Rapid Multi-Band Patch Antenna Yield Estimation Using Polynomial Chaos-Kriging

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Abstract. Yield estimation of antenna systems is important to check their robustness with respect to the uncertain sources. Since the Monte Carlo sampling-based real physics simulation model evaluations are computationally intensive, this work proposes the polynomial chaos-Kriging (PC-Kriging) metamodeling technique for fast yield estimation. PC-Kriging integrates the polynomial chaos expansion (PCE) as the trend function of Kriging metamodel since the PCE is good at capturing the function tendency and Kriging is good at matching the observations at training points. The PC-Kriging is demonstrated with an analytical case and a multi-band patch antenna case and compared with direct PCE and Kriging metamodels. In the analytical case, PC-Kriging reduces the computational cost by around 42% compared with PCE and over 94% compared with Kriging. In the antenna case, PC-Kriging reduces the computational cost by over 60% compared with Kriging and over 90% compared with PCE. In both cases, the savings are obtained without compromising the accuracy.

Keywords: Yield estimation, microstrip multi-band patch antenna, Monte Carlo sampling, PCE, Kriging, PC-Kriging.

1 Introduction

Yield is the metric for checking the reliability of antenna system with respect to the uncertainties due to the manufacturing process [1, 2]. In particular, yield is the percentage of designs satisfying the design specifications. The process of yield estimation can be completed by running arbitrary number of high-fidelity simulation models [1], such as full-wave electromagnetic (EM) model [3], using Monte Carlo sampling (MCS) [4]. The high-fidelity physics model evaluations are typically time-consuming, making the MCS-based yield estimation computationally prohibitive.

Metamodeling techniques [5, 6] are widely used to alleviate the computational burden. There are generally two types of metamodels, data-fit metamodels [7] and multi-fidelity metamodels [8]. Data-fit metmodels utilize the high-fidelity physics-based simulation model evaluations as training points, while the multi-fidelity metamodels can make use of physics-based simulation models of varying degree of accuracy. Multi-

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fidelity metamodels can be efficient when fast and good low-fidelity models are available. Data-fit metamodeling is more versatile because only one level of simulation model is needed.

Advanced data-fit metamodels have been successfully used for antenna system modeling and design at reduced computational costs. Rama Sanjeeva Reddy et al. [9] introduced the radial basis function neural network into design of multiple function antenna arrays and obtained a success rate as high as 98%. Koziel et al. [10] constructed the fast data-fit Kriging metamodel as part of multi-objective design optimization of antennas handling arbitrary number of objective functions. Du et al. [11] introduced the PCE method for statistical metamodeling of the far field radiated by antennas undergoing random disturbances and validated the PCE model with a deformable canonical antenna.

This work introduces the PC-Kriging metamodel [12] for the yield estimation of multi-band patch antenna systems. PCE [13] is well-known for capturing the tendency of the objective function, whereas Kriging [14] handles the observation values at training points well. The PC-Kriging technique aims at integrating the advantages of both metamodeling methods expecting fewer training points required for constructing a reliable and fast model in lieu of the computationally expensive high-fidelity simulation model. This work demonstrates the PC-Kriging technique for the yield estimation of a multi-band patch antenna case.

The remainder part of this paper is organized as follows. Section 2 provides the details formulating the yield estimation of antenna. Section 3 describes the metamodeling methodologies, including Kriging, PCE and PC-Kriging, utilized in this work. Then all metamodeling techniques are demonstrated and compared on numerical examples in Section 4. This papers ends with conclusion in Section 5.

2 Yield Estimation of Antennas

Let the antenna response of interest, evaluated using EM simulation models, be denoted by $\mathbf{R}(\mathbf{x})$, and $\mathbf{x}$ is the vector containing deterministic/uncertain design parameters. Let $\mathbf{x}^0$ represent the nominal design under ideal conditions. Let $d\mathbf{x}$ be the disturbance due to the manufacturing tolerances or uncertainties existing in the antenna system, and can be sample using pre-define empirical probabilistic distributions. Therefore, the actual designs taking the tolerances and uncertainties under consideration can be represented as $\mathbf{x}^0 + d\mathbf{x}$.

Now a counting function $H(\mathbf{x})$ can be set up as [2]

$$H(\mathbf{x}) = \begin{cases} 1, & \text{if } \mathbf{R}(\mathbf{x}) \text{ satisfied the design specifications} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Then the yield at the nominal design introduced above, i.e., the percentage of satifying designs out of the total designs, can be given as

$$Y(\mathbf{x}^0) = [\sum_{j=1}^{N} H(\mathbf{x}^0 + d\mathbf{x}^j)] / N, \tag{2}$$

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where $d\mathbf{x}^j$, $j = 1, 2, \ldots, N$, are the disturbances with pre-assigned empirical probabilistic distributions as introduced above.

3 Methods

This section describes the mathematical details of formulating the metamodeling methods, including Kriging, PCE and PC-Kriging. This work considers the response feature approach proposed by Koziel et al. [2], which can reduce the complexity of problem constructing metamodel for response of interest at specific frequencies rather than modeling the whole signal.

3.1 Kriging

Kriging metamodeling technique is a type of Gaussian process regression, which takes the training points as the realization of the unknown random process. The generalized Kriging formulation [14] is the sum of a trend function $\mathbf{f}^{\mathrm{T}}(\mathbf{x})\boldsymbol{\beta}$ and a Gaussian deviation term $Z(\mathbf{x})$ as follows

$$M^{Kr}(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\boldsymbol{\beta} + Z(\mathbf{x}), \tag{3}$$

where $\mathbf{f}(\mathbf{x}) = [f_0(\mathbf{x}), \ldots, f_{p\text{-}1}(\mathbf{x})]^T \in \mathbb{R}^p$ is defined with a set of the regression basis functions, $\boldsymbol{\beta} = [\beta_0(\mathbf{x}), \ldots, \beta_{p\text{-}1}(\mathbf{x})]^T \in \mathbb{R}^p$ denotes the vector of the corresponding coefficients, and $Z(\mathbf{x})$ denotes a stationary random process with zero mean, variance and nonzero covariance. In this work, Gaussian exponential correlation function is adopted with the form

$$R[\mathbf{x}, \mathbf{x}'] = \sigma^2 \exp\left[-\sum_{k=1}^{m} \theta_k \left|x_k - x_k'\right|^2\right], \tag{4}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_m]^T$ denotes the vectors of unknown hyperparameters to be tuned. The Kriging predictor for any untried $\mathbf{x}$ can be written as

$$M^{Kr}(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{M}_S - \mathbf{F}\hat{\boldsymbol{\beta}}), \tag{5}$$

where a linear trend function $\mathbf{f} = [1, x_1, x_2, \ldots, x_{\mathrm{m}}]^{\mathrm{T}}$ is used in this work, $F_{ij} = f_j(x_i)$ where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N+1$, N is the total number of training points, $\hat{\boldsymbol{\beta}}$ comes from generalized least squares estimation, $\mathbf{r}$ is the correlation vector between the point to be predicted ($\mathbf{x}_{\mathrm{pred}}$) and training set points, here $r_i = R(\mathbf{x}_{\mathrm{pred}}, \mathbf{x}_i; \boldsymbol{\theta})$, $\mathbf{R}$ is the correlation matrix among training points with $R_{ik} = R(\mathbf{x}_{\mathrm{i}}, \mathbf{x}_{\mathrm{k}}; \boldsymbol{\theta})$ where $i, k = 1, 2, \ldots, N$, $\mathbf{M}_{\mathrm{S}}$ is the model response of the training points. $\boldsymbol{\beta}$ and σ^2 are given by

$$\boldsymbol{\beta} = (\mathbf{F}^T\mathbf{R}^{-1}\mathbf{F})^{-1}\mathbf{G}^T\mathbf{R}^{-1}\mathbf{M}_S, \tag{6}$$

and

$$\sigma^2 = 1/N(\mathbf{M}_S - \mathbf{G}\boldsymbol{\beta})^T\mathbf{R}^{-1}(\mathbf{M}_S - \mathbf{G}\boldsymbol{\beta}). \tag{7}$$

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The maximum likelihood estimation on $\boldsymbol{\theta}$ is found by solving

$$\boldsymbol{\theta} = \arg\min\left(\frac{1}{2}\log(\det(R)) + \frac{N}{2}\log(2\pi\sigma^2) + N/2\right). \tag{8}$$

3.2 Polynomial Chaos Expansion

PCE has the generalized formulation as follows [13]

$$M^{PC}(\mathbf{x}) = \sum_{i=1}^{\infty} \alpha_i \boldsymbol{\Psi}_i(\mathbf{x}), \tag{9}$$

where $\mathbf{x} \in \mathbb{R}^m$ is a vector with random independent components described by a probability density function f_X, $M^{PC}(\mathbf{x})$ is a map of $\mathbf{x}$, i is the index of ith polynomial term, $\boldsymbol{\Psi}_i$ are multivariate polynomial basis functions, whereas α_i are their corresponding expansion coefficient. In practice, a truncated form of the PCE is used

$$M^{PC}(\mathbf{x}) = \sum_{i=1}^{P} \alpha_i \boldsymbol{\Psi}_i(\mathbf{x}), \tag{10}$$

where $M^{PC}(\mathbf{x})$ is the approximate truncated PCE model, and P is the total number of sample points, which can be calculated as

$$P = \frac{(p+n)!}{p!n!}, \tag{11}$$

where p is the order of the PCE, and n is the total number of random input variables. The coefficient vector $\boldsymbol{\alpha}$ is found by solving a least-squares minimization problem

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \Psi(\mathbf{x}) - M(\mathbf{x})]. \tag{12}$$

In this work, the least-angle regression (LARS) method is used to solve (12) by adding an L_1 penalty term

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \Psi(\mathbf{x}) - M(\mathbf{x})] + \lambda \|\boldsymbol{\alpha}\|_1, \tag{13}$$

where λ is a penalty factor, $\|\boldsymbol{\alpha}\|_1$ is the L_1 norm of the coefficients of PCE.

3.3 Polynomial Chaos-Kriging

PC-Kriging [12] is a recently developed class of metamodels that integrates the PCE and Kriging metamodels. In particular, PCE is utilized as the trend function for the Kriging metamodel. The modeling flow is as follows:

1. Obtain observations (training points) from the physics-based simulation model.
2. Generate a PCE model following Section 3.2.
3. In Step 2, LARS technique selects the “important” basis terms, meaning those most correlated with the model response.

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4. Plug those “important” basis terms into (5), then construct the Kriging model.

4 Numerical Examples

The proposed PC-Kriging metamodeling technique is demonstrated on two numerical examples in this section. The first example is the modeling of a short column function which was first utilized by Eldred et al. [15] for demonstrating uncertainty quantification. The second example is a multi-band patch antenna system which has normal distributions of zero mean and standard deviation of 0.08 mm modeling the disturbances [2].

4.1 Short Column Function

The short column function [15] models a structural column with uncertainties due to the material properties. The function is given as

$$f(\mathbf{x}) = 1 - \frac{4M}{bh^2Y} - \frac{P^2}{b^2h^2Y^2}, \tag{14}$$

where b is the width of the cross section and equals 5 mm, h is the depth of the cross section and equals 15 mm, Y, M and P are the uncertain parameters in this case and $Y \sim$ Lognormal(5, 0.5) MPa is the yield stress, $M \sim$ Normal(2,000, 400) MNm is the bending moment, and $P \sim$ Normal(500, 100) MPa is the axial force.

In this case, we set up the 1% of standard deviation (σ) of the testing points as the accepted root mean squared error (RMSE). Figure 1 shows the plot of the RMSE of all three metamodeling techniques versus the number of training points. The plot shows that all metamodeling approaches can reduce the RMSE when increasing the total number of training points. The Kriging, PCE and PC-Kriging metamodels, however, need different number of samples to reach the 1%σ_{testing} accuracy. In particular, Kriging needs around 1,200 training points and PCE around 120 training points, whe-

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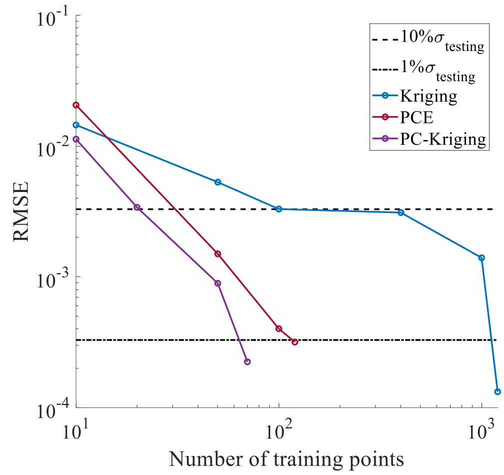

Fig. 1. Metamodeling accuracy versus the computational cost.

reas PC-Kriging requires only around 70 training points. Thus, PC-Kriging needs around 42% fewer samples than PCE and around 94% fewer than Kriging. In this case, the PC-Kriging metamodel at each number of training points utilizes a 14th degree of the PCE as the trend function.

4.2 Multi-Band Patch Antenna

The geometry of the microstrip dual-band patch antenna utilized in this work is given in Fig. 2. The antenna is implemented on a 0.762 mm thick Taconic RF-35 dielectric substrate ($\varepsilon_r = 3.5$). The independent geometry parameters are $\mathbf{x} = [L\ l_1\ l_2\ l_3\ l_4\ W\ w_1\ w_2\ g]^T$. The EM model **R** is implemented in CST [1, 2]. The nominal design, corresponding to the antenna resonances allocated at the frequencies 2.4 GHz and 5.8 GHz, is $\mathbf{x}^0$ = [14.18 3.47 12.44 5.06 15.56 0.65 8.29 5.60]^T (all dimensions in mm).

The antenna yield is estimated for the following specs: $|S_{11}| \leq -10$ dB for both 2.4 GHz and 5.8 GHz. It is assumed that Gaussian distribution of the geometry deviation vector $d\mathbf{x}$ has a zero mean and a standard variance of 0.08 mm. The parametric study on the convergence of the yield value versus the number of training points is shown in Fig. 3. The PCE, Kriging and PC-Kriging metamodeling approaches are compared with the direct Monte Carlo sampling technique involving 500 EM evaluations of **R**.

As shown in Table 1, to reach satisfactory yield estimations, the PCE and Kriging require around 200 and 50 training points, respectively. The proposed PC-Kriging requires only 20 training points. Thus, in this case the PC-Kriging needs over 90% fewer samples than PCE and more than 60% fewer samples than Kriging.

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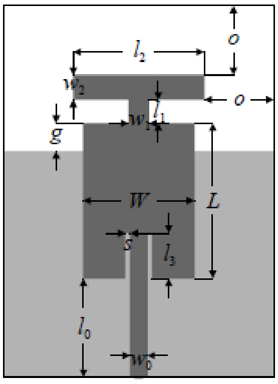

Fig. 2. Geometry of the dual-band patch antenna.

Table 1. Computational cost for satisfactory yield estimation of the multi-band patch antenna.

Geometry	Methodology	Yield Estimation	Number of Samples
	EM Model	0.490	500
Gaussian	PCE	0.580	200
σ = 0.08 mm	Kriging	0.532	50
	PC-Kriging	0.528	20

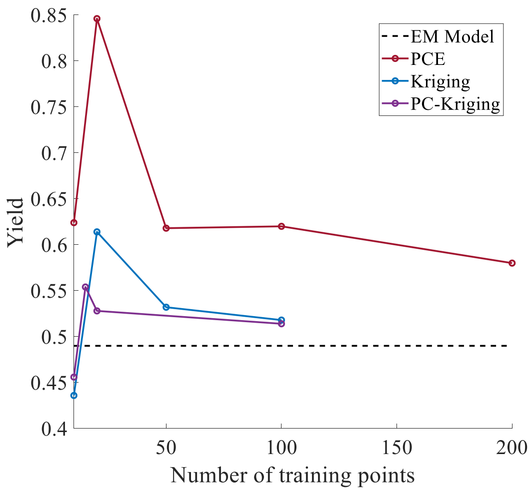

Fig. 3. Convergence of yield estimation as a function of the number of training points for the considered metamodeling techniques as well as direct EM-based Monte Carlo simulation.

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5 Conclusion

The PC-Kriging metamodeling technique has been proposed for rapid multi-band patch antenna yield estimation. PC-Kriging aims at combining the advantages of both PCE and Kriging metamodels for a further reduction on the computational cost. The results of multi-band patch antenna yield estimation show that PC-Kriging can be used to estimate the yield at a significantly lower computational cost than using Kriging or PCE. Further studies are needed to fully determine how the well proposed approach works. Future work will also consider problems of higher complexity.

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