

Stochastic volatility and early warning indicator

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Abstract. We extend Merton’s framework by adopting stochastic volatility to propose an early warning indicator for banks’ credit risk. Bayesian inference is employed to estimate the parameters of Heston model. We provide empirical evidence and demonstrate the comparative strength of our risk measure over others.

Keywords: Early warning indicator · Credit risk · Heston model · Bayesian inference.

1 Introduction

Monitoring the risk borne by the banking sector and detecting early warning signals were propelled to the forefront of regulation after the catastrophic financial crisis of 2007–08 much of which was attributable to banks such as Lehman Brothers. In finance, Merton’s probability of default (PoD) is regarded as an informative and reliable credit risk measure [1].⁴ Under this model, the firm’s equity is considered as a call option with a strike price equal to the face value of the firm’s debt [1,2,3,4]. Merton model assumes that the value of the firm’s assets follows a lognormal diffusion process that has a constant volatility; however, it is restricted in terms of being able to adequately describe the real world [5].

Hence, we adopt the concept of time-varying volatility, specifically Heston model [6,7,8,9,10] in which volatility is driven by its own mean-reverting stochastic process where log-returns of an asset exhibit heavy tails. Bu and Liao employed stochastic volatility and jumps to explain the time variation in credit default swaps, a proxy for credit risk [11]. Fulop and Li suggested a simulation-based parameter learning methodology to estimate parameters, and applied their approach to stochastic volatility and jump models [12]. Based on these studies,

⁴ Network approach is also applied to assess the credit risk of banks. Angelini et al. and Khashman employed neural network using the real-world credit approval data of Italy and Germany to evaluate banks’ credit risk [13,14]. González-Avella et al. adopted network topology, i.e., loans are interpreted as links between banks (nodes), to examine the interbank credit risk with financial contagion [15].

we propose an indicator that delivers early warning signals for banks' credit risk, and compare the performance of our measure with others.

This paper is organized as follows. In the second section, we adopt stochastic volatility to probability of undercapitalization (PoU) and propose an early warning indicator. The third section explains the parameter estimation strategy, and we discuss the application of our risk indicator for two US banks in the fourth section. Finally, the fifth section concludes.

2 Stochastic volatility and the effect of capital buffer

Among the pool of stochastic volatility models, we pick out Heston model due to its semi-closed form solution and realistic assumptions such as mean-reversion of variance and statistical dependence between an asset and its volatility. The value of a firm at time t , V_t , is assumed to evolve with a stochastic variance, σ_t^2 , that follows a Cox, Ingersoll and Ross process [16,17]

$$dV_t = \mu V_t dt + \sigma_t V_t dW_{1,t} \quad \text{and} \quad d\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + \sigma_v \sigma_t^2 dW_{2,t},$$

where μ is the growth rate of firm value, κ is the mean reversion speed for the variance, θ is the mean reversion level for the variance, σ_v is the volatility of the variance, and $W_{i,t}$ (for $i = 1, 2$) is a standard Brownian motion. The Feller condition, $2\kappa\theta > \sigma_v^2$, is imposed to ensure that the variance is strictly positive [18]. It is further assumed that the asset value and its variance are driven by a correlated stochastic component of $d\langle W_{1,t}, W_{2,t} \rangle = \rho dt$. When the asset return and the variance are positively correlated ($\rho > 0$), the distribution of return has a fat right tail [7].

A firm's asset consists of equity and debt. In particular, a bank's equity is considered as an European call option with a strike price equal to the obligated debt payment L at the maturity T as $E_T = \max\{V_T - L, 0\}$. Thus, the calculation of PoD with Heston model is as follows. For the simplicity of notation, all subscripts are suppressed, and the proof is provided in the Appendix.

Proposition: *Let $x_t = \log V_t$ and $v_t = \sigma_t^2$. PoD admits a semi-analytical expression*

$$PoD = P(V_T \leq L) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{e^{-iu \log L} \varphi(u; 0, x(0), v(0))}{iu} \right] du,$$

where $\varphi(u; t, x, v)$ takes an exponential linear form as

$$\varphi(u; t, x, v) = \exp(A(T-t, u) + B(T-t, u)v + iux) \quad \text{for} \quad 0 \leq t \leq T,$$

$$A(t, u) = \left(i\mu u - \frac{\kappa\theta\lambda_2}{a} \right) t + \frac{\kappa\theta}{a} \log \frac{1-l}{1-le^{dt}} \quad \text{and} \quad B(t, u) = -\frac{\lambda_2}{a} \cdot \frac{1-e^{dt}}{1-le^{dt}}.$$

The terms are defined as

$$\begin{aligned} a &= \frac{1}{2}\sigma_v^2, & b &= iu\sigma_v\rho - \kappa, & c &= -\frac{1}{2}(u^2 + iu), \\ d &= \sqrt{b^2 - 4ac}, & \lambda_2 &= \frac{b-d}{2} & \text{and} & l = \frac{b-d}{b+d}. \end{aligned}$$

Unlike general firms, banks are subject to capital adequacy requirement, however PoD simply focuses on a firm's debt-paying ability. Hence, Chan-Lau and Sy proposed distance-to-capital (DC) to address banks' undercapitalization risk [19]. PoU and DC are considered as more conservative measures than PoD and distance-to-default [20,21]. A bank is regarded as undercapitalized once $V_T - L < c \cdot V_T$ holds at time T after debt payment and PoU of a bank can be computed as

$$\text{PoU} = P\left(V_T < \frac{L}{1-c}\right) = \text{PoD} + P\left(L \leq V_T < \frac{L}{1-c}\right).$$

We further propose an early warning indicator, namely the effect of capital buffer (ECB). When bank failure looms, the elevated possibility of insolvency risk eats up the capital buffer, and the regulation on a bank's capital plays a lesser role in governing risk. Hence, the ECB drops to small numbers, which can be interpreted as warning signals,

$$\text{ECB} = \frac{\text{PoU} - \text{PoD}}{\text{PoU}}.$$

3 Estimation strategy

A firm's value and its variance are not directly observable, thus, we need to estimate these variables from equity prices. However, the observed equity prices may be contaminated by the microstructure of noise [22]

$$\log S_t = \log \hat{S}_t(V_t, \sigma_t^2) + \delta \nu_t, \quad (1)$$

where ν_t is *i.i.d.* standard normal random variable. Thus, the fundamental component of equity price is a function of V_t and σ_t^2 [7]

$$\begin{aligned} \hat{S}_t(V_t, \sigma_t^2) &= V_t P_1 - L e^{-r(T-t)} P_2, \\ P_j &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{\exp(-iuL)}{iu} \times \exp(C_j + D_j \sigma_t^2 + iu \ln V_t) \right] du, \end{aligned}$$

where $u \in \mathcal{R}$ is the characteristic index, and C_j and D_j are known functions of the model parameters for $j = 1, 2$.

The estimation can be simplified as the input of observed equity prices $y_{1:t} = \{\log S_1, \dots, \log S_t\}$, the output of a parameter set $\Theta = \{\mu, \theta, \kappa, \sigma_v, \rho, \delta\}$, and the latent states $x_{1:t} = \{(V_1, \sigma_1^2), \dots, (V_t, \sigma_t^2)\}$. Then, we apply the sequential Bayesian inference to estimate the parameters and hidden states [12], hence, our objective is to find the joint posterior distribution $p(x_t, \Theta | y_{1:t})$ of states and parameters at each time t . Since there is no analytical solution of the joint posterior distribution, we need to draw samples from this distribution. The underlying idea of sampling is to break up the interdependence of hidden states and fixed parameters

$$p(x_t, \Theta | y_{1:t}) = p(x_t | y_{1:t}, \Theta) p(\Theta | y_{1:t}).$$

Thus, the procedure of sampling from the posterior distribution can be divided into: (i) state filtering $p(x_t|y_{1:t}, \Theta)$; and (ii) parameter learning $p(\Theta|y_{1:t})$. State filtering estimates the probability of latent state variables for a given static parameter set, and we can derive the recursion of the filtering density (the parameter set Θ is suppressed in this step)

$$p(x_t|y_{1:t}) \propto p(y_t|x_t) \int p(x_t|x_{t-1})p(x_{t-1}|y_{1:t-1})dx_{t-1},$$

where $p(x_t|y_{1:t-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|y_{1:t-1})dx_{t-1}$. Suppose that we have a weighted sample to represent the target distribution $p(x_{t-1}|y_{1:t-1})$ at time $t-1$, i.e., $\{(x_{t-1}^{(i)}, \omega_{t-1}^{(i)}), i = 1, \dots, M\}$, where $\omega_{t-1}^{(i)} \doteq p(y_{t-1}|x_{t-1}^{(i)})$. Then, a new weighted sample $\{(x_t^{(i)}, \omega_t^{(i)}), i = 1, \dots, M\} \sim p(x_t|y_{1:t})$ can be drawn by a recursive approach: (i) obtain a new sample $x_t^{(i)}$ from $p(x_t|x_{t-1}^{(i)})$; and (ii) assign a weight $\omega_t^{(i)} = p(y_t|x_t^{(i)})$ for each $x_t^{(i)}$.

Parameter learning evaluates the probability of parameter set Θ for the given observed equity prices. For each parameter particle $\{\Theta^{(j)}, j = 1, \dots, N\}$, the likelihood is

$$\hat{p}(y_{1:t}|\Theta) = \prod_{l=2}^t p(y_l|y_{1:l-1}, \Theta)p(y_1|\Theta), \quad (2)$$

where $p(y_l|y_{1:l-1}, \Theta) = \int p(y_l|x_l, \Theta)p(x_l|y_{1:l-1}, \Theta)dx_l$. From state filtering, we already have a sample of $\{x_l^{(i)}, i = 1, \dots, M\} \sim p(x_l|y_{1:l-1}, \Theta)$, so

$$\hat{p}(y_l|y_{1:l-1}, \Theta) = \frac{1}{M} \sum_{i=1}^M p(y_l|x_l^{(i)}, \Theta), \quad (3)$$

then, we can calculate the posterior distribution of Θ

$$p(\Theta|y_{1:t}) = p(y_{1:t}|\Theta)p(\Theta). \quad (4)$$

The transition density $p(x_t|x_{t-1})$ and the likelihood of measurement $p(y_t|x_t)$ are necessary for both state filtering and parameter learning. The transition law is determined by

$$\log V_{t+\tau} = \log V_t + (\mu - \frac{1}{2}\sigma_t^2)\tau + \sigma_t\sqrt{\tau}\varepsilon_{1,t}, \quad (5)$$

$$\sigma_{t+\tau}^2 = \sigma_t^2 + \kappa(\theta - \sigma_t^2)\tau + \sigma_v\sigma_t\sqrt{\tau}\varepsilon_{2,t}, \quad (6)$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ follow $N(0, 1)$ with correlation ρ , and τ is the time interval of one period. The likelihood of measurement y_t is determined by Equation (1).

The process of obtaining the posterior distribution from the real data is as follows. Assume that we have $\{(V_{t-1}^{(i)}, \sigma_{t-1}^{2(i)}); i = 1, \dots, M\} \sim p(x_{t-1}|y_{t-1})$, then we can obtain $p(x_t|y_t)$ from the following steps: (i) draw the next volatility $\sigma_t^{2(i)}$ from $p(\sigma_t^2|\sigma_{t-1}^{2(i)})$ in Equation (6) and error term $\nu_t^{(i)} \sim N(0, 1)$; and (ii) solve the equation $S_t e^{-\delta\nu_t^{(i)}} = \hat{S}_t(V_t^{(i)}, \sigma_t^{2(i)}|P_{1,t-1}, P_{2,t-1})$ for $V_t^{(i)}$, which is a rearrangement

of Equation (1). To solve V_t , Duan and Fulop found an approximate inversion, which can estimate the solution without solving nonlinear equations [22].

Using the above sample, we can obtain the posterior density of measurement from the following steps: (i) calculate $P_{1,t}$ and $P_{2,t}$ based on sample $\sigma_t^{2(i)}$ and $V_t^{(i)}$; (ii) evaluate $\hat{S}_t(V_t^{(i)}, \sigma_t^{2(i)} | P_{1,t}, P_{2,t})$; (iii) calculate the probability density $p(y_t | x_t)$, which follows a normal distribution, i.e., $\log S_t \sim N(\log \hat{S}_t, \delta^2)$, taken from Equation (1); and (iv) calculate the posterior density of measurement based on Equations (2)–(4).

4 Application to banks

To demonstrate the performance of the ECB, we apply it to Lehman Brothers and Bank of America for the period between 1 April 2006 and 29 August 2008. The capital adequacy ratio c is set as 6.25% for investment banks following the capital rules applied by the Securities and Exchange Commission, and 4% for commercial banks, which is the tier 1 capital adequacy ratio in the Basel Accords. We assume that banks' capital completely consists of equity. Both PoD and PoU show similar movements as displayed in figures 1 and 2, and these measures deliver warnings prior to the bankruptcy of Lehman Brothers and the bailout to Bank of America. In reality, the US government provided 25 and 20 billion USD on October 2008 and January 2009, through Troubled Asset Relief Program (preferred stock purchase) to Bank of America. The gap between PoU and PoD indicates the capital buffer (effect of capital adequacy requirement).

Moreover, the shareholders of a bank are considered to be offered put options on the bank's assets through the bank safety net since the depositors' repayment is guaranteed in case of bank run through deposit insurance scheme, which is provided by the Federal Deposit Insurance Corporation in the US. Thus, it is suggested that shareholders had exploited the bank safety net prior to crises through various risk-taking activities [23,24], leading to increases in put value, which can be another early warning indicator. The put value can be calculated from the contingent claim model

$$\hat{S}'_t(V_t, \sigma_t^2) = V_t(P_1 - 1) - L_t e^{-r(T-t)}(P_2 - 1).$$

In the case of Lehman Brothers, the ECB gave an early warning signal in mid-2007, approximately a year earlier than the put value. For Bank of America, the ECB started to decline from the end of 2007, delivering a warning signal in mid-2008, however the put value failed to deliver any warnings. Put differently, the put value of bank safety net is insufficient as an early warning indicator unlike the ECB.

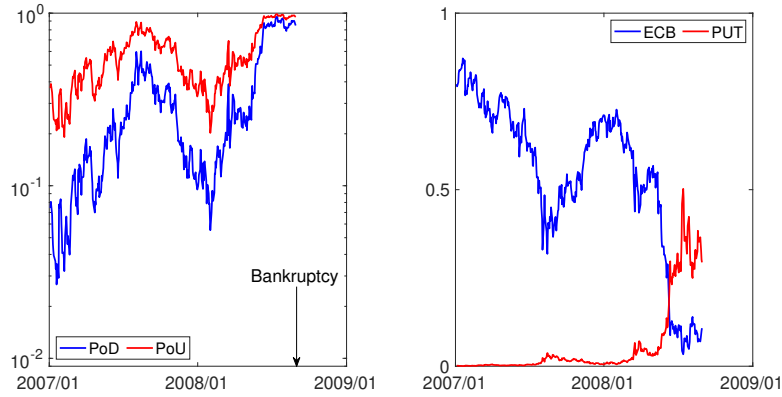


Fig. 1. Credit risk and early warning indicator (Lehman Brothers)

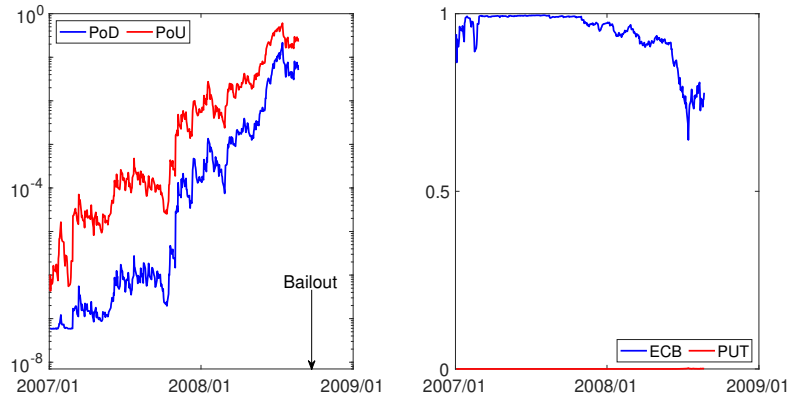


Fig. 2. Credit risk and early warning indicator (Bank of America)

5 Concluding remarks

We extend the Merton model by incorporating stochastic volatility and the concept of undercapitalization to evaluate credit risk of banks in a more realistic manner. We elect Heston model, in which asset return distribution exhibits non-lognormal properties such as heavy tails. We employ Bayesian inference to estimate parameters. Then, capital adequacy requirement is adopted to better illustrate banks' credit risk, and we further propose an early warning indicator, namely the ECB. The application of the ECB to Lehman Brothers and Bank of America demonstrates the comparative strength of our early warning indicator compared to the put option value of the bank safety net.

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Appendix. Proof of the proposition

According to Ito's formula, the dynamics of $x(t)$ are given by

$$dx(t) = \left(\mu - \frac{1}{2}v(t) \right) dt + \sqrt{v(t)}dW_V(t).$$

Let $\varphi(u; t, x, v) = E[e^{iux(T)} | x(t) = x, v(t) = v]$. Then, by Gil-Pelaez inversion formula [25], we have

$$\begin{aligned} P(V(T) < L) &= P(x(T) < \log L) \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left[\frac{e^{-iu \log L} \varphi(u; t, x(0), v(0))}{iu} \right] du. \end{aligned}$$

By Feynman-Kač theorem, φ solves the following boundary value problem

$$\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial x} \left(\mu - \frac{1}{2}v \right) + \frac{\partial \varphi}{\partial v} \kappa(\theta - v) + \frac{1}{2} \frac{\partial^2 \varphi}{\partial x^2} v + \frac{1}{2} \frac{\partial^2 \varphi}{\partial v^2} \sigma_v^2 v + \sigma_v v \rho \frac{\partial^2 \varphi}{\partial x \partial v} = 0,$$

$$\varphi(u; T, x, v) = e^{iux}.$$

Following the guess by Heston [7], we assume that φ takes an exponential linear form

$$\varphi(u; t, x, v) = \exp(A(T-t, u) + B(T-t, u)v + iux).$$

Because of $\varphi(u; T, x, v) = e^{iux}$ for any x and v , we have boundary conditions for A and B as

$$A(0, u) = B(0, u) = 0.$$

Denoting $\tau = T - t$ and plugging the "guessed" form into a partial differential equation, we get

$$-\left(\frac{\partial A}{\partial \tau} + \frac{\partial B}{\partial \tau} v \right) + iu \left(\mu - \frac{1}{2}v \right) + B\kappa(\theta - v) - \frac{1}{2}u^2 v + \frac{1}{2}B^2 \sigma_v^2 v + iu\sigma_v v \rho B = 0.$$

As this holds for any v , we get the following two ODEs

$$\frac{\partial A}{\partial \tau} = i\mu u + B\kappa\theta,$$

$$\frac{\partial B}{\partial \tau} = -\frac{1}{2}iu - B\kappa - \frac{1}{2}u^2 + \frac{1}{2}B^2 \sigma_v^2 + iu\sigma_v \rho B.$$

The ODE for B takes the form of Riccati equation:

$$\frac{\partial B}{\partial \tau} = \frac{1}{2}\sigma_v^2 B^2 + (iu\sigma_v \rho - \kappa)B - \frac{1}{2}(iu + u^2) \equiv aB^2 + bB + c,$$

where

$$a = \frac{1}{2}\sigma_v^2, \quad b = iu\sigma_v\rho - \kappa \quad \text{and} \quad c = -\frac{1}{2}(u^2 + iu).$$

According to the solution of Riccati equation, the solution to the ODE for B is given by $B = -\frac{h'}{ah}$, where $h(\tau)$ solves the following ODE

$$h'' - bh' + ach = 0.$$

Denote $d = \sqrt{b^2 - 4ac}$, then h takes the form

$$h(\tau) = D_1e^{\lambda_1\tau} + D_2e^{\lambda_2\tau},$$

where $\lambda_1 = \frac{b+d}{2}$ and $\lambda_2 = \frac{b-d}{2}$. Letting $H = \frac{D_1}{D_2}$ and plugging h into B , we get

$$B(\tau, u) = -\frac{\lambda_1 D_1 e^{\lambda_1 \tau} + \lambda_2 D_2 e^{\lambda_2 \tau}}{a(D_1 e^{\lambda_1 \tau} + D_2 e^{\lambda_2 \tau})} = -\frac{\lambda_1 H e^{\lambda_1 \tau} + \lambda_2 e^{\lambda_2 \tau}}{a(H e^{\lambda_1 \tau} + e^{\lambda_2 \tau})}.$$

Recall the boundary condition, $B(0, u) = 0$. It follows immediately $-H = \frac{\lambda_2}{\lambda_1} \triangleq l$. Thus, we further have

$$B(\tau, u) = -\frac{-\lambda_2 e^{\lambda_1 \tau} + \lambda_2 e^{\lambda_2 \tau}}{a\left(-\frac{\lambda_2}{\lambda_1} e^{\lambda_1 \tau} + e^{\lambda_2 \tau}\right)} = -\frac{\lambda_2}{a} \cdot \frac{1 - e^{d\tau}}{1 - l e^{d\tau}}.$$

To solve A , note that the indefinite integral is

$$\int B(\tau, u) d\tau = -\frac{\lambda_2}{a} \int \frac{1 - e^{d\tau}}{1 - l e^{d\tau}} d\tau = -\frac{\lambda_2 \tau}{a} + \frac{\lambda_2}{ad} \left(1 - \frac{1}{l}\right) \log(1 - l e^{d\tau}) + \text{const.}$$

Hence,

$$A(\tau, u) = i\mu u \tau - \frac{\kappa \theta \lambda_2}{a} \tau + \frac{\kappa \theta \lambda_2}{ad} \left(1 - \frac{1}{l}\right) \log(1 - l e^{d\tau}) + \text{const.}$$

Recall the boundary condition $A(0, u) = 0$. Thus we can solve for the constant term and further simplify the expression to

$$A(\tau, u) = \left(i\mu u - \frac{\kappa \theta \lambda_2}{a}\right) \tau + \frac{\kappa \theta}{a} \log\left(\frac{1 - l}{1 - l e^{d\tau}}\right),$$

and the proof is complete.

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