Explicit Size-Reduction-Oriented Design of a Compact Microstrip Rat-Race Coupler Using Surrogate-Based Optimization Methods

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Abstract. In this paper, an explicit size reduction of a compact rat-race coupler implemented in a microstrip technology is considered. The coupler circuit features a simple topology with a densely arranged layout that exploits a combination of high- and low-impedance transmission line sections. All relevant dimensions of the structure are simultaneously optimized in order to explicitly reduce the coupler size while maintaining equal power split at the operating frequency of 1 GHz and sufficient bandwidth for return loss and isolation characteristics. Acceptable levels of electrical performance are ensured by using a penalty function approach. Two designs with footprints of 350 mm^2 and 360 mm^2 have been designed and experimentally validated. The latter structure is characterized by 27% bandwidth. For the sake of computational efficiency, surrogate-based optimization principles are utilized. In particular, we employ an iterative construction and re-optimization of the surrogate model involving a suitably corrected low-fidelity representation of the coupler structure. This permits rapid optimization at the cost corresponding to a handful of evaluations of the high-fidelity coupler model.

Keywords: Microwave Couplers, Rat-Race Couplers, Coupler Optimization, Surrogate-Based Optimization, Computer-Aided Design, Compact Coupler, Compact Microstrip Resonant Cells.

1   Introduction

Design of compact microwave structures is an important yet challenging task because size reduction stays in conflict with other objectives concerning electrical performance of the circuit [1]-[4]. In case of many classes of structures such as couplers, several criteria have to be handled at the same time (e.g., power split error, achieving a specific operating frequency, minimization of return loss, etc.) [3]-[5]. Another problem is that

due to considerable electromagnetic (EM) cross-couplings present in highly compressed layouts of miniaturized structures [6]-[10], equivalent network models (typically used as design tools) are highly inaccurate [3], [9]. Reliable evaluation of the circuit performance can only be realized by means of full-wave EM analysis, which is computationally expensive [4], [5]. Consequently, design through numerical optimization—although highly desirable—is very difficult. On one hand, manual design approaches (e.g., parameter sweeps) do not allow for simultaneous control of the structure size and electrical responses [2]. On the other hand, conventional optimization algorithms exhibit high computational cost due to a large number of EM simulations necessary for convergence [11].

In this paper, an explicit size reduction of a compact microstrip coupler is considered. Small size of the circuit is partially obtained through tightly arranged layout based on a combination of high- and low-impedance transmission lines which allows efficient utilization of the available space. Furthermore, geometrical dimensions of the circuit are obtained through numerical optimization oriented towards explicit size reduction. Surrogate-based methods [12]-[16] are used to speed up the design process. More specifically, we utilize variable-fidelity models and space mapping technology [12], [16] to construct the surrogate model, further utilized as a prediction tool that iteratively guides the optimization process towards the optimum design. Simultaneous control of the coupler size and its electrical performance parameters is achieved by means of a penalty function approach. The optimized coupler structure exhibits small size of 350 mm^2 and acceptable performance in terms of power split as well as bandwidth. Only slight loosening of the size constraint (to 360 mm^2) leads to considerable bandwidth improvement to 270 MHz. Both designs have been fabricated and experimentally validated.

2 Design Optimization Procedure

In this section, an optimization procedure utilized to obtain a minimum-size coupler design is discussed. Specifically, we formulate design optimization problem and describe utilized design optimization algorithm. The numerical results and comparison of the structure with the state-of-the-art couplers are given in Section 4, whereas its experimental validation is given in Section 5.

2.1 Problem Formulation

The primary objective is to minimize the coupler size $A(\boldsymbol{x})$. On the other hand, the design process is also supposed to ensure sufficient electrical performance of the structure. We consider the following requirements [4]:

- $d_S = |S_{21.f}(\boldsymbol{x}) - S_{31.f}(\boldsymbol{x})| \leq \varepsilon$ at the operating frequency (here, we set $\varepsilon = 0.2$ dB);
- $S_{\max} = \max(\min\{S_{11.f}(\boldsymbol{x}), S_{41.f}(\boldsymbol{x})\}) \leq S_m$ (we assume $S_m = -25$ dB);
- $f_{S11.f}(\boldsymbol{x})$ and $f_{S41.f}(\boldsymbol{x})$, i.e., the frequencies realizing minimum of $S_{11.f}(\boldsymbol{x})$ and $S_{41.f}(\boldsymbol{x})$, respectively, are as close to the operating frequency f_0 as possible.

The design optimization problem is formulated as [11]

$$x^* = \arg\min_x U\left(\mathbf{R}_f(\mathbf{x})\right) \tag{1}$$

where $\mathbf{R}_f$ is a high-fidelity EM simulation model of the structure as described above, whereas $\mathbf{x}^*$ is the optimum design to be found. In order to take into account all of these goals the objective function is defined as follows

$$U(\mathbf{x}) = A(\mathbf{x}) + \beta_1\left(\max\left\{(d_S - \varepsilon)/\varepsilon, 0\right\}\right)^2 +$$
$$+ \beta_2\left(\max\left\{(S_{\max} - S_m)/\,|\,S_m\,|, 0\right\}\right)^2 + \tag{2}$$
$$+ \beta_{f1}\left|(f_{S_{11.f}}(\mathbf{x}) - f_0)/f_0\right|^2 + \beta_{f2}\left|(f_{S_{41.f}}(\mathbf{x}) - f_0)/f_0\right|^2$$

This formulation is supposed to ensure (with certain tolerance) equal power split (controlled by d_S) as well as sufficient return loss and isolation (controlled by $S_{\max}$) at the operating frequency. The coefficients β_1, β_2, β_{f1}, and β_{f2} are chosen so that the corresponding penalty functions take noticeable values (when compared to $A(\mathbf{x})$) for relative violations larger than a few percent.

2.2 Surrogate-Based Coupler Optimization

For the sake of computational efficiency the design process is executed using surrogate-based optimization methods with variable-fidelity EM models [11]. More specifically, direct solving of (1) is replaced by an iterative procedure

$$\mathbf{x}^{(i+1)} = \arg\min_x U\left(\mathbf{R}_s^{(i)}(\mathbf{x})\right) \tag{3}$$

that yields a series $\mathbf{x}^{(i)}$, $i = 0, 1, \ldots$, of approximations to $\mathbf{x}^*$, with $\mathbf{R}_s^{(i)}$ being a surrogate model at iteration i. Here, the surrogate is constructed by suitable correction of the low-fidelity model $\mathbf{R}_c$ as mentioned in the previous section. The model correction is realized using space mapping [11]. In this work, we utilized frequency scaling and additive response correction. Frequency scaling is realized by evaluating the low-fidelity model at a set of frequencies that are transformed with respect to the original frequency sweep $F = [f_1 \ldots f_m]$ (at this the high-fidelity model is simulated) as follows $F' = [\alpha_0 + \alpha_1 f_1 \ldots \alpha_0 + \alpha_1 f_m]$. Here, α_0 and α_1 are coefficients found (using nonlinear regression) so as to minimize the misalignment between the scaled low- and high-fidelity models, i.e., $\|\mathbf{R}_c'(\mathbf{x}^{(i)}) - \mathbf{R}_f(\mathbf{x}^{(i)})\|$. The additive response correction is applied on the top of frequency scaling so that we have $\mathbf{R}_s^{(i)}(\mathbf{x}) = \mathbf{R}_c'(\mathbf{x}) + [\mathbf{R}_f(\mathbf{x}^{(i)}) - \mathbf{R}_c'(\mathbf{x}^{(i)})]$. The correction term $[\mathbf{R}_f(\mathbf{x}^{(i)}) - \mathbf{R}_c'(\mathbf{x}^{(i)})]$ ensured zero-order consistency between the surrogate and the high-fidelity model at the current iteration point $\mathbf{x}^{(i)}$.

3 Numerical Results and Comparisons

Consider a rectangular-shaped, equal-split rat-race coupler (RRC) is shown in Fig. 1. It consists of two horizontal and four vertical compact microstrip resonant cells (CMRSs) [9]. The cells contain folded high-impedance lines interconnected with low-impedance stubs, which allows obtaining complementary geometry that ensures tight filling of the structure interior and thus good utilization of available space. This is critical for achieving considerable miniaturization rate. On the other hand, the circuit contains a relatively

small number of geometry parameters which facilitates its further design optimization process.

The coupler is implemented on a Taconic RF-35 substrate (ε_r = 3.5, tanδ = 0.0018, h = 0.762 mm). The geometry parameters are $\boldsymbol{x}$ = [w_1 w_2 w_3 d_1 d_2 l_1]^T, whereas w_0 = 1.7 is fixed (all dimensions in mm). The design procedure involves fine and coarsely discretized EM models of the RRC, both evaluated in CST Microwave Studio [17]. The high-fidelity model $\boldsymbol{R}_f$ contains ~700,000 mesh cells and its simulation time on a dual Intel E5540 machine is 52 minutes. The low-fidelity model $\boldsymbol{R}_c$ has ~150,000 cells (simulation time 4 minutes).

The considered structure has been designed using the above outlined methodology. The final design (here, denoted as design A) is $\boldsymbol{x}_A^*$ = [4.979 0.179 1.933 0.197 0.164 2.568]^T. The footprint of the optimized circuit is only 350 mm^2. Obtained frequency characteristics of the structure are shown in Fig. 2. In the next step, for the sake of improved coupler performance, the area constraint has been increased to 360 mm^2 and the circuit has been re-optimized. The parameter vector of an alternative design (denoted as coupler B) is $\boldsymbol{x}_B^*$ = [4.395 0.244 2.263 0.199 0.233 2.499]^T. The frequency responses of the structure are shown in Fig. 3.
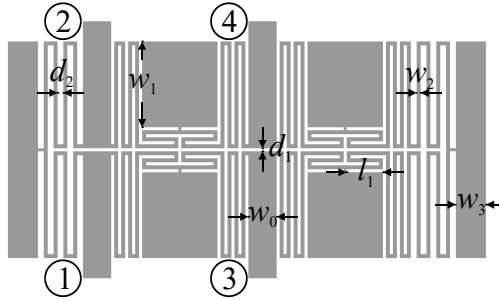

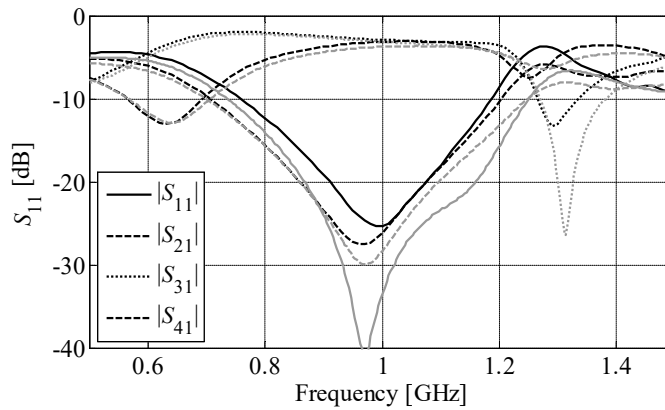**Fig. 1.** Geometry of the considered compact microstrip rat-race coupler.

Fig. 2. Simulated (black) and measured (gray) characteristics of the design A; layout area 350 mm^2.

Utilization of variable-fidelity simulation models in combination with space mapping technology permits low cost of the optimization process, equivalent to less than twenty evaluations of the high-fidelity coupler model for both designs (A and B).

Both coupler designs have been compared with other state-of-the-art structures [9], [19]-[22] in terms of the bandwidth and miniaturization rate (expressed in terms of the guided wavelength λ_g defined for the operating frequency and the given substrate parameters). The results collected in Table 1 indicate that both coupler realizations provide competitive miniaturization while ensuring broader bandwidth than other structures with similar sizes.

4   Experimental validation

Both coupler designs have been fabricated and measured. Photograph of manufactured coupler A is shown in Fig. 4, whereas the comparison of its simulated and measured frequency characteristics is provided in Fig. 2. The obtained results indicate that the operational bandwidth of the structure defined as the frequency range for which both the reflection and isolation are below the level of –20 dB is 170 MHz for simulation and 220 MHz for measurement. Moreover, the simulated and measured power split error at $f_0 = 1$ GHz is 0.25 dB and 0.59 dB, respectively. The phase difference between ports 2 and 3 (see Fig. 1) is shown in Fig. 5a. Its simulated and measured value is about 8.7° which can be considered acceptable. The deviation from 0° is due to lack of phase control mechanism during the optimization process.

Comparison of the simulated and measured scattering parameters of coupler B is shown in Fig. 3. It should be noted that the slightly increased size has resulted in increase of –20 dB bandwidth to 270 MHz and 290 MHz for simulation and measurement, respectively.

Table 2. A comparison of competitive compact coupler designs

Coupler	Bandwidth %	Dimensions mm × mm	Effective λ_g	Miniaturization %[*]
Design [19]	39.0	32.4 × 51.9	0.20 × 0.32	53.6
Design [20]	17.2	38.5 × 38.5	0.19 × 0.19	73.8
Design [21]	16.8	22.4 × 22.4	0.14 × 0.14	85.8
Design [9]	20.2	22.8 × 17.0	0.13 × 0.09	91.5
Design [22]	15.1	6.67 × 52.5	0.04 × 0.28	92.2
Design A	17.0	12.1 × 29.0	0.07 × 0.16	92.2
Design B	27.0	11.2 × 32.2	0.06 × 0.18	92.1

[*] w.r.t. conventional RRC (effective λ_g: 0.26 × 0.53, size: 4536 mm^2) [9].

The simulated power split error and phase difference (cf. Fig. 5b) at f_0 are 0.2 dB and 4.7°, whereas measured values are 0.7 dB and 5.6°, respectively. One should emphasize that the considered RRC structure is sensitive for fabrication inaccuracies which is the reason of noticeable discrepancies between the simulated and the measured responses [9]. The key electrical properties of both coupler designs have been gathered in Table 2.
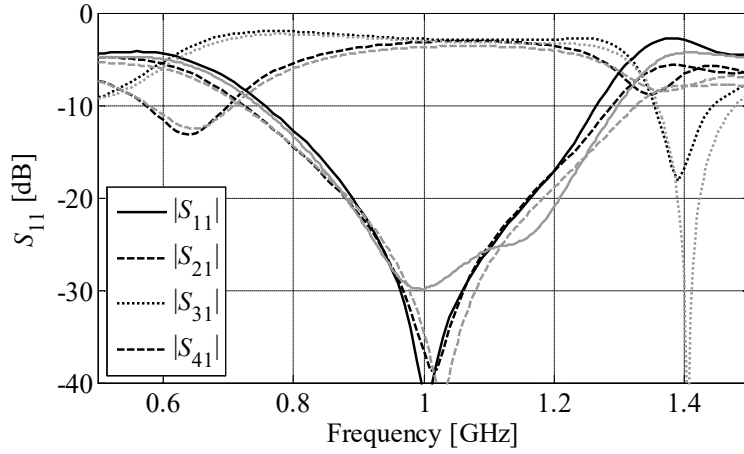

Fig. 3. Simulated (black) and measured (gray) responses of the design B; layout area constraint $A(x) \leq 360$ mm^2.

Table 2. Key features of couplers A and B: simulation vs measurements

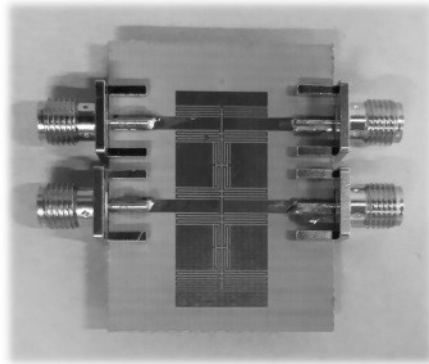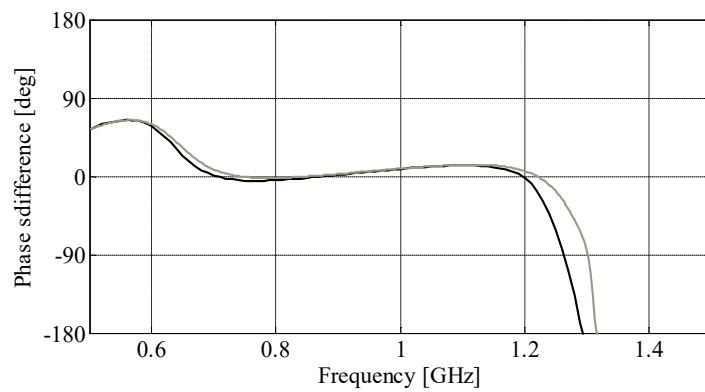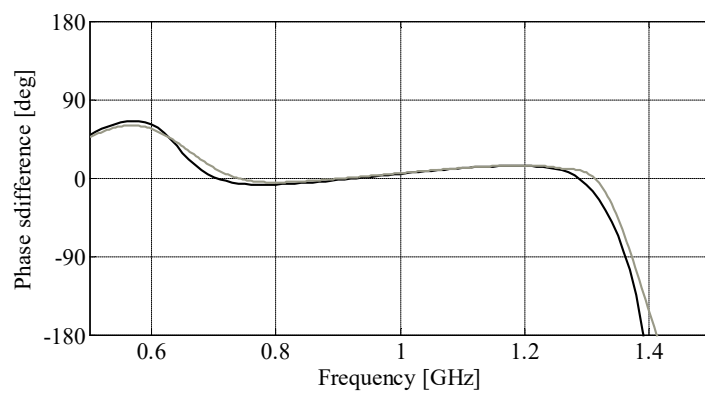| $f_0 = 1$ GHz	Coupler A		Coupler B	
	simulated	measured	simulated	measured
$|S_{11}|$	–25.3 dB	–33.4 dB	–41.7 dB	–29.9 dB
$|S_{21}|$	–3.17 dB	–3.73 dB	–3.05 dB	–3.70 dB
$|S_{31}|$	–2.92 dB	–3.14 dB	–2.85 dB	–3.02 dB
$|S_{41}|$	–26.2 dB	–28.3 dB	–36.8 dB	–34.7 dB
Bandwidth	170 MHz	220 MHz	270 MHz	290 MHz
$\angle S_{21} - \angle S_{31}$	8.48°	8.92°	4.73°	5.57°

Fig. 4. Photograph of the fabricated coupler prototype (design A).

Fig. 5. Comparison of simulated and measured phase difference of the proposed compact couplers: (a) design A; and (b) design B.

5   Conclusions

In this work, an explicit size reduction of a compact coupler structure implemented in microstrip technology has been considered. Due to highly-packed geometry of the considered structure, as well as appropriate handling of all design requirements, a very small size of 350 mm^2 can be achieved (with 17-percent bandwidth). At the same time, optimization for electrical performance (with the maximum size constrained to 360 mm^2) leads to bandwidth increase to 27 percent with respect to the operating frequency of 1 GHz. Utilization of variable-fidelity electromagnetic simulations as well as space mapping technology allowed us to maintain low cost of the optimization process. Here it is equivalent to less than twenty evaluations of the high-fidelity model of the coupler under design. The structure has been favorably compared with benchmark compact couplers. Simulation results are supported with measurement data. Future work will focus on utilization of the method for design of compact multi-band coupler structures.

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