally. At the same time, the right panel shows the critical regime, corresponding to decrease in entropy, has an interval of increasing entropy before it – this interval corresponds to transitional regime and show how number of edges change due to increasing number of stressed nodes. For this reason, fund infusion return nodes to prevailing stressed state from critical, and we see increase in entropy until some critical point.

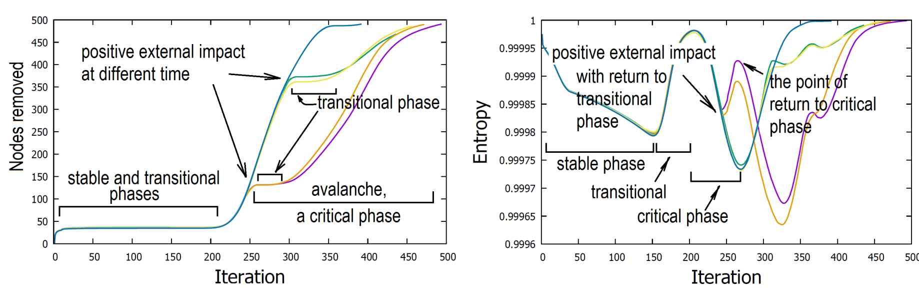

Fig. 3. Functional properties are often evaluated as a number of nodes. Transitions observed in functional properties (**left**) do not reflect hidden dynamics in topological properties (**right**)

These observations, in the combination with results of current paper, say that system has its capacity against external impacts. In this case, this combination is

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prevailing, so topology is not so significant. Infusion brings additional capacity to the system, allowing to avoid failures for a number of iterations when summarised external impact $\sum_{i=1}^{N} g_i(t)$ is fixed $\forall t$. In practice, this time can be appropriate for changes in managerial approaches or strategies, nevertheless, it is obvious, that in current case external impact must be balanced by other resources to provide stable system evolution.

5 Conclusion and future work

This study formally demonstrates, how micro-level dynamics of complex agent networks results in global patterns at system-level, in particular in the case of default cascades in interbank networks. This allows to see, how local effects are accumulated, and how this affects times between regimes of evolution. In addition, this gives a base for the exploration of which factors will have more influence and how to control it.

Cascading behaviour, observed via system state variables, is preceded by changes in inner dynamics, related to co-evolution of node states and structure of their interaction. This can be detected aforehand by means of topological features, like entropy of Laplacian spectrum, in the case of correspondence between node states and micro-level dynamics resulting in structural changes. In this way, tipping points are related to the share of nodes in each category. The consideration of real-valued state set for nodes, instead of discrete states, allows for consideration of system capacity against external impacts, which has a connection with lower levels. In the context of homogeneous structure of interactions, with no weak-connected components, the most effect is due to relation between external impact and overall systemic capacity, opposing to it.

The considered agent-based network evolves under factors, comprising node states dynamics, network topology, and external effects. These factors are considered in the estimations of the number of iterations before starting points of critical and transitional regimes. The regimes are introduced to distinguish cascading dynamics (critical regime) and early-warning tipping point, associated with the start of transitional regime. In addition, the above mentioned regimes are associated with real-valued node states, broken into three semi-intervals and triggering corresponding types of local dynamics. Therefore, we show the correspondence between dynamics at different scales and present formal model, providing inter-scale connection and prediction of tipping points.

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